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Effects of various nonhomogeneous parameters on propagation of surface waves in viscoelastic media of higher order under gravity

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ABSTRACT

The present study reveals the effects of non-homogeneity, viscous, gravity, magnetic and thermal fields in the wave velocity equations corresponding to Stoneley, Rayleigh and Love waves respectively. The theory of generalized surface waves has firstly been developed and then it has been employed to study the surface waves. The wave velocity equations have been obtained for Stoneley waves, Rayleigh waves and Love waves, and are in well agreement with the corresponding classical result in the absence of viscosity, temperature, gravity, magnetism as well as non-homogeneity of the material medium.

Keywords: Inhomogeneous media, Variable density, Surface waves, Viscosity, Gravity.

INTRODUCTION

When seismic waves propagate underground, they are influenced not only by the anisotropy of the media, but also by intrinsic viscosity of media given by Carcione [1]. Therefore, in order to accurately describe the underground propagation of the seismic waves and then more precisely guide seismic data acquisition, processing and interpretation, media models should be chosen that can simultaneously imitate anisotropic characteristics of formation and viscoelastic characteristics for numerical simulation and analysis of wave fields As a result, the theory of surface waves has been developed by Stoneley [2], Bullen [3], Ewing et. al. [4], Hunters and Jeffreys [5].

The effect of gravity on wave propagation in an elastic solid medium was first considered by Bromwich [6], treating the force of gravity as a type of body force. Love [7] extended the work of Bromwich investigated the influence of gravity on superfacial waves and showed that the Rayleigh wave velocity is affected by the gravity field. Sezawa [8] studied the dispersion of elastic waves propagated on curved surfaces.

The transmission of elastic waves through a stratified solid medium was studied by Thomson [9]. Haskell [10] studied the dispersion of surface waves in multilayered media. A source on elastic waves is the monograph of Ewing, Jardtezky and Press [11]. Biot [12] studied the influence of gravity on Rayleigh waves, assuming the force of gravity to create a type of initial stress of hydrostatic nature and the medium to be incompressible. Taking into account, the effect of initial stresses and using Biot's theory of incremental deformations, Dey modified the work of Jones [13]. De and Sengupta [14] studied many problems of elastic waves and vibrations under the influence of gravity field. Sengupta and Acharya [15] studied the influence of gravity on the propagation of waves in a thermoelastic layer. Brunelle [16] studied the surface wave propagation under initial tension of compression. Wave propagation in a thin two-layered laminated medium with stress couples under initial stresses was studied by Roy [17]. Datta [18] studied the effect of gravity on Rayleigh wave propagation in a homogeneous, isotropic elastic solid medium. Goda [19] studied the effect of inhomongeneity and anisotropy on Stoneley waves. Recently Abd-Alla and Ahmed [20] studied the Rayleigh waves in an orthotropic thermoelastic medium under gravity field and initial stress. Recently, Kakar et al. [21] investigated surface waves in non homogeneous, general magneto-thermo, viscoelastic media of higher order.

In this work, the problem of nth order viscoelastic surface waves under gravity involving time rate of strain, the medium being isotropic and non-homogeneous has been studied under the influence of gravity, magnetic field and temperature. Biot's theory of incremental deformations has been used to obtain the wave velocity equation for Stoneley, Rayleigh and Love waves. Further these equations are in complete agreement with the corresponding classical results in the absence of viscosity, gravity, magnetic and thermal field, non-homogeneity of the material medium.

2 FORMULATION OF THE PROBLEM

Let M_1 and M_2 be two non-homogeneous, viscoelastic, isotropic, semi-finite media (Fig.1). They are perfectly welded in-contact to prevent any relative motion or sliding before and after the disturbances and that the continuity of displacement, stress etc. hold good across the common boundary surface. Further the mechanical properties of M_1 are different from those of M_2. These media extend to an infinite great distance from the origin and are separated by a plane horizontal boundary and M_2 is to be taken above M_1.

Let Oxyz be a set of orthogonal Cartesian co-ordinates and let O be the any point on the plane boundary and Oz points vertically downward to the medium M_1. We consider the possibility of a type of wave traveling in the direction Ox, in such a manner that the disturbance is largely confined to the neighborhood of the boundary which implies that wave is a surface wave.

It is assume that at any instant, all particles in any line parallel to Oy having equal displacement and all partial derivatives with respect to y are zero. Further let us assume that u, v, w is the components of displacements at any point (x, y, z) at any time t.

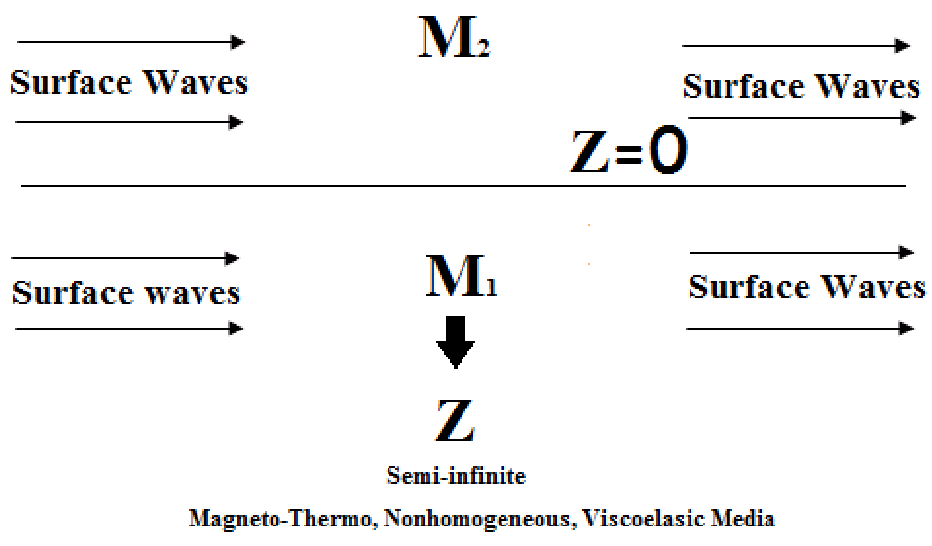

Fig.1 Geometry of the problem

It is also assume that gravitational field produces a hydrostatic initial stress is produced by a slow process of creep where the shearing stresses tend to become small or vanish after a long period of time. The equilibrium conditions of initial stress are

$$\frac{\partial \tau}{\partial x}=0, \frac{\partial \tau}{\partial z}+\rho g=0 \tag{1}$$

