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Advances in Applied Science Research, 2015, 6(9):128-143

ISSN: 0976-8610
CODEN (USA): AASRFC

Effects of thermal radiation and chemical reaction on MHD free convection flow past a moving vertical plate with heat source and convective surface boundary condition

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ABSTRACT

This paper analyzes the heat and mass transfer effects on MHD steady two-dimensional laminar free convective boundary layer flow of a viscous incompressible radiating fluid, past a moving vertical plate in the presence of chemical reaction. The left surface of the plate is in contact with hot fluid, while the stream of cold fluid flows along the right surface. The governing boundary layer equations of continuity, momentum, energy and concentration are transformed into nonlinear ordinary differential equations using similarity transformations, and then solved by Runge-Kutta method along with shooting technique. The effects of various material parameters on the velocity, temperature and concentration as well as the skin friction coefficient, Nusselt number and Sherwood number are represented in graphs and tables and discussed in detail. It is found that there is a good agreement between the present results and existing results, in the reduced cases

Key words: Boundary layer flow, Heat and Mass Transfer, Internal heat generation, Steady, Shooting method.

INTRODUCTION

Coupled heat and mass transfer finds applications in a variety of engineering applications, such as the migration of moisture through the air contained in fibrous insulation and grain storage installations, filtration, chemical catalytic reactors and processes, spreading of chemical pollutants in plants and diffusion of medicine in blood veins. Free convection flow of an incompressible viscous fluid past an infinite or semi-infinite vertical plate has been studied since long because of its technological importance. Muthucumaraswamy [1] have investigated the effects of heat and mass transfer on a continuously moving isothermal vertical surface with uniform suction. Subhashini et al. [2] analyzed the effect of mass transfer on the flow past a vertical porous plate. Unsteady free convective flow on taking into account the mass transfer phenomenon past an infinite vertical porous plate with constant suction was studied by Soundalgekar and Wavre [3]. Soundalgekar [4] presented the effects of mass transfer and free convection currents on the flow past an impulsively started vertical plate.

Magneto hydrodynamic flow has applications in meteorology, solar physics, cosmic fluid dynamics, astrophysics, geophysics and in the motion of earth's core. Shanker and Kishan [5] presented the effect of mass transfer on the MHD flow past an impulsively started infinite vertical plate. Bhaskara Reddy and Bathaiah [6,7] analyzed the Magnetohydrodynamic free convection laminar flow of an incompressible Viscoelastic fluid. Later, he was studied the MHD combined free and forced convection flow through two parallel porous walls. Elabashbeshy [8] studied heat and mass transfer along a vertical plate in the presence of magnetic field.

Analysis of transport processes and their interaction with chemical reaction has the greatest contributions to many areas of chemical science. The effect of chemical reaction on different configurations of the problem has been investigated by many authors. Gangadhar and Bhaskar Reddy [9] has considered the problem of chemically reacting MHD boundary layer flow of heat and mass transfer over a moving vertical plate in a porous medium with suction. Das *et al*. [10] have studied the effect of mass transfer flow past an impulsively started infinite vertical plate with heat flux and chemical reaction. The chemical reaction effect on heat and mass transfer flow along a semi-infinite horizontal plate has been studied by Anjalidevi and Kandaswamy [11] and later it was extended for Hiemenz flow by Seddeek et al. [12] and for polar fluid by Patil and Kulkarni[13]. Salem and Abd El-Aziz [14] have reported the effect of hall currents and chemical reaction on hydromagnetic flow of a stretching vertical surface with internal heat generation and absorption.

The heat source/sink effects in thermal convection, are significant where there may exist a high temperature differences between the surface (e.g. space craft body) and heat generation is also important in the context of exothermic or endothermic chemical reactions the ambient fluid. Sparrow and Cess [15] provided one of the earliest studies using a similarity approach for stagnation point flow with heat source/sink which vary in time. Pop and Soundalgekar[16] studied unsteady free convection flow past an infinite plate with constant suction and heat source.

