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Effects of variable viscosity and nonlinear radiation on MHD flow with heat transfer over a surface stretching with a power-law velocity

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ABSTRACT

An analysis has been carried out to discuss the nonlinear MHD, steady, two dimensional, laminar boundary layer flows with heat transfer characteristics of an incompressible, viscous, electrically conducting and radiating fluid with variable viscosity over a surface stretching with power-law velocity in the presence of a variable magnetic field and nonlinear radiation effects. The fluid is assumed to be a gray, emitting, absorbing but non-scattering medium. Governing nonlinear partial differential equations are transformed to nonlinear ordinary differential equations by utilizing suitable similarity transformation. Then the resulting nonlinear ordinary differential equations are solved numerically using the Nachtsheim-Swigert iteration shooting technique for satisfaction of asymptotic boundary conditions by Runge-kutta fourth order method. The numerical results for velocity and temperature distribution are obtained for different values of viscosity measuring parameter, velocity exponent parameter, magnetic interaction parameter, surface temperature parameter, radiation parameter and Prandtl number. Values for skin friction coefficient and dimensionless rate of heat transfer are also obtained numerically for variation of physical parameters.

Keywords: Nonlinear Radiation, MHD flow, Stretching surface, power-law velocity & variable viscosity.

Nomenclature

B(x) - Variable applied magnetic induction.
M - Magnetic interaction parameter.
Pr - Prandtl Number.
T - Temperature of the fluid.
m - Velocity exponent parameter.
p - Pressure of the fluid.
u, v - Velocity component of fluid in x and y direction.
ρ - Density of the fluid.
ν - Kinematic viscosity.
α - Thermal conductivity.
η - Similarity variable
Ψ - Stream function.
θ - Dimensionless temperature.
B_O - Constant applied magnetic induction.
C_p - Specific heat at constant pressure.
T_w - Temperature of the heated surface.
T_∞ - Temperature of the ambient fluid.
q_r - Component of radiative flux.

INTRODUCTION

The study of two-dimensional boundary layer flow and heat transfer over a nonlinear stretching surface with variable viscosity is very important as it finds many practical applications in geophysics, particularly, geothermal energy extraction and underground storage systems. In addition, they also find very useful applications in the design of insulation systems employing porous media.

Previous studies on heat transfer were mostly based on the constant physical properties of the ambient fluid. However, it is known that these properties may change with temperature, especially for fluid viscosity. To accurately predict the flow and heat transfer rates, it is necessary to take into account this variation of viscosity. Due to these aspects, Gary et al.[1] has concluded that when the variable viscosity is included, the flow characteristics change substantially compared to the constant viscosity. Pop et al.[2] studied the influence of variable viscosity on laminar boundary layer flow and heat transfer due to a continuously moving plate.

Heat transfer in a viscous fluid over a stretching sheet with viscous dissipation and internal heat generation was discussed by Vajravelu and Hadjinicolaou [3].Further, Kafoussius and Williams [4] have investigated the effects of temperature-dependent viscosity on the mixed convection flow past a vertical flat plate in the region near the leading edge using the local nonsimilarity method. Boundary layer flow and heat transfer on a continuous flat surface moving in a parallel free stream with variable fluid properties were investigated by Hassanien [5]. Hossain et al. [6] have investigated the natural convection flow from a vertical wavy surface with variable viscosity.

Hossain and Munir [7] investigated the mixed convection flow from a vertical flat plate for a temperature dependent viscosity. In the above study, the viscosity of the fluid has been considered to be inversely proportional to a linear function of the temperature. Goswani [8] has studied the influence of variable viscosity on MHD flow and heat transfer for a continuous moving flat plate. Anjali Devi and Thiagarajan [9] analyzed the effects of variable viscosity on nonlinear MHD flow and heat transfer over a surface stretching with power law velocity. Mukhopadhyay et al. [10] studied the effects of MHD boundary layer flow over a heated stretching sheet with variable viscosity. A numerical reinvestigation on MHD boundary layer flow over a heated stretching sheet with variable viscosity was analysed by Asterios Pantokratoras [11]. Tomer et al. [12] analysed the influence of variable viscosity on convective Heat transfer along an inclined plate embedded in Porous Medium with an Applied Magnetic Field. Yasir Khan et al. [13] investigated the effects of variable viscosity and thermal conductivity on a thin film flow over a shrinking/stretching sheet.

Interaction of forced convection with thermal radiation plays a significant role in many practical applications. In particular, radiative heat transfer is more important with rising temperature levels and may be dominant over conduction and convection at high temperature. The application of thermal radiation effects involve furnaces, rocket nozzles, engines, nuclear reactors and during atmospheric reentry of space vehicles.

Further, radiation effects on heat transfer over a stretching surface are important in the context of space technology and processes involving high temperature. Saundalgekar and Takhar [14] studied the effects of radiation on the natural convection flow of a gas past a semi-infinite flat plate. Later, Hossain and Takhar [15] analyzed the effect of radiation using the Rosseland diffusion approximation which leads to nonsimilar solutions for the forced and free convection flow of an optically dense viscous incompressible fluid past a heated vertical plate with uniform free stream and uniform surface temperature.

Siddheshwar and Mahabaleswar [16] investigated the effects of radiation and heat source on MHD flow of a viscoelastic liquid and heat transfer over a stretching sheet. Mukhopadhyay and Layek [17] analysed the effects of thermal radiation and variable fluid viscosity on free convective flow and heat transfer past a porous stretching surface. Mostafa A. A. Mahmoud [18] analysed, thermal radiation effect on unsteady MHD free convection flow past a vertical plate with temperature-dependent viscosity. The effect of thermal radiation on MHD mixed convective heat transfer adjacent to a vertical continuously stretching sheet in the presence of variable viscosity was studied by Salem and Rania Fathy [19]. Seddeek and Almushigeh [20] investigated the effects of radiation and variable viscosity on MHD free convective flow and mass transfer over a stretching sheet with chemical reaction. Effect of radiation with temperature dependent viscosity and thermal conductivity on unsteady stretching sheet through porous media was analysed by Abdou [21].

V. Sri Hari Babu and G. V. Ramana Reddy [22] analyzed the Mass transfer effects on MHD mixed convective flow from a vertical surface with ohmic heating and viscous dissipation.

Satya Sagar Saxena and G. K. Dubey [23] studied the effects of MHD free convection heat and mass transfer flow of viscoelastic fluid embedded in a porous medium of variable permeability with radiation effect and heat source in slip flow regime. Unsteady MHD heat and mass transfer free convection flow of polar fluids past a vertical moving porous plate in a porous medium with heat generation and thermal diffusion was analysed by Satya Sagar Saxena and G. K. Dubey [24].

But so far, no contribution has been made to analyse the effects of variable viscosity and nonlinear radiation on a forced convection flow of an electrically conducting fluid past a surface stretching with power-law velocity which motivated for the present study.

1.2 Formulation of the problem

Forced convection flow with nonlinear radiation along a horizontal stretching surface which is kept at uniform temperature T_w and stretching with a power law velocity $u_w = u_o x^m$ (where u_o and m are constants) through a fluid with variable viscosity is considered. The fluid is assumed to be incompressible, viscous, gray, emitting, absorbing and electrically conducting, but non scattering medium at temperature T_∞. A variable magnetic field is applied normal to the horizontal stretching surface. Cartesian coordinate system is chosen. The x-axis is taken in the direction of the main flow along the stretching porous sheet with velocity components u and v in these directions. Here the surface is issued from a thin slit at $x = 0$, $y = 0$ and subsequently being stretched.