The dynamical equations of motion for three-dimensional non-homogeneous, isotropic, viscoelastic solid medium in Cartesian co-ordinates with Eq. (1) are

$$\frac{\partial \tau_{11}}{\partial x}+\frac{\partial \tau_{12}}{\partial y}+\frac{\partial \tau_{13}}{\partial z}+\rho g \frac{\partial w}{\partial x}=\rho \frac{\partial^2 u}{\partial t^2}, \tag{2a}$$

$$\frac{\partial \tau_{12}}{\partial x}+\frac{\partial \tau_{22}}{\partial y}+\frac{\partial \tau_{23}}{\partial z}+\rho g \frac{\partial w}{\partial y}=\rho \frac{\partial^2 v}{\partial t^2}, \tag{2b}$$

$$\frac{\partial \tau_{13}}{\partial x}+\frac{\partial \tau_{23}}{\partial y}+\frac{\partial \tau_{33}}{\partial z}-\rho g\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)=\rho \frac{\partial^2 w}{\partial t^2}. \tag{2c}$$

Where ρ be the density of the material medium and $\tau_{ij}=\tau_{ji}\ \forall$ i, j are the stress components. Let us consider that the medium is a perfect electric conductor, we take the linearized Maxwell equations governing the electromagnetic field, taking into account absence of the displacement current (in system-international unit) in the form

$$\vec{\nabla}\cdot\vec{E}=0, \vec{\nabla}\cdot\vec{B}=0, \vec{\nabla}\times\vec{E}=-\frac{\partial \vec{B}}{\partial t}, \vec{\nabla}\times\vec{B}=\mu_e\varepsilon_e\frac{\partial \vec{E}}{\partial t}. \tag{3}$$

Where, $\vec{E}$, $\vec{B}$, μ_e and ε_e are electric field, magnetic field induction, permeability and permittivity of the medium. The value of magnetic field intensity is

$$\vec{H}(0,0,H)=\vec{H}_0+\vec{H}_i \tag{4}$$

We consider an orthotropic elastic solid under constant primary magnetic field $\vec{H}$ acting on y-axis and $\vec{H}_i$ is the perturbation in the magnetic field intensity.

It is assumed that prior to the existence of any disturbance both the media are everywhere at the constant absolute temperature T_0.

The stress-strain relations for general isotropic, thermo, viscoelastic medium, according to Voigt are [22]

$$\tau_{ij}=2D_\mu e_{ij}+(D_\lambda\Delta-D_\beta T+H_0^2 DD_{m_e})\delta_{ij} \tag{5}$$

where,

$\Delta=\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}$ and D_λ, D_μ, D_β are elastic constants

Introducing Eq. (5) in Eq. (2a), Eq. (2b), Eq. (2c), we get

$$D_\lambda\frac{\partial \Delta}{\partial x}+\Delta\frac{\partial D_\lambda}{\partial x}+2D_\mu\frac{\partial^2 u}{\partial x^2}+2\frac{\partial u}{\partial x}\frac{\partial D_\mu}{\partial x}-D_\beta\frac{\partial T}{\partial x}+D_\mu\frac{\partial}{\partial z}\left[\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right]+$$

$$\left[\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right]\frac{\partial D_\mu}{\partial z}\ H_0^2 D_{m_e}\frac{\partial D}{\partial x}+H_0^2 D\frac{\partial D_{m_e}}{\partial x}+\rho g\frac{\partial w}{\partial x}=\rho\frac{\partial^2 u}{\partial t^2}, \tag{6a}$$

$$D_\mu \nabla^2 v + \frac{\partial v}{\partial x}\frac{\partial D_\mu}{\partial x} + \frac{\partial v}{\partial z}\frac{\partial D_\mu}{\partial z} = \rho \frac{\partial^2 v}{\partial t^2}, \tag{6b}$$

$$D_\mu \frac{\partial}{\partial x}\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right)+\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right)\frac{\partial D_\mu}{\partial x}+2D_\mu\frac{\partial^2 w}{\partial z^2}+2\frac{\partial w}{\partial z}\frac{\partial D_\mu}{\partial z}+D_\lambda\frac{\partial \Delta}{\partial z}$$
$$+\Delta\frac{\partial D_\lambda}{\partial z}-D_\beta\frac{\partial T}{\partial z}-T\frac{\partial D_\beta}{\partial z}\,H_0^2 D_{m_e}\frac{\partial D}{\partial z}+H_0^2 D\frac{\partial D_{m_e}}{\partial z}-\rho g\frac{\partial u}{\partial x}=\rho\frac{\partial^2 w}{\partial t^2}. \tag{6c}$$

We assume that the non-homogeneities for the media M_1 and M_2 are given by

$$D_\lambda = \sum_{K=0}^{n} \lambda_K e^{mz}\frac{\partial^K}{\partial t^K},\ D_\mu = \sum_{K=0}^{n} \mu_K e^{mz}\frac{\partial^K}{\partial t^K},\ D_\beta = \sum_{K=0}^{n} \beta_K e^{mz}\frac{\partial^K}{\partial t^K},\ D_{m_e} = \sum_{K=0}^{n}(\mu_e)_K e^{mz}\frac{\partial^K}{\partial t^K},$$

$\rho = \rho_0\, e^{mz}$

and

$$D'_\lambda = \sum_{K=0}^{n} \lambda'_K e^{lz}\frac{\partial^K}{\partial t^K},\quad D_\mu = \sum_{K=0}^{n} \mu'_K e^{lz}\frac{\partial^K}{\partial t^K},\quad D_\beta = \sum_{K=0}^{n} \beta_K e^{lz}\frac{\partial^K}{\partial t^K},\quad D'_{\mu_e} = \sum_{K=0}^{n}(\mu'_e)_K e^{mz}\frac{\partial^K}{\partial t^K},$$

$$\rho' = \rho'_0\, e^{lz} \tag{7}$$

where λ_0, M_0, λ'_0, μ'_0 are elastic constants, whereas β_0, β'_0 are thermal parameters are ρ_0, ρ'_0, m, n are constants. λ_K, μ_K (K = 0,1,2, ... n) are the parameters associated with Kth order viscoelasticity and β_K and $(\mu_e)_K$ (K = 1, 2, ..., n) are the thermal and magnetic parameters associated with Kth order. T is the absolute temperature over the initial temperature T_0.