The effect of radiation on MHD flow and heat transfer problem has become more important industrially. At high operating temperature, radiation effect can be quite significant. Many processes in engineering areas occur at high temperature and knowledge of radiation heat transfer becomes very important for design of reliable equipment, nuclear plants, gas turbines and various propulsion devices or aircraft, missiles, satellites and space vehicles. Based on these applications, Cogley *et al*. [17] showed that in the optically thin limit, the fluid does not absorb its own emitted radiation but the fluid does absorb radiation emitted by the boundaries. Satter and Hamid [18] investigated the unsteady free convection interaction with thermal radiation in a boundary layer flow past a vertical porous plate. Vajravelu [19] studied the flow of a steady viscous fluid and heat transfer characteristic in a porous medium by considering different heating processes. Hossain and Takhar [20] have considered the radiation effect on mixed convection boundary layer flow of an optically dense viscous incompressible fluid along a vertical plate with uniform surface temperature. Raptis [21] investigate the steady flow of a viscous fluid through a porous medium bounded by a porous plate subjected to a constant suction velocity by the presence of thermal radiation. Makinde [22] examined the transient free convection interaction with thermal radiation of an absorbing emitting fluid along moving vertical permeable plate. The effect of the chemical reaction and radiation absorption on the unsteady MHD free convection flow past a semi – infinite vertical permeable moving plate with heat source and suction has been studied by Ibrahim *et al.* [23]. Recently Bakr [24] presented an analysis on MHD free convection and mass transfer adjacent to moving vertical plate for micropolar fluid in a rotating frame of reference in presence of heat generation/ absorption and a chemical reaction. Suneetha *et al.* [25] have investigated the radiation effects on the MHD free convection flow past an impulsively started vertical plate with variable surface temperature and concentration. Chandrakala [26] discussed the free convection flow of a viscous incompressible fluid with uniform heat flux in the presence of thermal radiation. Aliakbar *et al.* [27] analysed the influence of thermal radiation on MHD flow of Maxwellian fluids above stretching sheets. Anur Ishak [28] presented the MHD Boundary Layer Flow Due to an Exponentially Stretching Sheet with Radiation Effect. Bala Anki Reddy and Bhaskar Reddy [29]studied the thermal radiation effects on hydro-magnetic flow due to an exponentially stretching sheet.

The object of present paper is to study the effects of thermal radiation and chemical reaction effects on MHD free convection flow past a moving vertical plate with heat source and convective surface boundary condition in the presence of heat generation. The governing boundary layer equations are reduced to ordinary differential equations using similarity transformations and the resulting equations are then solved numerically using Runge-Kutta fourth order method along with shooting technique. A parametric study is conducted to illustrate the influence of various governing parameters on the velocity, temperature and concentration as well as the skin-friction coefficient (surface shear stress), the local Nusselt number (surface heat transfer coefficient), the plate surface temperature and local Sherwood number (surface concentration gradient) and discussed in detail.

MATERIALS AND METHODS

MATHEMATICAL ANALYSIS

A steady two-dimensional boundary layer flow of a stream of cold viscous incompressible, electrically conducting and radiating fluid past a moving vertical flat plate in the presence of heat generation and chemical reaction, is considered. The flow is assumed to be in the direction of *x*-axis along the plate and y-axis is normal to it. The left surface of the plate is heated by convection from a hot fluid at temperature T_f that gives the heat transfer coefficient h_f, and T_∞ is the temperature of the fluid away from the plate. The cold fluid in contact with the right surface of the plate generates heat internally at the volumetric rate Q_0. A uniform magnetic field strength B_0 is applied in the transverse direction of *y*-axis. The fluid is assumed to be slightly conducting, so that the magnetic Reynolds number is much less than unity, and hence the induced magnetic field is negligible in comparison with applied magnetic field. It is assumed that there is no applied voltage which implies the absence of electrical field. The fluid is considered to be a gray, absorbing emitting radiation but non-scattering medium and the Rosseland approximation is used to describe the heat flux in the energy equation.