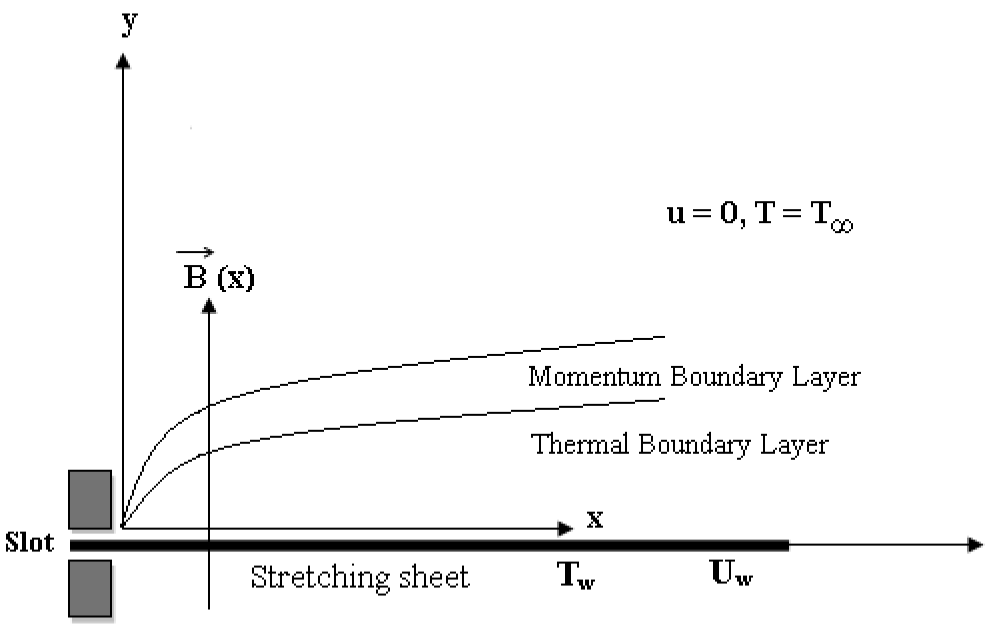

Fig.1 Schematic diagram of the problem

The x-axis runs along the continuous surface in the direction of motion and y-axis perpendicular to it.
The following assumptions are made:

- The flow is two-dimensional, steady and laminar.
- The fluid properties are assumed to be constant, except for the fluid viscosity.
- The usual boundary layer assumptions are made [Ali.M.E [25]].
- Magnetic Reynolds number is assumed to be small. Under this assumption, the induced magnetic field is assumed to be negligible.
- Since the flow is steady, $curl\vec{E} = 0$. Also $div\vec{E} = 0$ in the absence of surface charge density. Hence $\vec{E} = 0$ is assumed.
- The viscous and Joule's dissipation are considered to be negligible.
- The radiation heat flux in the x-direction is considered to be negligible in comparison to that in the y-direction.

The fluid viscosity is taken as

$$\frac{1}{\mu} = d_2\left(T - T_r\right) \tag{1}$$

where $d_2 = \dfrac{d_1}{\mu_\infty}$ and $T_r = T_\infty - \dfrac{1}{d_1}$ (2)

Here d_1, d_2 and T_r are constants, and their values depend on the reference state and the thermal property of the fluid.

The continuity, momentum, and energy conservation equations under the boundary layer assumptions are given by:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{3}$$

$$\rho\left[u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right] = \frac{\partial}{\partial y}\left[\mu\frac{\partial u}{\partial y}\right] - \sigma B^2(x)u \tag{4}$$

where $B(x) = B_o x^{\frac{(m-1)}{2}}$

$$\rho C_p\left[u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right] = K\frac{\partial^2 T}{\partial y^2} - \frac{\partial q_r}{\partial y} \tag{5}$$

with the associated boundary conditions

$$\begin{array}{llll} u = u_w = u_o x^m, \; v = 0, & T = T_w & at\; y = 0 & (u_0 > 0) \\ u = 0, & T = T_\infty & as\; y \to \infty & \end{array} \tag{6}$$

where u_w is the velocity of the stretching surface, the quantities u and v are the velocity components in the x and y directions respectively, u_o is a dimensional constant, B_o is a constant magnetic field and all the other quantities have their usual meanings. The radiative heat flux term is simplified by using the Rosseland diffusion approximation (Hossian et al. [6]) and accordingly

$$q_r = -\frac{16\sigma^* T^3}{3\alpha^*}\frac{\partial T}{\partial y} \tag{7}$$

where σ^* is the Stefan-Boltzmann constant, α^* is the Rosseland mean absorption coefficient.

The equation of continuity is satisfied if we choose a stream function $\psi(x, y)$ such that

$$u = \frac{\partial \psi}{\partial y} \text{ and } v = -\frac{\partial \psi}{\partial x} \tag{8}$$

Introducing the usual similarity transformation (Ali [25])

$$\eta(x, y) = y\sqrt{\frac{m+1}{2}}\sqrt{\frac{u_o x^{m-1}}{\nu}} \tag{9}$$

$$\psi(x, y) = \sqrt{\frac{2}{m+1}}\sqrt{\nu u_o x^{m+1}}\, f(\eta) \tag{10}$$

$$\theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty},\ \theta_w=\frac{T_w}{T_\infty} \tag{11}$$

where θ_w is surface temperature parameter.

It is obtained that

$$u=u_w f'(\eta) \tag{12}$$

$$v=-\sqrt{\frac{m+1}{2}}\sqrt{\frac{\nu u_w}{x}}\left[f(\eta)+\frac{m-1}{m+1}\eta f'(\eta)\right] \tag{13}$$

Equations (4.4) and (4.5) can be written as

$$f'''-\left(\frac{\theta-\lambda}{\lambda}\right)\left[ff''-\left(\frac{2m}{m+1}\right)f'^2-M^2f'\right]-\frac{f''\theta'}{\theta-\lambda}=0 \tag{14}$$

$$\left\{1+\frac{4}{3R^*}(1+(\theta_w-1)\theta)^3\right\}\theta''+\frac{4}{R^*}(1+(\theta_w-1)\theta)^2(\theta_w-1)\theta'^2+\Pr f\theta'=0 \tag{15}$$

where $\lambda=\dfrac{T_r-T_\infty}{T_w-T_\infty}$ is the viscosity measuring parameter.

$M=\sqrt{\dfrac{2\sigma B_o^{\,2}}{\rho u_o(m+1)}}$ is the magnetic interaction parameter

$R^*=\dfrac{K\alpha^*}{4\sigma^*T_\infty^3}$ is the Radiation parameter

$Pr=\dfrac{\mu c_p}{K}$ is the Prandtl number.

Associated boundary conditions are given by

$$\begin{aligned}&f(0)=0,\quad f'(0)=1,\quad \theta(0)=1\\&f'(\infty)=0,\quad \theta(\infty)=0\end{aligned} \tag{16}$$

1.3 Numerical solution of the problem

The present work is concerned with the effects of variable viscosity and nonlinear radiation on MHD flow with heat transfer over a surface stretching with a power-law velocity. The numerical solutions of the problem are obtained by solving the non-linear differential equations (14) and (15) subject to (16) using fourth order Runge- Kutta based shooting method along with Nachtsheim-Swigert iteration technique for satisfaction of asymptotic boundary conditions. The problem mainly depends on the values for $f''(0)$ and $\theta'(0)$ which are guessed initially. The success of the procedure depends very much on how good this guess is. The initial guesses are made properly in order to obtain the convergence.

RESULTS AND DISCUSSION

Numerical solutions of the problem are obtained for various values of the physical parameters involved in the problem such as Magnetic interaction parameter *M*, velocity exponent parameter m, radiation parameter *R**, surface temperature parameter θ_w, viscosity measuring parameter λ and Prandtl number *Pr*. The numerical results are displayed with the help of graphical illustrations.