Due to temperature rise of the material medium, it has been observed that all the parameters representing elastic property, the effect of viscosity and thermal field depends on the temperature and ultimately depends on time t. In a thermo viscoelastic solid, the thermal parameters β_K (K = 0, 1, ... n) are given by
$\beta_K = (3\lambda_K + 2\mu_K)\,\alpha_t$, where α_t be the coefficient of linear expansion of solid.

$$\left(G_\lambda+G_\mu+H_0^2G_{m_e}\right)\frac{\partial\Delta}{\partial x}+G_\mu\nabla^2 u+mG_\mu\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right)-G_\beta\frac{\partial T}{\partial x}+\rho g\frac{\partial w}{\partial x}=\rho_0\frac{\partial^2 u}{\partial t^2} \tag{8a}$$

$$G_\mu \nabla^2 v + mG_\mu \frac{\partial v}{\partial z} = \rho_0 \frac{\partial^2 v}{\partial t^2}, \tag{8b}$$

$$\left(G_\lambda+G_\mu+H_0^2G_{m_e}\right)\frac{\partial\Delta}{\partial z}+\Delta G_\lambda m+2G_\mu m+2G_\mu m\frac{\partial w}{\partial z}-mG_\beta TmH_0^2\,DG_{m_e}-\rho g\frac{\partial u}{\partial x}=\rho_0\frac{\partial^2 w}{\partial t^2} \tag{8c}$$

where,

$$G_\lambda = \sum_{K=0}^{n}\lambda_K\frac{\partial^K}{\partial t^K},\ G_\mu = \sum_{K=0}^{n}\mu_K\frac{\partial^K}{\partial t^K},\ G_\beta = \sum_{K=0}^{n}\beta_K\frac{\partial^K}{\partial t^K},\ G_{\mu_e} = \sum_{K=0}^{n}(\mu_e)_K\frac{\partial^K}{\partial t^K},\ \nabla^2 = \frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}. \tag{9}$$

To investigate the surface wave propagation along the direction of Ox, we introduce displacement potential ϕ (x, z, t) and ψ (x, z, t) which are related to the displacement components as follows:

$$u=\frac{\partial\phi}{\partial x}-\frac{\partial\psi}{\partial z},\ w=\frac{\partial\phi}{\partial z}+\frac{\partial\psi}{\partial x}. \tag{10}$$

Substituting Eq. (10) in Eqs (8a), (8b) and (8c), we get

$$G_R\,\nabla^2\phi+mG_S\left(2\frac{\partial\phi}{\partial z}+\frac{\partial\psi}{\partial x}\right)-G_L T+g\frac{\partial\psi}{\partial x}=\frac{\partial^2\phi}{\partial t^2}, \tag{11a}$$

$$G_S\,\nabla^2 v+mG_S\frac{\partial v}{\partial z}=\frac{\partial^2 v}{\partial t^2}, \tag{11b}$$

$$G_S\,\nabla^2\psi+mG_P\frac{\partial\phi}{\partial x}+2m\,G_S\frac{\partial\psi}{\partial z}-g\frac{\partial\phi}{\partial x}=\frac{\partial^2\psi}{\partial t^2}, \tag{11c}$$

Where,

$$U_{KR}^2=\frac{\lambda_K+2\mu_K+H_0^2(\mathrm{m}_e)_K}{\rho_0},\ U_{KS}^2=\frac{\mu_K}{\rho_0},\ U_{KP}^2=\frac{\lambda_K+H_0^2(\mathrm{m}_e)_K}{\rho_0},\ U_{KL}^2=\frac{\beta_K}{\rho_0}$$

and

$$G_R=\sum_{K=0}^{n}U_{KR}^2\frac{\partial^K}{\partial t^K},\ G_S=\sum_{K=0}^{n}U_{KS}^2\frac{\partial^K}{\partial t^K},\ G_P=\sum_{K=0}^{n}U_{KP}^2\frac{\partial^K}{\partial t^K},\ G_L=\sum_{K=0}^{n}U_{KL}^2\frac{\partial^K}{\partial t^K}. \tag{12}$$

To determine T, Fourier's law of heat conduction

$$p\,\nabla^2 T=C_v\frac{\partial T}{\partial t}+T_0 G_L\frac{\partial}{\partial t}\left(\nabla^2\phi\right), \tag{13}$$

where K be the thermal conductivity and obeys the law as given by $K = K_0\, e^{mz}$,

$p=\dfrac{K_0}{\rho_0}$ and C_V be the specific heat of the body at constant volume.

Further, similar relations in medium M_2 can be found out by replacing λ_K, μ_K, β_K, ρ_0 by λ'_K, μ'_K, β'_K, ρ'_0 and so on.

3 SOLUTION OF THE PROBLEM

Now our main objective to solve Eq. (11a), Eq. (11b), Eq. (11c) and Eq. (13).