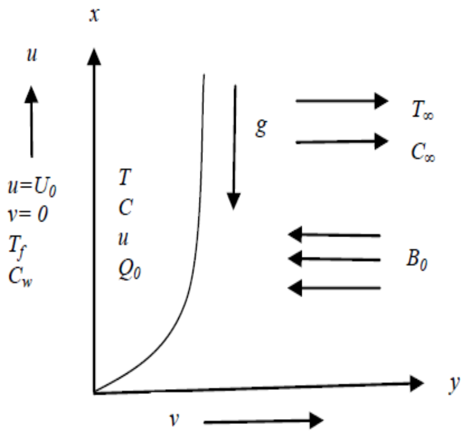

Fig.1: Schematic diagram of the physical model

Under the above assumptions and Boussinesq approximation the continuity, momentum, energy, and concentration equations describing the flow can be written as

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B_0^2}{\rho}+g\beta(T-T_\infty)+g\beta^*(C-C_\infty) \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{Q_0}{\rho C_p}(T-T_\infty)-\frac{1}{\rho C_p}\frac{\partial q_r}{\partial y} \tag{3}$$

$$u\frac{\partial C}{\partial x}+\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2}-Kr'C \tag{4}$$

where *u* and *v* denote the fluid velocity in the *x*- and *y*-directions respectively. *T* and *C* are the temperature and concentration of the fluid respectively, υ is the kinematic viscosity, α is the thermal diffusivity, *D* is the mass diffusivity, β is the thermal expansion coefficient, $\beta^*$is the solutal expansion coefficient, ρ is the fluid density, *g* is the gravitational acceleration, σ is the electrical conductivity, Q_0 is the heat source, C_p is the specific heat at constant pressure, q_r is the radiative heat flux and *Kr'* is the chemical reaction rate of the species concentration.

The third and fourth terms on the RHS of the momentum equation (2) denote the thermal and concentration buyoncy effects, respectively. Also the second and third terms on the RHS of the energy equation (3) represent the heat absorption and thermal radiation effects, respectively. The last term of the species equation (4) represents the chemical reaction effect. It is assumed that the physical properties, that is, viscosity, heat capacity, thermal diffusivity, and the mass diffusivity of the fluid remain invariant throughout the fluid.

The boundary conditions for the velocity, temperature and concentration fields at the plate surface and far into the cold fluid are

$$\begin{aligned} & u(x,0) = U_0,\ v = (x,0), \\ & -k\frac{\partial T}{\partial y}(x,0) = h_f\left[T_f - T(x,0)\right], \\ & C(x,0) = C_w = Ax^{\lambda} + C_\infty, \\ & u(x,\infty) = 0,\ T(x,\infty) = T_\infty,\ C(x,\infty) = C_\infty, \end{aligned} \tag{5}$$

where C_w is the species concentration at the plate surface, A is the constant, λ is the power index of the concentration, U_0 is the plate velocity, k is the thermal conductivity coefficient, and C_∞ is the concentration of the fluid away from the plate.

By using the Rosseland approximation(Brewster[30]), the heat flux q_r is given by

$$q_r = \frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y} \tag{6}$$

where σ^* is the Stephen-Boltzmann constant and k^* the mean absorption. It should be noted that by using the Rosseland approximation, the present analysis is limited to optically thick fluids. If the temperature differences with in the flow are sufficiently small, then equation (3) can be linearized by expanding T^4 into the Taylor series about T_∞, which after neglecting higher order terms takes the form

$$T^4 = 4T_\infty^3 T - 3T_\infty^4 \tag{7}$$

In view of the equations (2.6) and (2.7), the equation (2.3) reduces to

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \frac{Q_0}{\rho C_p}(T - T_\infty) + \frac{16\sigma^* T_\infty^3}{3k^*\rho c_p}\frac{\partial^2 T}{\partial y^2} \tag{8}$$

In order to write the governing equations and the boundary conditions in dimensionless form, the following dimensionless quantities are introduced.

$$\begin{aligned} & \eta = y\sqrt{\frac{U_0}{\upsilon x}}, \quad u = U_0 f'(\eta), \quad v = -\frac{1}{2}\sqrt{\frac{\upsilon U_0}{x}} f(\eta) + \frac{U_0 y}{2x} f'(\eta), \\ & \theta(\eta) = \frac{T - T_\infty}{T_f - T_\infty},\ \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty},\ Ha_x = \frac{\sigma B_0^2}{\rho U_0},\ Gr_x = \frac{g\beta(T_f - T_\infty)x}{U_0^2}, \\ & Gc_x = \frac{g\beta^*(C_w - C_\infty)x}{U_0^2}, \quad Bi_x = \frac{h_f}{k}\sqrt{\frac{\upsilon x}{U_0}},\ \Pr = \frac{\upsilon}{\alpha},\ Sc = \frac{\upsilon}{D},\ S_x = \frac{Q_0 x}{U_0 \rho C_p}, \\ & Kr_x = \frac{Kr' x}{U_0},\ Nc = \frac{C_\infty}{C_w - C_\infty},\ R = \frac{4\sigma^* T_\infty^3}{3k^* k} \end{aligned} \tag{9}$$