In the absence of radiation the results have been compared with that of Anjali Devi & Thiagarajan [9] which are illustrated through Figs. 2 to 5 respectively. From these figures, it can be clearly seen that the results are in good agreement with that of Anjali Devi & Thiagarajan [9].

In order to illustrate the numerical results pertaining to velocity and temperature, Figs. 6 to 8 are drawn.

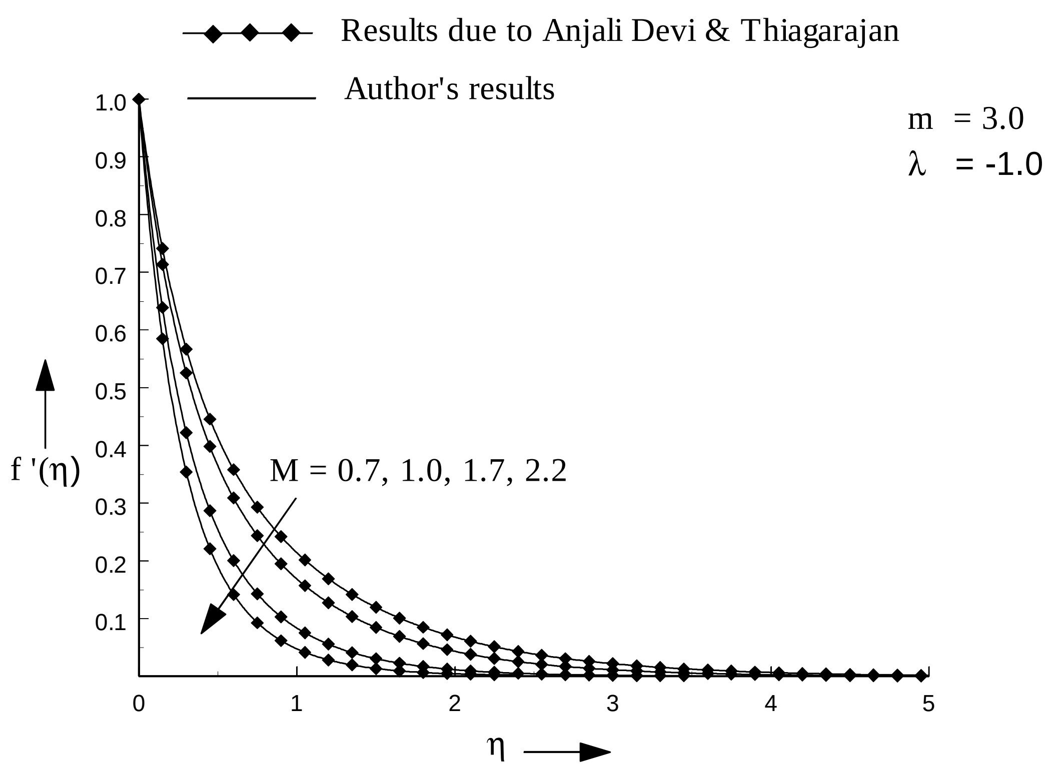

Fig. 2 Velocity profiles for different M

Figure 6 displays the plot of dimensionless velocity $f'(\eta)$ for different values of magnetic interaction parameter *M*. It is noted that as magnetic interaction parameter *M* increases, velocity $f'(\eta)$ decreases elucidating the fact that the effect of magnetic field is to decelerate the velocity.

The effect of magnetic field *M* over the dimensionless temperature $\theta(\eta)$ is shown with the help of Fig. 7. Increasing magnetic interaction parameter *M* is to reduce the temperature.

Figure 8 illustrates the effect of velocity exponent parameter *m* over the dimensionless velocity field $f'(\eta)$. It is observed that the effect of velocity exponent parameter is to reduce the velocity, elucidating the fact that the boundary layer thickness decreases as *m* increases.

The effect of radiation parameter R^* over the dimensionless temperature $\theta(\eta)$ is seen through Fig.9. It is noted that the effect of radiation parameter is to reduce the temperature. It elucidates that the thermal boundary layer thickness decreases as R^* increases.

The effect of surface temperature parameter θ_w over the dimensionless temperature $\theta(\eta)$ is shown in Figure 10. Increasing surface temperature parameter θ_w *is* to increase the temperature.

Prandtl number variation over the dimensionless temperature distribution when the surface is stretching with a power-law velocity is elucidated through Fig.11. As Prandtl number *Pr* increases, temperature $\theta(\eta)$ decreases, illustrating the fact that the effect of Prandtl number is to decrease the temperature in the presence of magnetic field. Furthermore, the effect of Prandtl number is to reduce the thickness of thermal boundary layer.