For this, we seek the solutions in the following forms.

$$(\phi,\psi,T,v)=[f(z),\ j(z),\ T_1(z),\ h(z)]\,e^{i\alpha(x-ct)} \tag{14}$$

Using Eq. (12) in Eq. (9a), Eq. (9b), Eq. (9c) and Eq. (11), we get a set of differential equations for the medium M_1 as follows:

$$\frac{d^2 f}{dz^2} + 2mf_1^{\,2}\frac{df}{dz} + h_1^2 f + (i\alpha m f_1^{\,2} + i\alpha g)\, j - g_1^{\,2} T_1 = 0,$$
$$\frac{d^2 h}{dz^2} + m\frac{dh}{dz} + K_1^{\,2} h = 0,$$
$$\frac{d^2 g}{dz^2} + 2m\frac{dg}{dz} + K_1^{\,2} j + (i\alpha m l_1^{\,2} - i\alpha g)\, f = 0,$$
$$\frac{d^2 T_1}{dz^2} + AT_1 + B\left(\frac{d^2 f}{dz^2} - \alpha^2 f\right) = 0, \qquad (15)$$

where,

$$f_1{}^2 = \frac{\sum_{K=0}^{n} U_{KS}^2 (-i\alpha c)^K}{\sum_{K=0}^{n} U_{KR}^2 (-i\alpha c)^K}, \quad h_1{}^2 = \frac{\alpha^2 c^2}{\sum_{K=0}^{n} U_{KR}^2 (-i\alpha c)^K} - \alpha^2, \quad K_1{}^2 = \frac{\alpha^2 c^2}{\sum_{K=0}^{n} U_{KS}^2 (-i\alpha c)^K} - \alpha^2,$$

$$l_1{}^2 = \frac{\sum_{K=0}^{n} U_{KP}^2 (-i\alpha c)^K}{\sum_{K=0}^{n} U_{KS}^2 (-i\alpha c)^K}, \quad g_1{}^2 = \frac{\sum_{K=0}^{n} U_{KL}^2 (-i\alpha c)^K}{\sum_{K=0}^{n} U_{KR}^2 (-i\alpha c)^K}, \quad A = \frac{C_v\, i\alpha c}{p} - \alpha^2, \quad B = \frac{i\alpha c T_0}{p} G_L \qquad (16)$$

and those for the medium M_2 are given by

$$\frac{d^2 f}{dz^2} + 2lf_1^{\prime 2}\frac{df}{dz} + h_1^{\prime 2} f + (i\alpha l f_1^{\prime 2} + i\alpha g)\, j - g_1^{\prime 2} T_1 = 0,$$
$$\frac{d^2 h}{dz^2} + l\frac{dh}{dz} + K_1^{\prime 2} h = 0,$$
$$\frac{d^2 g}{dz^2} + 2l\frac{dg}{dz} + K_1^{\prime 2} j + (i\alpha l . l_1^{\prime 2} - i\alpha g)\, f = 0,$$
$$\frac{d^2 T_1}{dz^2} + A' T_1 + B'\left(\frac{d^2 f}{dz^2} - \alpha^2 f\right) = 0, \qquad (17)$$

where,

$$f_1{}'^2 = \frac{\sum_{K=0}^{n} U_{KS}^{\prime 2} (-i\alpha c)^K}{\sum_{K=0}^{n} U_{KR}^{\prime 2} (-i\alpha c)^K}, \quad h_1{}'^2 = \frac{\alpha^2 c^2}{\sum_{K=0}^{n} U_{KR}^{\prime 2} (-i\alpha c)^K} - \alpha^2, \quad K_1{}'^2 = \frac{\alpha^2 c^2}{\sum_{K=0}^{n} U_{KS}^{\prime 2} (-i\alpha c)^K} - \alpha^2,$$

$$l_1{}'^2 = \frac{\sum_{K=0}^{n} U_{KP}^{\prime 2} (-i\alpha c)^K}{\sum_{K=0}^{n} U_{KS}^{\prime 2} (-i\alpha c)^K}, \quad g_1{}'^2 = \frac{\sum_{K=0}^{n} U_{KL}^{\prime 2} (-i\alpha c)^K}{\sum_{K=0}^{n} U_{KR}^{\prime 2} (-i\alpha c)^K}, \quad B' = \frac{i\alpha c T_0}{p'} G'_L \qquad (18)$$

Eq. (15) and Eq. (17) must have exponential solutions in order that f, j, T_1, h will describe surface waves, and they must become varnishing small as z $\rightarrow \infty$.

Hence for the medium M_1

$$\phi(x,z,t) = \left\{A_1 e^{-\lambda_1 z} + B_1 e^{-\lambda_2 z} + C_1 e^{-\lambda_3 z}\right\} e^{i\alpha(x-ct)}$$
$$\psi(x,z,t) = \left\{A_2 e^{-\lambda_1 z} + B_2 e^{-\lambda_2 z} + C_2 e^{-\lambda_3 z}\right\} e^{i\alpha(x-ct)}$$
$$T(x,z,t) = \left\{A_3 e^{-\lambda_1 z} + B_3 e^{-\lambda_2 z} + C_3 e^{-\lambda_3 z}\right\} e^{i\alpha(x-ct)}$$
$$v(x,z,t) = C e^{-\lambda_4 z + i\alpha(x-ct)} \tag{19a}$$

and similarly for the medium M_2 are given by

$$\phi(x,z,t) = \left\{A'_1 e^{-\lambda'_1 z} + B'_1 e^{-\lambda'_2 z} + C'_1 e^{-\lambda'_3 z}\right\} e^{i\alpha(x-ct)}$$
$$\psi(x,z,t) = \left\{A'_2 e^{-\lambda'_1 z} + B'_2 e^{-\lambda'_2 z} + C'_2 e^{-\lambda'_3 z}\right\} e^{i\alpha(x-ct)}$$
$$T(x,z,t) = \left\{A'_3 e^{-\lambda'_1 z} + B'_3 e^{-\lambda'_2 z} + C'_3 e^{-\lambda'_3 z}\right\} e^{i\alpha(x-ct)}$$
$$v(x,z,t) = C' e^{-\lambda'_4 z + i\alpha(x-ct)} \tag{19b}$$

Where λ_j and λ'_j (j = 1, 2, 3) are the real roots of the eqns.