where prime denotes differentiation with respect to η, η- the similarity variable, $f(\eta)$ - the dimensionless stream function, $f'(\eta)$ - the dimensionless velocity, $\theta(\eta)$- the dimensionless temperature, $\varphi(\eta)$ - the dimensionless concentration Ha_x - the local magnetic field parameter, Gr_x - the local thermal Grashof number, Gc_x - the modified Grashof number, Bi_x - the local convective heat transfer parameter, Pr - the Prandtl number, Sc - the Schmidt number, S_x - the local heat source parameter, Kr_x - the local chemical reaction parameter, Nc - the concentration difference parameter and R - the radiation parameter.

In the view of the above similarity transformations, the equations (2), (3) and (6) reduce to

$$f''' + \frac{1}{2} ff'' - Ha_x f' + Gr_x \theta + Gc_x \varphi = 0 \tag{10}$$

$$(1+4R)\theta'' + \frac{1}{2}\Pr f\theta' + \Pr S_x \theta = 0 \tag{11}$$

$$\varphi'' + \frac{1}{2} Scf\varphi' - ScKr_x(\varphi + Nc) = 0 \tag{12}$$

The corresponding boundary conditions are

$$\begin{aligned} & f(0)=0, \\ & \theta'(0) = Bi_x\left[\theta(0)-1\right],\ \varphi(0)=1 \\ & f'(\infty)=0,\ \theta(\infty)=0,\ \varphi(\infty)=0 \end{aligned} \tag{13}$$

It can be noted that the local parameters Ha_x, Gr_x, Gc_x, Bi_x, and Kr_x and in (10) – (12) are functions of x and generate local similarity solution. In order to have a true similarity solution we assume the following relation [31]:

$$\begin{aligned} & h_f = \frac{a}{\sqrt{x}},\quad \sigma = \frac{b}{x},\quad \beta = \frac{c}{x}, \\ & \beta^* = \frac{d}{x},\quad Q_0 = \frac{e}{x},\quad Kr' = \frac{m}{x}, \end{aligned} \tag{14}$$

where a, b, c, d, e and m are the constants with appropriate dimensions. In view of relation (14) the parameters, Ha_x, Gr_x, Gc_x, Bi_x, and Kr_x, and are now independent of x and henceforth, we drop the index "x" for simplicity.

For the type of boundary layer flow under consideration, the skin-friction coefficient, Nusselt number and Sherwood number are important parameters. They are described as follows.

Knowing the velocity field, the shearing stress at the plate can be obtained, which in non-dimensional form (skin-friction coefficient) given by

$$C_f = \frac{2\tau_w}{\rho U_0^2} = \frac{2\mu}{\rho U_0^2}\left(\frac{\partial u}{\partial y}\right)_{y=0} = 2\,\mathrm{Re}_x^{-1/2} f''(0)$$

where Re_x is the Reynolds number and τ_w is the shear stress along the plate.

Knowing the temperature field, the heat transfer coefficient at the plate can be obtained, which in the non-dimensional form in terms of the Nusselt number, is given by

$$Nu = \frac{q_w x}{k(T_w - T_\infty)} = -\frac{x}{(T_w - T_\infty)}\left(\frac{\partial T}{\partial y}\right)_{y=0} = -\mathrm{Re}_x^{1/2}\theta'(0)$$

q_w is the surface heat.

Knowing the concentration field, the mass transfer coefficient at the plate can be obtained, which in the non-dimensional form in terms of the Sherwood number, is given by

$$Sh = \frac{q_m x}{k\left(C_w - C_\infty\right)} = -\frac{x}{\left(C_w - C_\infty\right)}\left(\frac{\partial C}{\partial y}\right)_{y=0} = -\mathrm{Re}_x^{1/2}\,\varphi'(0)$$

q_m is the surface mass.