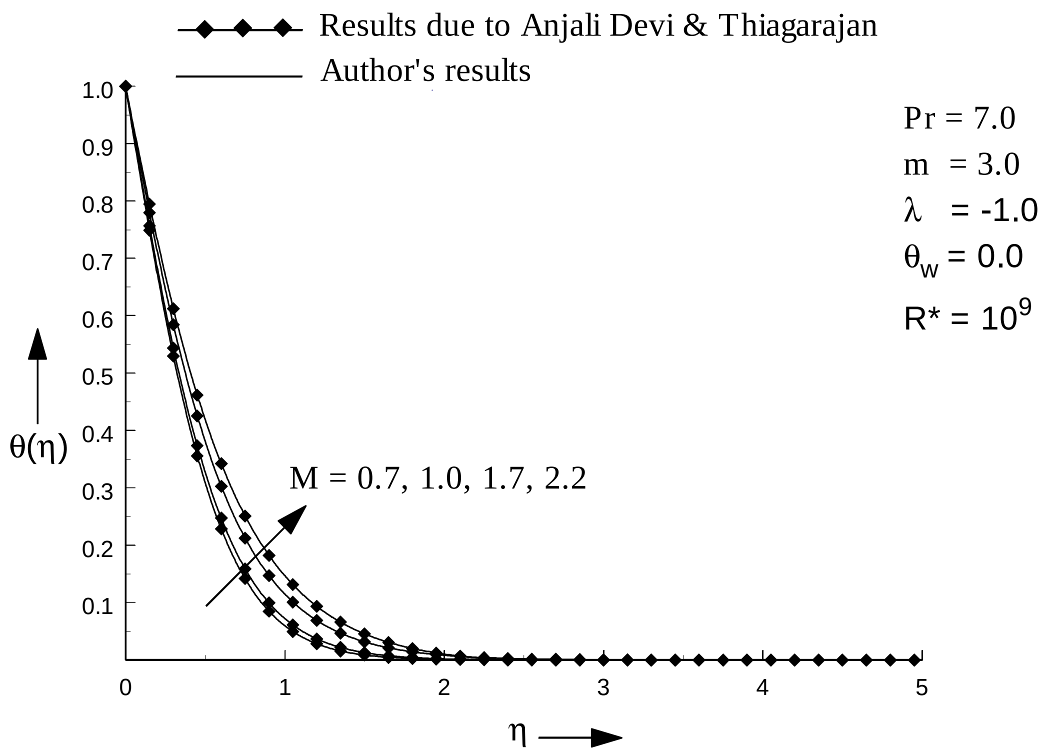

Fig. 3 Temperature profiles for different M

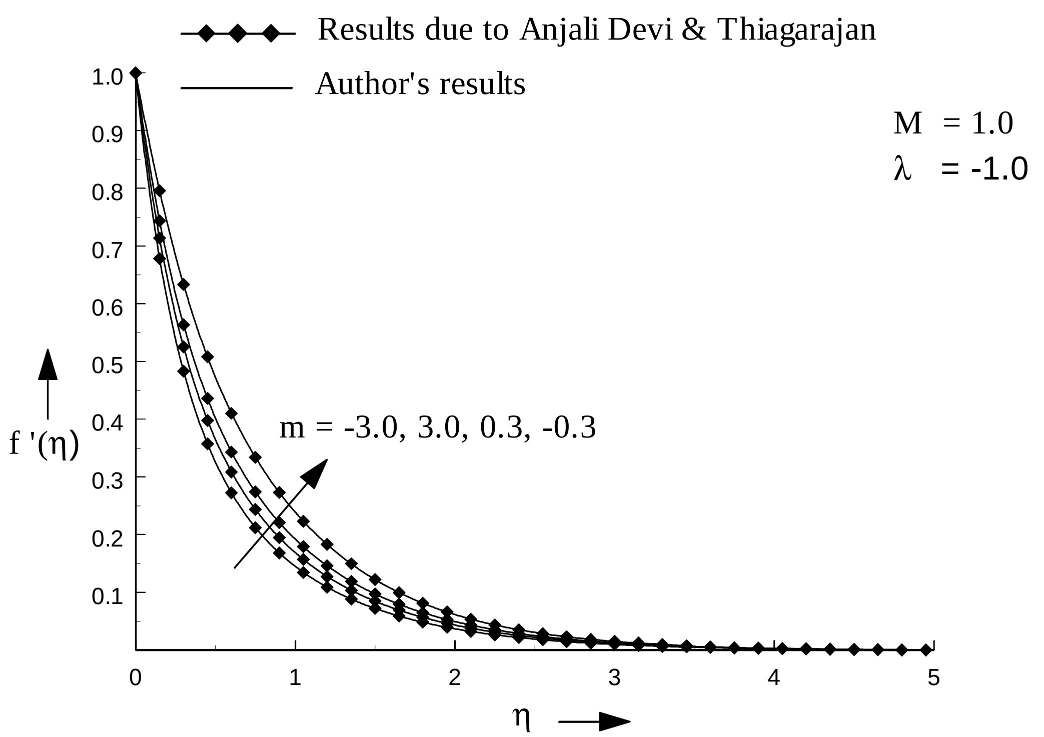

Fig. 4 Velocity profiles for different m

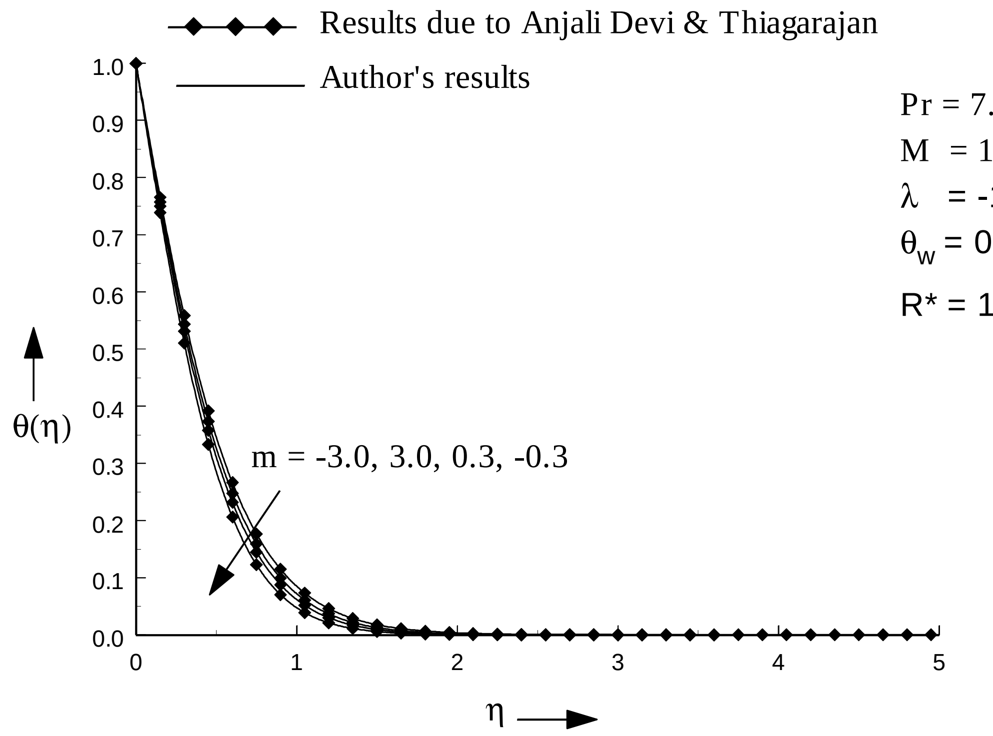

Fig. 5 Temperature profiles for different m

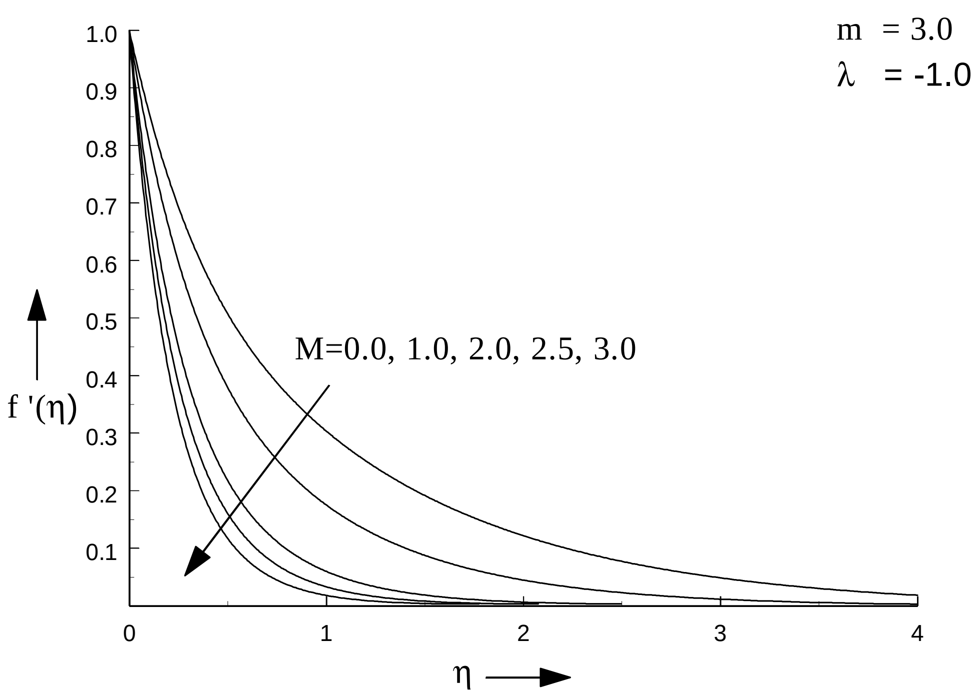

Fig.6 Velocity profiles for different M

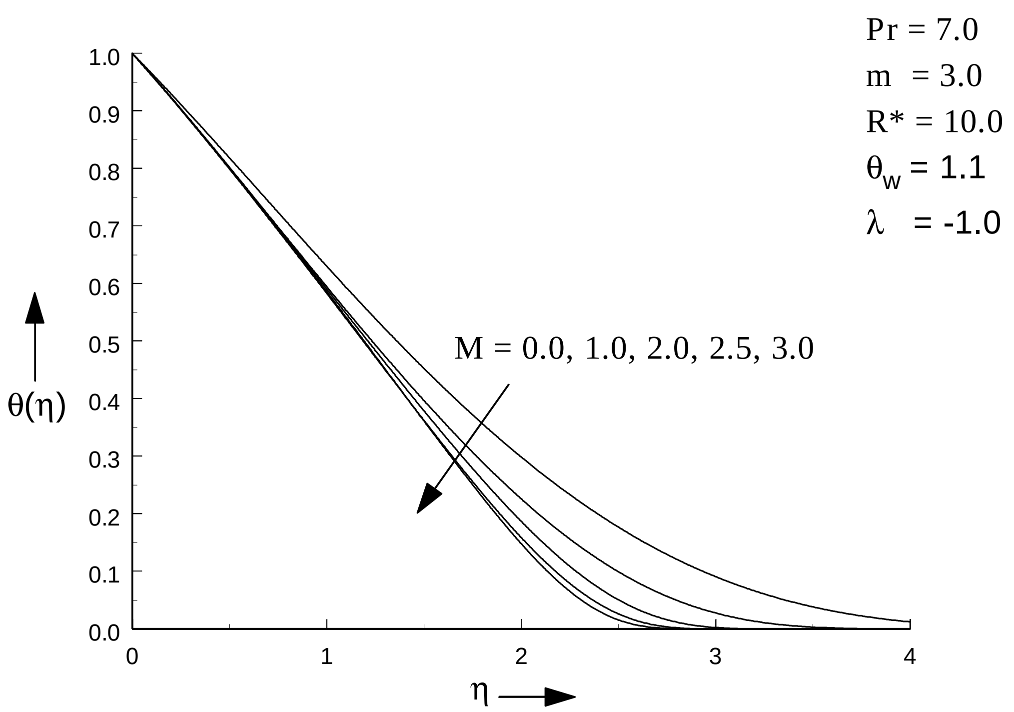

Fig. 7 Temperature profiles for different M

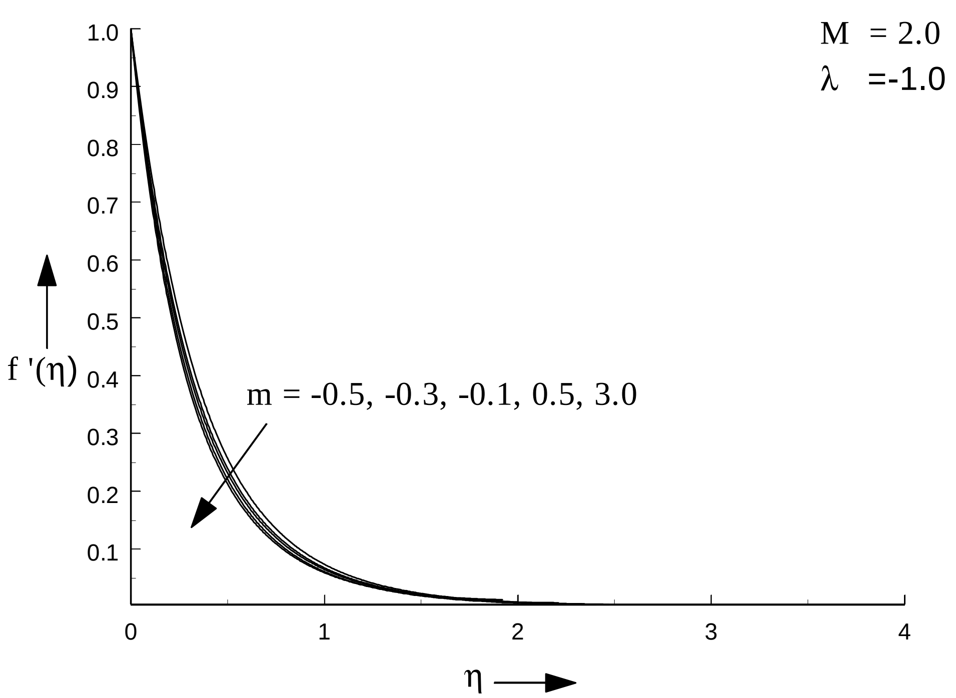

Fig. 8 Velocity profiles for different m

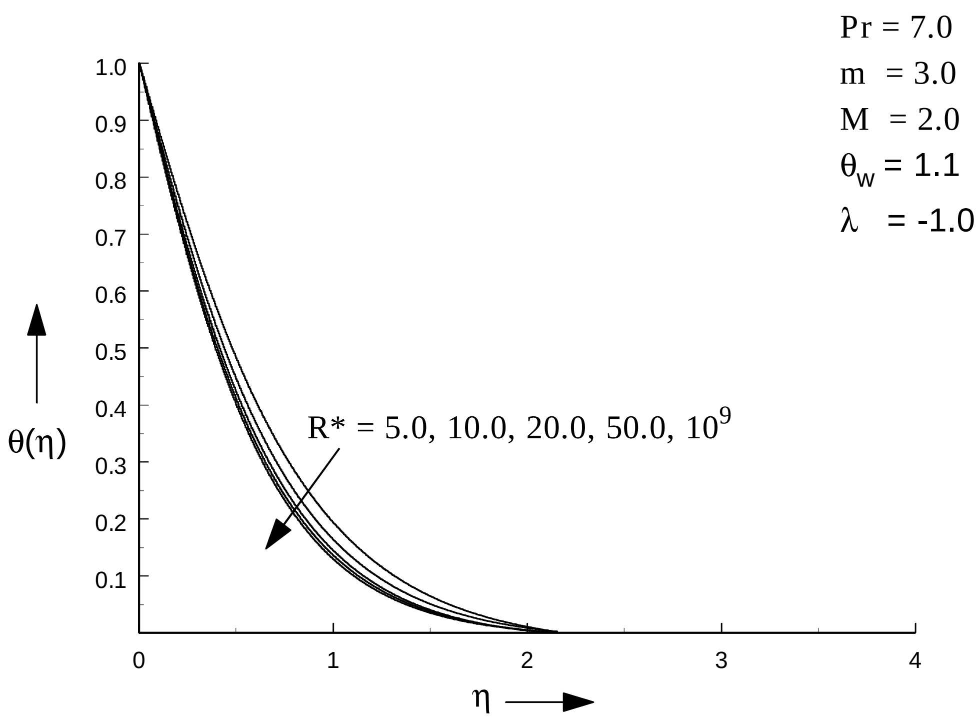

Fig.9 Temperature profiles for different R*

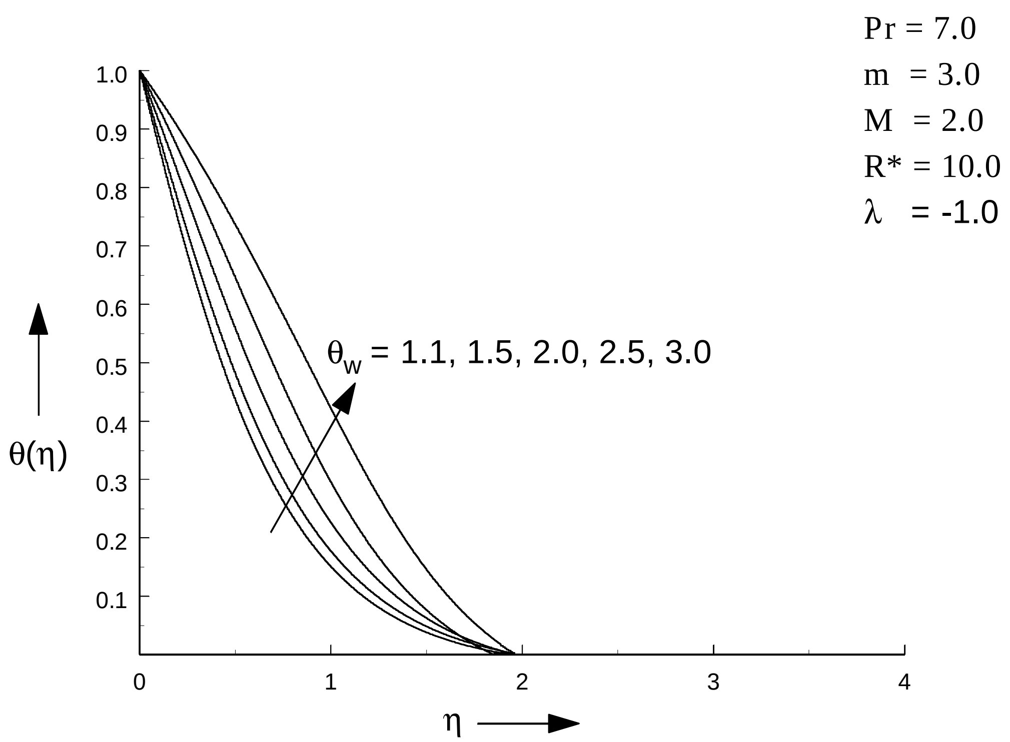

Fig.10 Temperature profiles for different θ_w

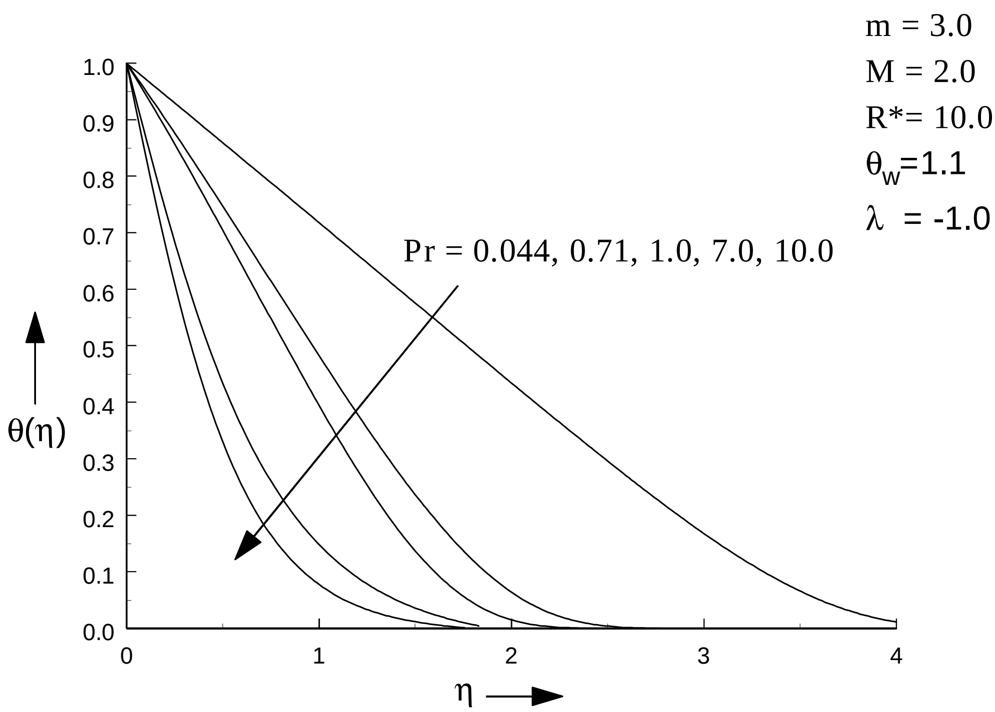

Fig.11 Temperature profiles for different Pr

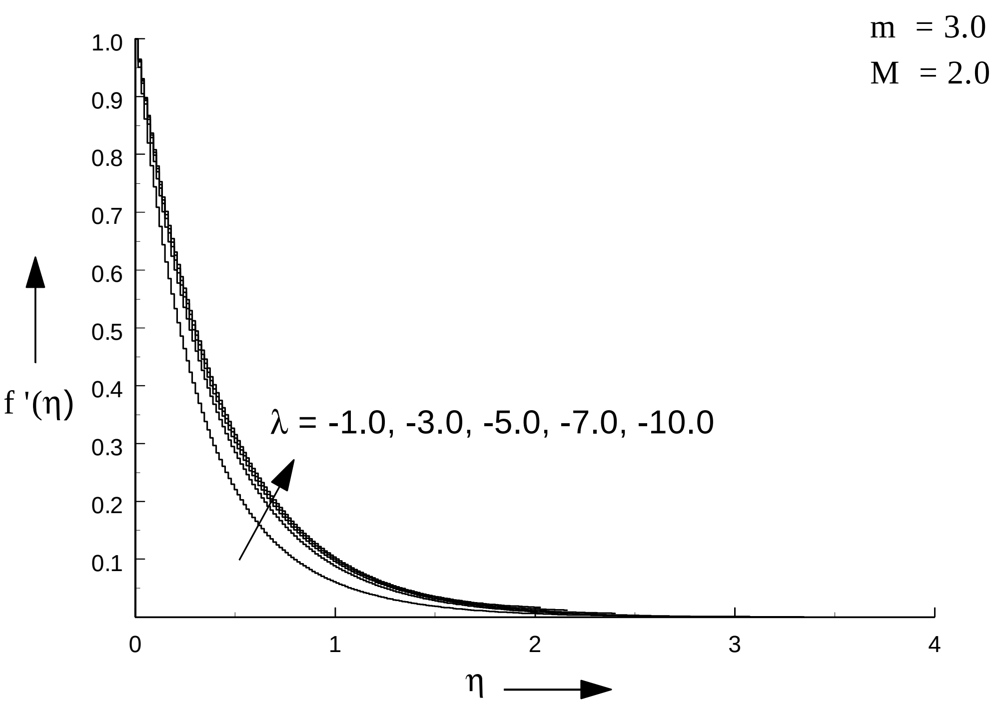

Fig.12 Velocity profiles for different λ

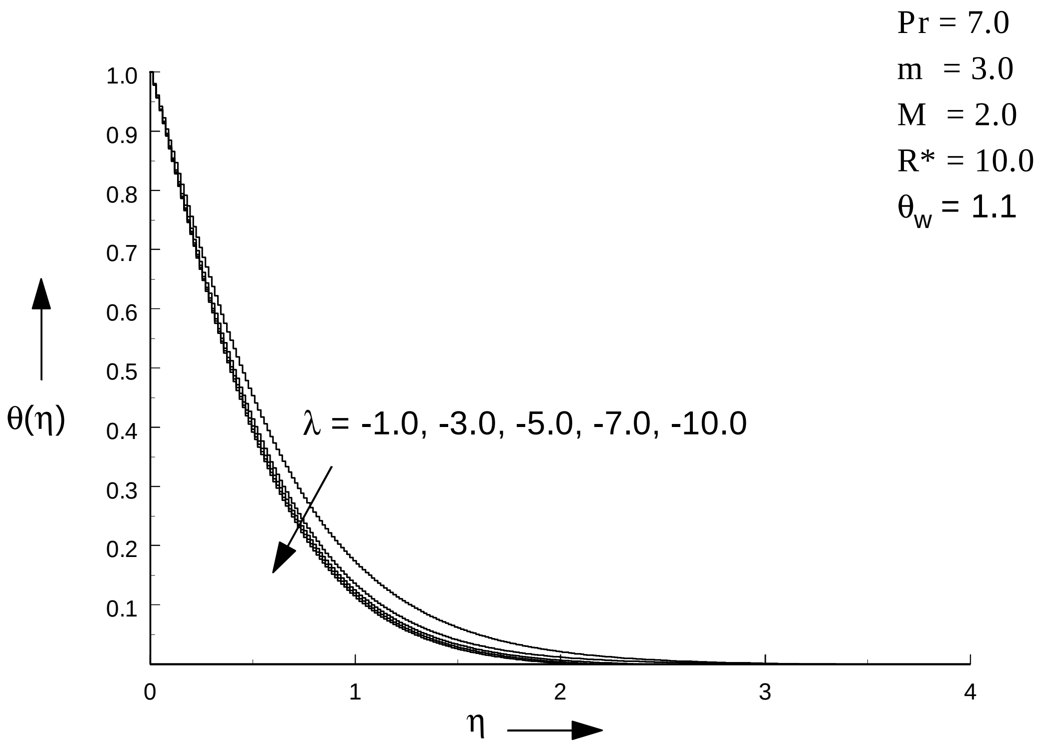

Fig.13 Temperature profiles for different λ

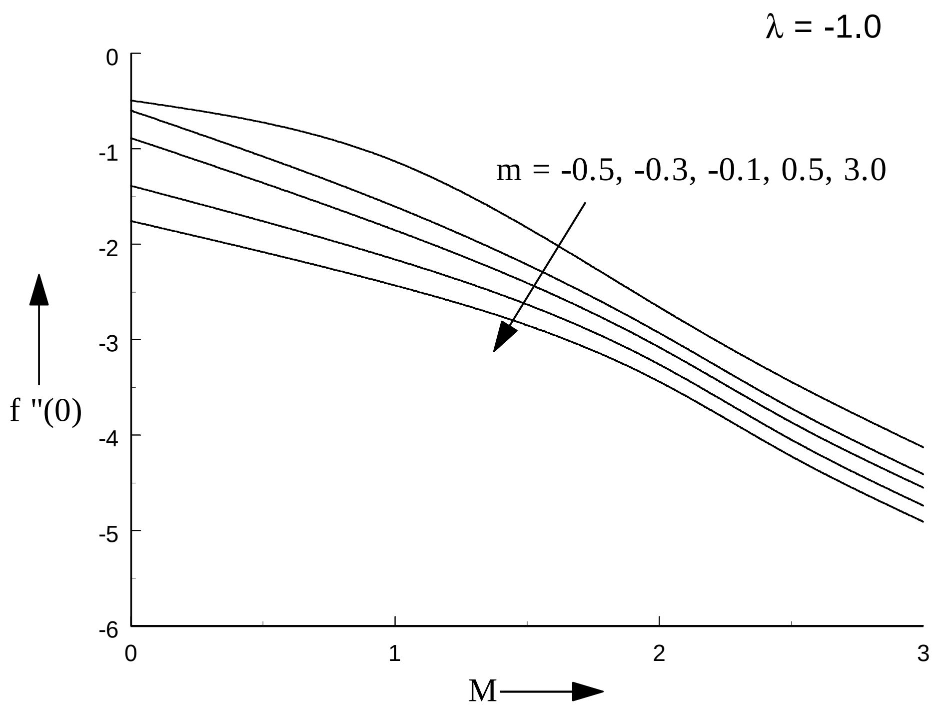