$$\lambda^6 + \xi_1 \lambda^5 + \xi_2 \lambda^4 + \xi_3 \lambda^3 + \xi_4 \lambda^2 + \xi_5 \lambda + \xi_6 = 0, \tag{20}$$

where,

$$\xi_1 = 2m\{1 + f_1^2\},\ \xi_2 = K_1^2 + A + 4m^2 + h_1^2 + Bg_1^2,\ \xi_3 = 2mA + 2f_1^2 m(K_1^2 + A) + 2mh_1^2 + 2mBg_1^2$$
$$\xi_4 = AK_1^2 + 4m^2A f_1^2 + (K_1^2 + A) h_1^2 + \alpha^2 m^2 l_1^2 f_1^2 + BK_1^2 g_1^2 - \alpha^2 Bg_1^2, \tag{21}$$

$$\xi_5 = 2mAK_1^2 f_1^2 + 2mAh_1^2 - 2m\alpha^2 Bg_1^2,\ \xi_6 = AK_1^2 h_1^2 + A\alpha^2 m^2 l_l^2 f_1^2 - \alpha^2 B K_1^2 g_1^2.$$

$$\lambda'^6 + \xi'_1 \lambda'^5 + \xi'_2 \lambda'^4 + \xi'_3 \lambda'^3 + \xi'_4 \lambda'^2 + \xi'_5 \lambda' + \xi'_6 = 0 \tag{22}$$

where,

$\xi'_1 = 2l\{1 + f_1'^2\}$, $\xi'_2 = K_1'^2 + A' + 4l^2 + h_1'^2 + B'g_1'^2$, $\xi'_3 = 2lA' + 2lf_1'^2 (K_1'^2 + A) + 2lh_1'^2 + 2l B'g_1'^2$,
$\xi'_4 = A'K_1'^2 + 4l^2A' f_1'^2 + (K_1'^2 + A') h_1'^2 + \alpha^2 l^2 l_1'^2 f_1'^2 + B' K_1'^2 g_1'^2 - \alpha^2 B' g_1'^2$, $\xi'_5 = 2lA'K_1'^2 f_1'^2 + 2lA'h_1'^2 - 2l \alpha^2 B'g_1'^2$, $\xi'_6 = A'K_1'^2 h_1'^2 + A'\alpha^2 l^2 l_l'^2 f_1'^2 - \alpha^2 B' K_1'^2 g_1'^2$.
(23)

and λ_4, λ'_4 = $\{m + (m^2 - 4K_1^2)^{½}\}/2$, $\{l + (l^2 - 4K_1'^2)^{½}\}/2$

Where the symbol used in eqns. (21) and (23) are given by eqns. (16) and (18).

The constants A_j, B_j, C_j (j = 1, 2, 3) are related with A'_j, B'_j, C'_j (j = 1, 2, 3) in Eq. (19a) and Eq. (19b) by means of first equations in Eq. (15) and Eq. (17).

Equating the coefficients of $e^{-\lambda_1 z}$, $e^{-\lambda_2 z}, e^{-\lambda_3 z}, e^{-\lambda'_1 z}$, $e^{-\lambda'_2 z}, e^{-\lambda'_3 z}$ to zero, after substituting Eq. (19a) and Eq. (19b) in the first and 3rd equations of Eq. (15) and Eq. (17) respectively, we get

$A_2 = \gamma_1 A_1$, $B_2 = \gamma_2 B_1$, $C_2 = \gamma_3 C_1$, and $A_3 = \delta_1 A_1$, $B_3 = \delta_2 B_1$, $C_3 = \delta_3 C_1$, (24)

where,

$\gamma_j = \dfrac{-i\alpha m l_1^2}{\lambda_j^2 - 2m\lambda_j + K_1^2}$ (j = 1, 2, 3),

$\delta_j = \dfrac{1}{g_1^2}\,[\lambda_j^2 - 2m f_1^2 \lambda_j + h_1^2 + i\,\alpha\, m f_1^2 \gamma_j]$, j = 1, 2, 3.

Similar result holds for medium M_2 and usual symbols replacing by dashes respectively.

4 BOUNDARY CONDITIONS

(i) The displacement components, temperature and temperature flux at the boundary surface between the media M_1 and M_2 must be continuous at all times and positions.

i.e. $\left[u,\, v,\, w,\, T,\, p\dfrac{\partial T}{\partial z}\right]_{M_1} = \left[u,\, v,\, w,\, T,\, p'\dfrac{\partial T}{\partial z}\right]_{M_2}$

(ii) The stress components τ_{31}, τ_{32}, τ_{33} must be continuous at the boundary z = 0.

i.e. $[\tau_{31},\, \tau_{32},\, \tau_{33}]_{M_1} = [\tau_{31},\, \tau_{32},\, \tau_{33}]_{M_2}$ at z = 0 respectively

Where,

$\tau_{31} = D_\mu\left(2\dfrac{\partial^2\phi}{\partial x\partial z} + \dfrac{\partial^2\psi}{\partial x^2} - \dfrac{\partial^2\psi}{\partial z^2}\right)$,

$\tau_{32} = D_\mu \dfrac{\partial v}{\partial z}$,

$\tau_{33} = D_\lambda \nabla^2\phi + 2D_\mu\left(\dfrac{\partial^2\phi}{\partial z^2} + \dfrac{\partial^2\phi}{\partial x\partial z}\right) - D_B T + D_{m_e} H_0^2 \nabla^2\phi$. (25)

Applying the boundary conditions, we get

$A_1(1 - i\gamma_1\zeta_1) + B_1(1 - i\gamma_2\zeta_2) + C_1(1 - i\gamma_3\zeta_3) - A'_1(1 - i\gamma'_1\zeta'_1)$ (26a)
$- B'_1(1 - i\gamma'_2\zeta'_2) - C'_1(1 - i\gamma'_3\zeta'_3) = 0$

$C = C'$ (26b)

$A_1(\gamma_1 + i\zeta_1) + B_1(\gamma_2 + i\zeta_2) + C_1(\gamma_3 + i\zeta_3) - A'_1(\gamma'_1 + i\zeta'_1) - B'_1(\gamma'_2 + i\zeta'_2) - C'_1(\gamma'_3 + i\zeta'_3) = 0$ (26c)

$\delta_1 A_1 + \delta_2 B_1 + \delta_3 C_1 = \delta'_1 A'_1 + \delta'_2 B'_1 + \delta'_3 C'_1$ (26d)

$p\lambda_1\delta_1 A_1 + p\lambda_2\delta_2 B_1 + p\lambda_3\delta_3 C_1 - p'\lambda'_1\delta'_1 A'_1 + p'\lambda'_2\delta'_2 B'_1 - p'\lambda'_3\delta'_3 C'_1 = 0$ (26e)