METHOD OF SOLUTION

It is noticed that the energy equation (11) in presence of the radiation parameter (*R*) in the fluid yields non-homogeneous differential equation which is coupled with momentum equation (10), and in general, difficult to solve analytically. In order to overcome this difficulty, we solve these equations numerically by fourth-order Runge-Kutta method in association with shooting technique (Jain *et al*.[32]. Firstly, these equations together with associated boundary conditions are reduced to first-order differential equations. Since equations to be solved are the third order for the velocity and second order for the temperature and concentration, the values of f', θ' and φ' are needed at η=0. Therefore, the shooting method is used to solve this boundary value problem. The step size $\Delta\eta$ =0.0001 is used to obtain the numerical solution with four decimal place accuracy as the criterion of convergence. From the process of numerical computation, the skin-friction coefficient, the Nusselt number, the Sherwood number and the plate surface temperature which are respectively proportional to $f''(0)$, $\theta'(0)$, $\varphi'(0)$ and $\theta(0)$ are also sorted out and their numerical values are presented in a tabular form.

RESULTS AND DISCUSSION

In order to get a physical insight into the problem, a parametric study is conducted to illustrate the effects of different governing parameters viz magnetic field parameter *Ha*, the thermal Grashof number *Gr*, the modified Grashof number *Gc*, the convective heat transfer parameter Bi_x, the Prandtl number *Pr*, the Schmidt number *Sc*, the heat source parameter *S*, the chemical reaction parameter *Kr*, and the concentration difference parameter *Nc* upon the nature of flow and transport, and the numerical results are depicted graphically in Figs. 2-22. Here the value of *Pr* is chosen as 0.72, which corresponds to air and the values of Sc are chosen as 0.62, which corresponds to water vapor. The other parameters are chosen arbitrarily.

The effects of magnetic parameter (*Ha*) on the velocity field in presence and absence of the source and chemical reaction parameter are shown in Fig.2. It illustrates that the velocity decreases with an increase in the magnetic parameter, because the magnetic parameter is found to retard the velocity at all points of the flow field. It is because that the application of transverse magnetic field will result in a resistive type force (Lorentz force) similar to drag force which tends to resist the fluid flow and thus reducing its velocity. Also, the boundary layer thickness decreases with an increase in the magnetic parameter. In presence of the source and chemical reaction parameter no such appreciable change is observed in Fig.2. Fig.3 display the effect of the magnetic parameter on the temperature. It is noticed that the temperature of the fluid increases as the magnetic parameter increase. This is due to the fact that the applied magnetic field tends to heat the fluid, and thus reduces the heat transfer from the wall. Further it can be seen that temperature increases due to presence of the heat source as well as the chemical reaction parameter. The effect of the magnetic parameter on the concentration field is illustrated in Fig.4. It can be seen that an increase in the magnetic parameter produces significant increase in the concentration boundary layer. However, it is interesting to note that the concentration decrease with an increase of both the heat source and chemical reaction parameters.

The effect of the convective heat transfer parameter (*Bi*) on the velocity is presented in Fig.5. It is seen that with an increase in the convective heat transfer parameter, a little increase in velocity near the boundary layer is observed. This is because the fluid adjacent to the right surface of the plate becomes lighter by hot fluid and rises faster. The boundary layer flows develop adjacent to vertical surface and the velocity reaches a maximum in the boundary layer. In presence of the source and chemical reaction parameter no change is observed. The effect of the convective heat parameter on the temperature distribution in presence and absence of the heat source and the chemical reaction are depicted in Fig.6. The thermal boundary layer thickness increases with an increase in the plate surface convective heat parameter. It can be observed that the amplitude of fluid temperature in presence of the heat source and chemical reaction is more in comparison to in absence of these parameters. The effect of the convective heat transfer parameter on concentration is presented in Fig.7. It is seen that with an increase in the convective heat transfer parameter, a little decrease in the concentration in absence of the heat source and chemical reaction is observed. In presence of the source and chemical reaction parameter concentration decreases rapidly.