Fig.14 Skin friction coefficient for different m

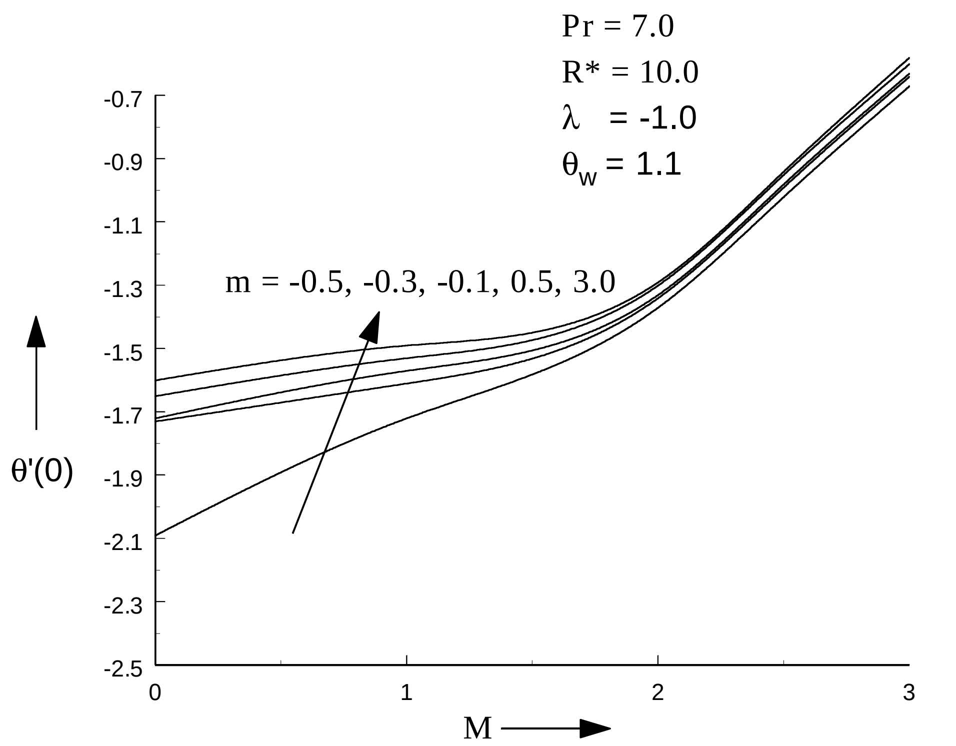

Fig.15 Dimensionless rate of heat transfer for different m

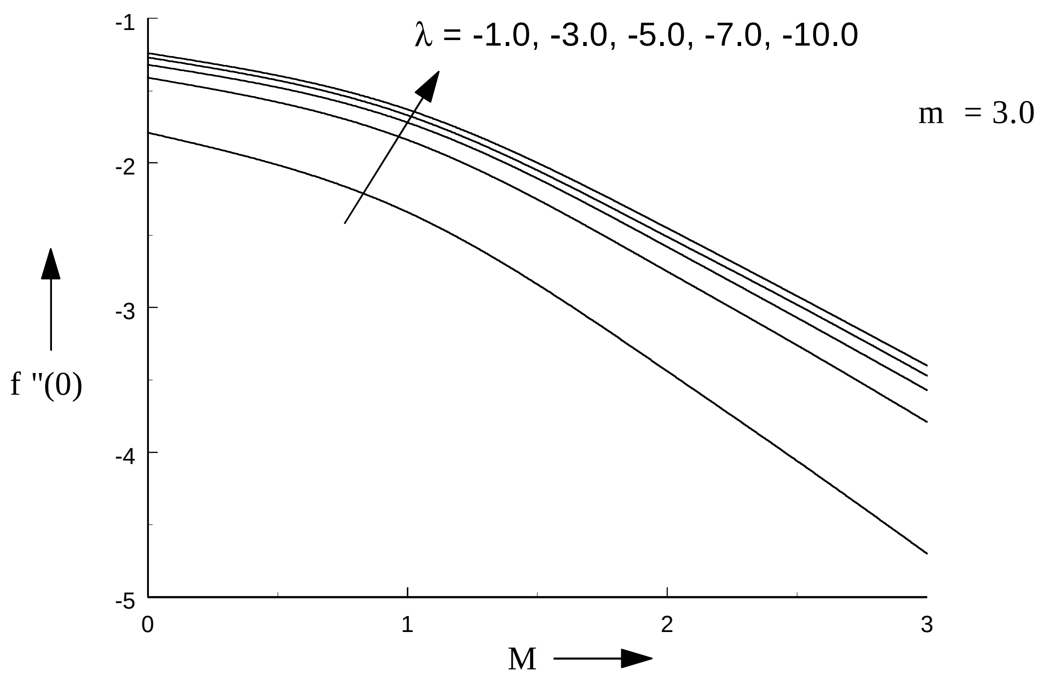

Fig.16 Skin friction coefficient for different λ

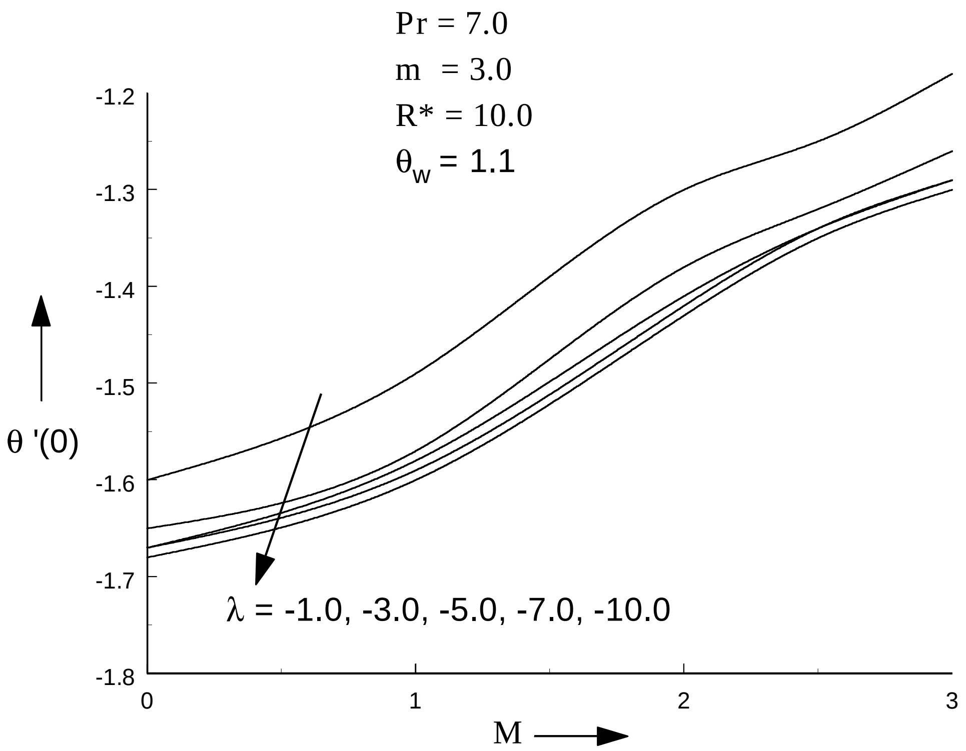

Fig.17 Dimensionless rate of heat transfer for different λ

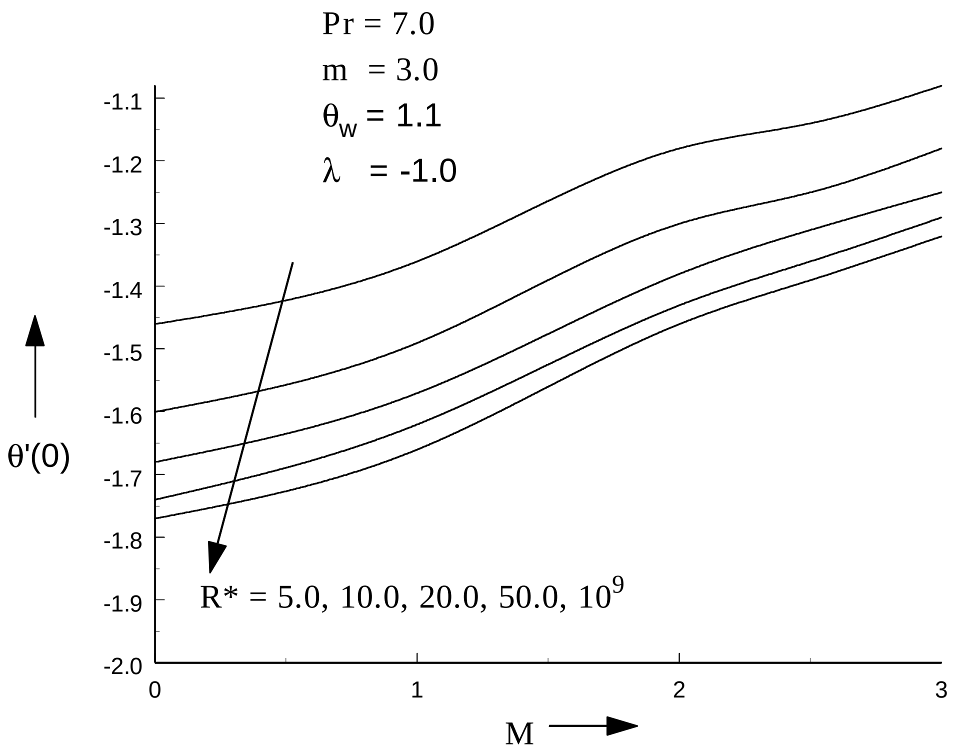

Fig.18 Dimensionless rate of heat transfer for different R*

The effect of viscosity measuring parameter λ over dimensionless velocity is seen through Fig.12. As viscosity measuring parameter λ increases in magnitude, velocity increases. Furthermore, it is interesting to note that the viscosity measuring parameter enhances the boundary layer thickness.

Figure 13 discloses the nondimensional temperature due to different values of the viscosity measuring parameter λ. It is clearly noted that increase in viscosity measuring parameter λ in magnitude, leads to decrease in temperature. In addition, the effect of increasing viscosity measuring parameter is to reduce the thickness of thermomagnetic layer.