$m_K^*[(2i\zeta_1 + \gamma_1 + \zeta_1^2\gamma_1)A_1 + (2i\zeta_2 + \gamma_2 + \zeta_2^2\gamma_2)B_1 + (2i\zeta_3 + \gamma_3 + \zeta_3^2\gamma_3)C_1]$

$= m'^{*}_{K}\,[(2i\,\zeta'_1 + \gamma_1 + \zeta_1'^2\,\gamma_1)\,A'_1 + (2i\,\zeta'_2 + \gamma_2 + \zeta_2'^2\,\gamma_2)\,B'_1$
$+ (2i\,\zeta'_3 + \gamma'_3 + \zeta_3'^2\,\gamma_3)\,C'_1]$ (26f)

$m^{*}_{K}\,[-\lambda_4 C] = m'^{*}_{K}\,[-\lambda'_4\,C']$ (26g)

$A_1\,[(l^{*}_{K} + (\mu_e)^{*}_{K}\,H_0^2)\,(\zeta_1^2 - 1) + 2\,m^{*}_{K}\,(\zeta_1^2 - i\zeta_1) - b^{*}_{K}\,\delta_1] + B_1\,[(l^{*}_{K} + (\mu_e)^{*}_{K}\,H_0^2)\,(\zeta_2^2 - 1) + 2\,m^{*}_{K}\,(\zeta_2^2 - i\zeta_2) - b^{*}_{K}\,\delta_2] + C_1\,[(l^{*}_{K} + (\mu_e)^{*}_{K}\,H_0^2)\,(\zeta_3^2 - 1) + 2\,m^{*}_{K}$

$(\zeta_3^2 - i\zeta_3) - b^{*}_{K}\,\delta_3] = A'_1\,[(l'^{*}_{K} + (\mu'_e)^{*}_{K}\,H_0^2)(\zeta_1'^2 - 1) + 2\,m'^{*}_{K}\,(\zeta_1'^2 - i\zeta'_1) - b'^{*}_{K}\,\delta'_1] + B'_1\,[(l'^{*}_{K} + (\mu'_e)^{*}_{K}\,H_0^2)\,(\zeta_2'^2 - 1) + 2\,m'^{*}_{K}\,(\zeta_2'^2 - i\zeta'_2) - b'^{*}_{K}\,\delta'_2] + C'_1[(l'^{*}_{K} + (\mu'_e)^{*}_{K}\,H_0^2)\,(\zeta_3'^2 - 1) + 2\,m'^{*}_{K}\,(\zeta_3'^2 - i\zeta'_3) - b'^{*}_{K}\,\delta'_3]$
(26h)

where, $\zeta_j = \dfrac{\lambda_j}{\alpha}$, $\zeta'_j = \dfrac{\lambda'_j}{\alpha}$, j = 1, 2, 3

and

$\lambda^{*}_{K} = \sum_{K=0}^{n} \lambda_K\,(-i\alpha c)^K$, $m^{*}_{K} = \sum_{K=0}^{n} \mu_K\,(-i\alpha c)^K$, $b^{*}_{K} = \sum_{K=0}^{n} \beta_K\,(-i\alpha c)^K$,

$(\mu_e)^{*}_{K} = \sum_{K=0}^{n} (\mu_e)_K\,(-i\alpha c)^K$, $l'^{*}_{K} = \sum_{K=0}^{n} \lambda'_K\,(-i\alpha c)^K$, $m'^{*}_{K} = \sum_{K=0}^{n} \mu'_K\,(-i\alpha c)^K$, $b'^{*}_{K} = \sum_{K=0}^{n} \beta'_K\,(-i\alpha c)^K$, $(\mu'_e)^{*}_{K} = \sum_{K=0}^{n} (\mu'_e)_K\,(-i\alpha c)^K$

From Eq. (26b) and Eq. (26g), we have C = C' = 0. Thus there is no propagation of displacement v. Hence SH-waves do not occur in this case.

Finally, eliminating the constants A_1, B_1, C_1, A'_1, B'_1, C'_1, from the remaining equations, we get

det (a_{ij})= 0, i, j = 1, 2, 3, 4, 5, 6. (27)

Where,
$a_{11} = 1 - i\gamma_1\,\zeta_1$, $a_{12} = 1 - i\gamma_2\zeta_2$, $a_{13} = 1 - i\gamma_3\zeta_3$, $a_{14} = (i\,\gamma'_1\,\zeta'_1 - 1)$,
$a_{15} = (i\,\gamma'_2\,\zeta'_2 - 1)$, $a_{16} = (i\,\gamma'_3\,\zeta'_3 - 1)$,
$a_{21} = \gamma_1 + i\zeta_1$, $a_{22} = \gamma_2 + i\zeta_2$, $a_{23} = \gamma_3 + i\zeta_3$, $a_{24} = (\gamma'_1 + i\,\zeta'_1)$, $a_{25} = (\gamma'_2 + i\zeta'_2)$,
$a_{26} = (\gamma'_3 + i\zeta'_3)$,
$a_{31} = \delta_1$, $a_{32} = \delta_2$, $a_{33} = \delta_3$, $a_{34} = -\delta'_1$, $a_{35} = -\delta'_2$, $a_{36} = -\delta'_3$,
$a_{41} = p\lambda_1\,\delta_1$, $a_{42} = p\lambda_2\,\delta_2$, $a_{43} = p\lambda_3\,\delta_3$, $a_{44} = -p'\,\lambda'_1\,\delta'_1$, $a_{45} = -p'\,\lambda'_2\,\delta'_2$,
$a_{46} = -p'\,\lambda'_3\,\delta'_3$,
$a_{51} = m^{*}_{K}\,(2i\,\zeta_1 + \gamma_1 + \gamma_1\,\zeta_1^2)$, $a_{52} = m^{*}_{K}\,(2i\,\zeta_2 + \gamma_2 + \gamma_2\,\zeta_2^2)$,
$a_{53} = m^{*}_{K}\,(2i\,\zeta_3 + \gamma_3 + \gamma_3\,\zeta_3^2)$,
$a_{54} = m'^{*}_{K}\,(2i\,\zeta'_1 + \gamma'_1 + \gamma'_1\,\zeta_1'^2)$, $a_{55} = m'^{*}_{K}\,(2i\,\zeta'_2 + \gamma'_2 + \gamma'_2\,\zeta_2'^2)$,
$a_{56} = m'^{*}_{K}\,(2i\,\zeta'_3 + \gamma'_3 + \gamma'_3\,\zeta_3'^2)$,
$a_{61} = (l^{*}_{K} + (\mu_e)^{*}_{K}\,H_0^2)\,(\zeta_1^2 - 1) + 2\,m^{*}_{K}\,(\zeta_1^2 - i\zeta_1) - b^{*}_{K}\,\delta_1$,