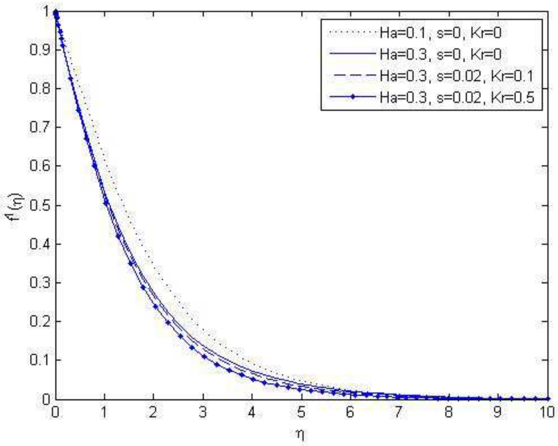

Fig.2: Velocity profiles for different values of Ha, S and Kr for Gr=0.1,Gc=0.1, Sc=0.62, Bi=0.1, Nc=0.01, Pr=0.72 and R=0.1

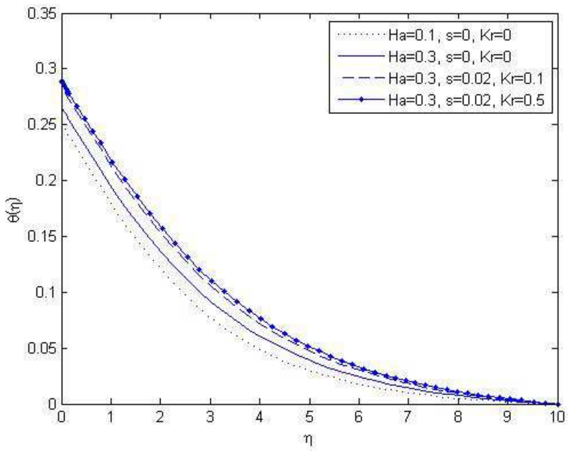

Fig.3: Temperaturre profiles for different values of Ha,S and Kr for Gr=0.1,Gc=0.1,Sc=0.62 , Bi=0.1, Nc=0.01, Pr=0.72 and R=0.1

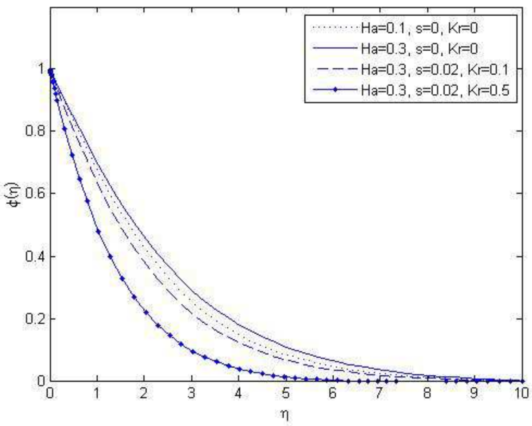

Fig.4: Concentration profiles for different values of Ha, S and Kr for Gr=0.1,Gc=0.1, Sc=0.62, Bi=0.1, Nc=0.01, Pr=0.72 and R=0.1

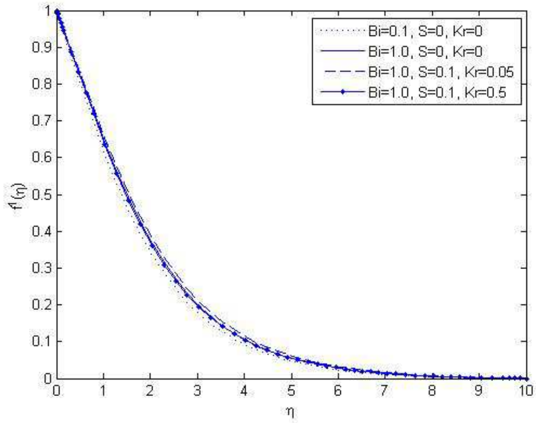

Fig5: Velocity profiles for different values of Bi,S and Kr for Ha=0.1, Gr=0.1,Gc=0.1, Sc=0.62,Nc=0.01 Pr=0.72 and R=0.1