Figure 14 displays the variation of skin friction coefficient $f''(0)$ against the magnetic interaction parameter *M* for different values of velocity exponent parameter *m*. It is seen that the skin friction coefficient $f''(0)$ decreases with increase of velocity exponent parameter *m* and it decreases for increasing magnetic interaction parameter *M*.

It is observed from Fig. 15 that the dimensionless rate of heat transfer $\theta'(0)$ increases with increase of velocity exponent parameter *m*. Further, it is noted that the dimensionless rate of heat transfer $\theta'(0)$ increases in magnitude for increasing magnetic interaction parameter *M*.

Figure 16 portrays the variation of skin friction coefficient $f''(0)$ against the magnetic interaction parameter *M* for different values of viscosity measuring parameter λ. It is inferred that the increase in magnitude of viscosity measuring parameter λ is to increase the skin friction coefficient and also the effect of increasing magnetic interaction parameter is to decrease the skin friction coefficient.

Figure 17 demonstrates the effect of magnetic interaction parameter *M* over the dimensionless rate of heat transfer $\theta'(0)$ for different values of viscosity measuring parameter λ. It is apparent that increasing viscosity measuring parameter λ (in magnitude) decreases the dimensionless rate of heat transfer $\theta'(0)$ and also an increase in magnetic interaction parameter increases the rate of heat transfer $\theta'(0)$.

Figure 18 displays the dimensionless rate of heat transfer $\theta'(0)$ against magnetic interaction parameter *M* for different R^* when the surface is nonlinearly stretching. It is seen that the dimensionless rate of heat transfer $\theta'(0)$ decreases with increase of radiation parameter R^* and increases with respect to magnetic interaction parameter *M*.

CONCLUSION

In the absence of radiation, the results are in good agreement with that of Anjali Devi and Thiagarajan [9]
The following conclusions are made in view of the above Results and Discussion.

- It is found that the effect of magnetic field is to decelerate the velocity and reduce the temperature.
- Temperature is found to reduce due to the effect of radiation parameter while it is found to increase with the increasing surface temperature parameter.
- The thermal boundary layer thickness decreases sharply with increasing Prandtl number.
- The increase in magnitude of viscosity measuring parameter λ is to increase the velocity and skin friction coefficient where as its effect is to decrease the temperature and dimensionless rate of heat transfer.
- The velocity and skin friction are decreased by the velocity exponent parameter. On the other hand, rate of heat transfer is enhanced by the velocity exponent parameter.
- It is observed that for increasing radiation parameter R^*, the dimensionless rate of heat transfer decreases.

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