$a_{62} = (l^*_K + (\mu_e)^*_K H_0^2)(\zeta_2{}^2 - 1) + 2\, m^*_K(\zeta_2{}^2 - i\zeta_2) - b^*_K \delta_2,$
$a_{63} = (l^*_K + (\mu_e)^*_K H_0^2)(\zeta_3{}^2 - 1) + 2\, m^*_K(\zeta_3{}^2 - i\zeta_3) - b^*_K \delta_3,$
$a_{64} = (l'^*_K + (\mu'_e)^*_K H_0^2)(\zeta_1'^2 - 1) + 2\, m'^*_K(\zeta_1'^2 - i\zeta'_1) - b'^*_K \delta'_1,$
$a_{65} = (l'^*_K + (\mu'_e)^*_K H_0^2)(\zeta_2'^2 - 1) + 2\, m'^*_K(\zeta_2'^2 - i\zeta'_2) - b'^*_K \delta'_2,$
$a_{66} = (l'^*_K + (\mu'_e)^*_K H_0^2)(\zeta_3'^2 - 1) + 2\, m'^*_K(\zeta_3'^2 - i\zeta'_3) - b'^*_K \delta'_3,$

From Eq. (27), we obtain velocity of surface waves in common boundary between two viscoelastic, non-homogeneous solid media under the influence of thermal and magnetic field, where the viscosity is of general nth order involving time rate of change of strain.

5 PARTICULAR CASES

Stoneley Waves:

It is the generalised form of Rayleigh waves in which we assume that waves are propagated along the common boundary of the two semi-infinite media M_1 and M_2. Thus Eq. (27) determine the wave velocity equation for Stoneley waves in the case of general magneto-thermo viscoelastic, non-homogeneous solid media of nth order involving time rate of strain. Clearly from Eq. (27), it is follows that the wave velocity equation for Stoneley waves depends upon the non-homogeneity of the material medium, temperature, gravity, magnetic and viscous field. This equation, of course, is in well agreement with the corresponding classical result, when the effects of thermal, gravity, magnetic and viscous field and non-homogeneity are absent.

Rayleigh Waves:

To investigate the possibility of Rayleigh waves in a thermo viscoelastic, non-homogeneous elastic media, we replace media M_2 by vacuum, in the proceeding problem, we also note the SH-waves do not occur in this case.
Since the temperature difference across the boundary is always small so thermal condition given by

$$\frac{\partial T}{\partial z} + hT = 0 \text{ at } z = 0 \text{ respectively} \quad (28)$$

Thus Eq. (26f) and Eq. (26h) reduces to,

$$(2i\,\zeta_1 + \gamma_1 + \gamma_1\,\zeta_1{}^2)\,A_1 + (2i\,\zeta_2 + \gamma_2 + \gamma_2\,\zeta_2{}^2)\,B_1 + (2i\,\zeta_3 + \gamma_3 + \gamma_3\,\zeta_3{}^2)\,C_1 = 0 \quad (29a)$$

$$[(l^*_K + (\mu_e)^*_K H_0^2)(\zeta_1{}^2 - 1) + 2\, m^*_K(\zeta_1{}^2 - i\zeta_1) - b^*_K \delta_1]\,A_1$$
$$+ [(l^*_K + (\mu_e)^*_K H_0^2)(\zeta_2{}^2 - 1) + 2\, m^*_K(\zeta_2{}^2 - i\zeta_2) - b^*_K \delta_2]\,B_1$$
$$+ [(l^*_K + (\mu_e)^*_K H_0^2)(\zeta_3{}^2 - 1) + 2\, m^*_K(\zeta_3{}^2 - i\zeta_3) - b^*_K \delta_3]\,C_1 = 0 \quad (29b)$$

From Eq. (27), we have

$$(\lambda_1 - h)\,\delta_1\,A_1 + (\lambda_2 - h)\,\delta_2\,B_1 + (\lambda_3 - h)\,\delta_3\,C_1 = 0 \quad (29c)$$

Eliminating A_1, B_1 and C_1 from Eq. (29a), Eq. (29b) and Eq. (29c), we get

$$\det(b_{ij}) = 0, \; i, j = 1, 2, 3. \quad (30)$$

Where,

$b_{11} = (2i\,\zeta_1 + \gamma_1 + \gamma_1\,\zeta_1{}^2)$, $b_{12} = (2i\,\zeta_2 + \gamma_2 + \gamma_2\,\zeta_2{}^2)$, $b_{13} = (2i\,\zeta_3 + \gamma_3 + \gamma_3\,\zeta_3{}^2)$,

$b_{21} = [(l^*_K + (\mu_e)^*_K H_0^2)(\zeta_1^2 - 1) + 2 m^*_K (\zeta_1^2 - i\zeta_1) - b^*_K \delta_1]$,
$b_{22} = [(l^*_K + (\mu_e)^*_K H_0^2)(\zeta_2^2 - 1) + 2 m^*_K (\zeta_2^2 - i\zeta_2) - b^*_K \delta_2]$,
$b_{23} = [(l^*_K + (\mu_e)^*_K H_0^2)(\zeta_3^2 - 1) + 2 m^*_K (\zeta_3^2 - i\zeta_3) - b^*_K \delta_3]$,
$b_{31} = (\lambda_1 - h)\delta_1$, $b_{32} = (\lambda_2 - h)\delta_2$, $b_{33} = (\lambda_3 - h)\delta_3$. (31)

Thus Eq. (30), gives the wave velocity equation for Rayleigh waves in a non-homogeneous, magneto-thermo viscoelastic solid media of nth order involving time rate of strain. From Eq. (30), it is follows that Dispersion equation of Rayleigh waves depends upon the non-homogeneity, the viscous, gravity, magnetic and thermal fields.