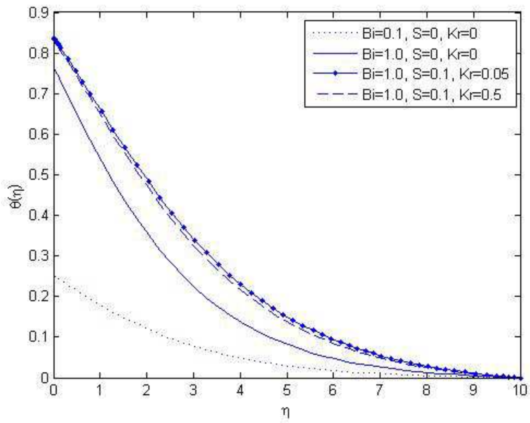

Fig.6: Temperature profiles for different values of Bi, S and Kr for Ha=0.1, Gr=0.1, Gc=0.1, Sc=0.62, Nc=0.01 Pr=0.72 and R=0.1

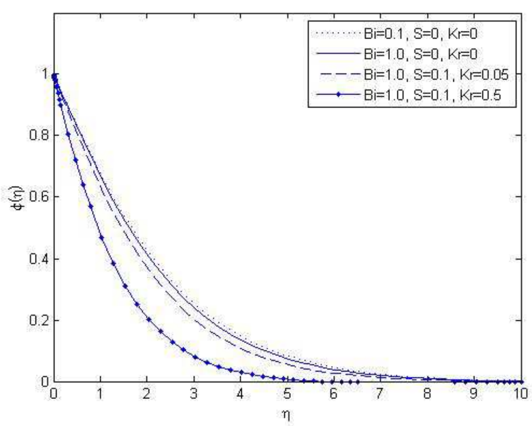

Fig.7: Concentration profiles for different values of Bi, S and Kr for Ha=0.1, Gr=0.1, Gc=0.1, Sc=0.62, Nc=0.01 Pr=0.72 and R=0.1

Fig.8 and Fig.9 illustrates the effect of the effect of the thermal Grashof number (*Gr*) and mass (solutal) Grashof number(*Gc*) on the velocity field. The thermal Grashof number signifies the relative effect of the thermal buyoncy force to the viscous hydrodynamic force and the mass (solutal) Grashof number(*Gc*) defines the ratio of species buyoncy force to the viscous hydrodynamic force in the boundary layer. It is observed that greater cooling of surface, with an increase in Gc and Gr result in an increase in the velocity. It is due to the fact that the increase in the values of Grashof number and modified Grashof number has the tendency to increase the thermal and mass buoyancy effect. The increase is also evident due to the presence of the source and chemical reaction parameters. Furthermore the velocity increases rapidly and suddenly falls near the boundary and then approaches the far field boundary condition due to favorable buoyancy force with the increase both *Gr* and *Gc*. Fig.10 and Fig.11 shows the steady state temperatures for different the Grashof number, the modified Grashof number, the internal heat source, and the chemical reaction parameters. It is observed that the thermal boundary layer decreases with an increase in the Grashof number and the modified Grashof number, but reverse effect is observed with the presence of the heat source and chemical reaction parameter. The effect of the buoyancy parameters (*Gr,Gc*) on the concentration field is illustrated in Fig.12 and Fig.13. It is noticed that the concentration boundary layer thickness decreases with an increase in the thermal and solutal Grashof numbers (*Gr* or *Gc*). It is due to the fact that an increase in the values of the Grashof number and the modified Grashof number has the tendency to increase the mass buoyancy effect. This gives rise to an increase in the induced flow and there by decreases the concentration.

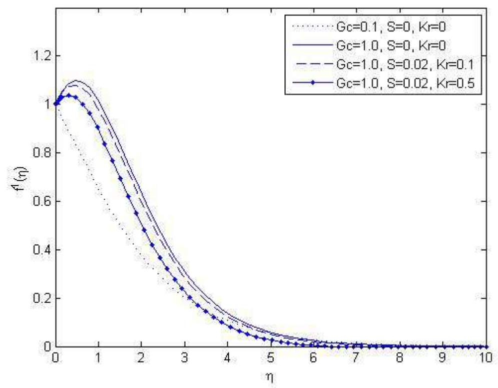

Fig.8: Velocity profiles for different values of Gc, S and Kr for Ha=0.1, Gr=0.1, Sc=0.62, Bi=0.1,Nc=0.01 Pr=0.72 and R=0.1