This equation, of course, is in complete agreement with the corresponding classical result by Bullen, when the effects of thermal, gravity, magnetic viscous field and non-homogeneity are absent.

Love Waves:
To investigate the possibility of love waves in a non-homogeneous, viscoelastic solid media, we replace medium M_2 is obtained by two horizontal plane surfaces at a distance H-apart, while M_1 remains infinite. For medium M_1, the displacement component v remains same as in general case given by Eq. (19). For the medium M_2, we preserve the full solution, since the displacement component along y-axis i.e. no longer diminishes with increasing distance from the boundary surface of two media.

Thus $$v' = C_1 e^{\lambda'_4 z + i\alpha(x-ct)} + C_2 e^{-\lambda'_4 z + i\alpha(x-ct)} \quad (32)$$

In this case, the boundary conditions are

(i) v and τ_{32} are continuous at z = 0
(ii) $\tau'_{32} = 0$ at z = –H.

Applying boundary conditions (i) and (ii) and using Eq. (19) and Eq. (26), we get

$$C = C_1 + C_2 \quad (33a)$$

$$-m^*_K \lambda_4 C = (\mu'_K)^* [\lambda'_4 C_1 - \lambda'_4 C_2]$$
(33b)

$$C_1 e^{-\lambda'_4 H} - C_2 e^{\lambda'_4 H} = 0 \quad (33c)$$

On eliminating the constants C, C_1 and C_2 from Eq. (33a), Eq. (33b) and Eq. (33c), we get

$$\tanh(\lambda'_4 H) = -\frac{\lambda_4 \mu^*_K}{\lambda'_4 (\mu'_K)^*}. \quad (34)$$

Thus Eq. (34) gives the wave velocity equation for Love waves in a non-homogeneous, magneto, thermo viscoelastic solid medium of nth order involving time rate of strain. Clearly it depends upon the non-homogeneity, gravity, magnetic and viscous fields and independent of thermal field.

CONCLUSION

1. The surface waves in a non-homogeneous, isotropic, viscoelastic solid medium under gravity of nth order including time rate of strain are investigated. It is observed that viscoelastic surface waves are affected by the time

rate of strain parameters. These parameters influence the wave velocity to an extent depending on the corresponding constants characterizing the magneto thermo and viscoelasticity of the material. So the results of this analysis become useful in circumstances where these effects cannot be neglected. These velocities depend upon the wave number 'α' confirming that these waves are affected by non-homogeneity of the material medium.

2. Love waves do not depends on temperature; these are only affected by viscous, gravity, magnetic fields and non-homogeneity of the material medium. In absence of all fields and non-homogeneity, the dispersion equation is in complete agreement with the corresponding classical result.

3. Rayleigh waves in a non-homogeneous, general magneto-thermo viscoelastic solid medium of higher order including time rate of change of strain we find that the wave velocity equation proves that there is dispersion of waves due to the presence of non-homogeneity, temperature, gravity, magnetic field and viscosity. The results are in complete agreement with the corresponding classical results in the absence of all fields and compression.

4. The wave velocity equation of Stoneley waves is very similar to the corresponding problem in the classical theory of elasticity. The dispersion of waves is due to the presence of non-homogeneity, gravity, magnetic field, temperature and viscoelasticity of the solid. Also, wave velocity equation of this generalized type of surface waves is in complete agreement with the corresponding classical result in the absence of all fields and non-homogeneity.

5. The solution of wave velocity equation for Stoneley waves cannot be determined by easy analytical methods however we can apply numerical techniques to solve this determinantal equation by choosing suitable values of physical constants for both media M_1 and M_2.

REFERENCES

[1] Carcione JM, *Geophysical Journal International*, **1990**, 101, 739-750.
[2] Stoneley R, *Proc. R. Soc. A*, 806, **1924**, 416-28.
[3] Bullen K.E, Theory of Seismology, Cambridge University Press, **1965**.
[4] Hunter, SC, Viscoelastic waves, Progress in solid mechanics, I. (Ed: Sneddon IN and Hill R) Cambridge University Press, **1970**.
[5] Jeffreys H, The Earth, Cambridge University Press, **1970**.
[6] Bromwich TJ, *Proc. London Math. Soc*., **1898**, 30, 98-120.
[7] Love AEH, Some problems of Geodynamics. Cambridge University Press, London, **1965**.
[8] Sezawa K, *Bull. Earthq. Res. Inst. Tokyo*, **1927**, 3, 1-18.
[9] Thomson WT, *J. Appl. Phys*., **1950**, 21, 89-93.
[10] Haskell, NA, *Bull. Seis. Soc. Amer.*, **1953**, 43, 17-34.
[11] Ewing WM, Jardetzky WS, Press F, Elastic waves in layered media, McGraw-Hill, New York, **1957**.
[12] M.A. Biot, Mechanics of incremental Deformations, J. Willy, **1965**.
[13] Jones JP, *J. Appl. Mechanics*, **1964**, 213-222.
[14] De SK, Sengupta PR, *J.Acanst. Soc. Am.*, **1974**, 55, 5-13.
[15] Sengupta PR, Acharya D, *Rev. Romm. Sci. Techmol. Mech. Appl., Tome,* **1979**, 24, 395-406.
[16] Brunelle EJ, *Bull. Seismol. Soc. Am.*, **1973**, 63, 1895-1899.
[17] Roy PP, *Acta Mechanisms*, **1984**, 54, 1-21.
[18] Datta BK, *Rev. Roumaine Sci. Tech. Ser. Mec. Appl*., **1986**, 31, 369-374.
[19] Goda MA, *Acta Mech.*, **1992**, 93, 89-98.
[20] Abd-Alla AM, Ahmed SM, *J. Earth, Moon Planets*, **1996**, 75, 185-197.
[21] Kakar R, Kakar S, and. Gupta KC, *International Journal of IT, Engineering and Applied Sciences Research*, **2012**, 1(1), 42-49.
[22] Voigt W, Abh. Gesch. Wiss, **1887**, 34.