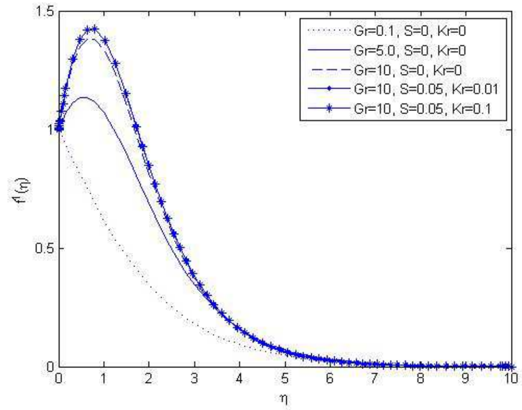

Fig.9: Velocity profiles for different values of Gr, S and Kr for Ha=0.1, Gc=0.1, Sc=0.62, Bi=0.1, Nc=0.01, Pr=0.72 and R=0.1

The effect of the Schmidt number (*Sc*) on the velocity, temperature and concentration are shown in Figs.14-16, respectively. The Schmidt number embodies the ratio of momentum to mass diffusivity. The Schmidt number quantifies the relative effectiveness of momentum and mass transport by diffusion in the hydrodynamic (velocity) and concentration (species) boundary layers. As the Schmidt number increases the concentration a decreases. This causes the concentration buoyancy effects to decreases yielding reduction in the fluid velocity. In presence of the source and chemical reaction parameter no such appreciable change is observed in Fig.14. The effect of the Schmidt number on the temperature distribution in presence and absence of the heat source and chemical reaction are depicted in Fig.15.The thermal boundary layer thickness increases with an increase of Schmidt number. It can be observed that the amplitude of fluid temperature in presence of the heat source and the chemical reaction is more in comparison to in absence of these parameters. Fig.16 depicts the effect of the Schmidt number on the concentration. Like temperature, the concentration value is higher at the surface and falls exponentially. The concentration decreases in presence of the source and the chemical reaction parameter.

Fig.17 shows that there is no significant effect on velocity profile with an increase in the Prandtl number (*Pr*) in presence and in absence of the source and chemical reaction parameter. From Fig.18, it is noticed that an increase in the Prandtl number results in decrease of the thermal boundary layer, and hence temperature decreases. The reason is that, smaller values of Pr are equivalent to increase the thermal conductivities, and therefore heat is able to diffuse away from the heated surface more rapidly than for higher values of *Pr*. But in presence of source and chemical reaction parameter a little decrease in temperature profile is observed. Fig.19 displays the effect of *Pr* on concentration profile against with the variation of source and chemical reaction parameters. The magnitude of concentration is higher at the plate and then decays to zero asymptotically.

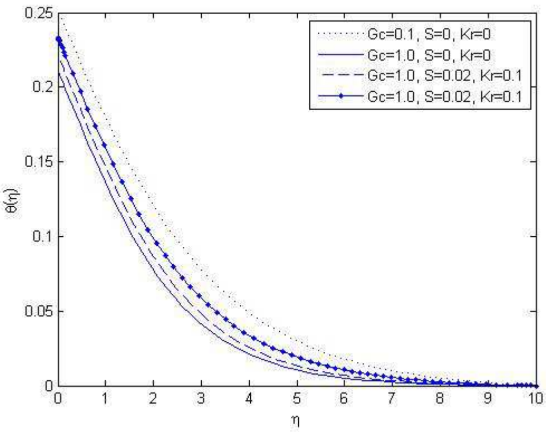

Fig.10: Temperature profiles for different values of Gc, S and Kr for Ha=0.1, Gr=0.1, Sc=0.62, Bi=0.1,Nc=0.01

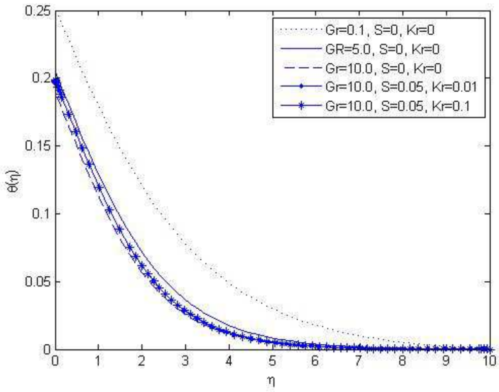

Fig.11: Temperature profiles for different values of Gr, S and Kr for Ha=0.1, Gc=0.1, Sc=0.62, Bi=0.1, Nc=0.01, Pr=0.72 and R=0.1

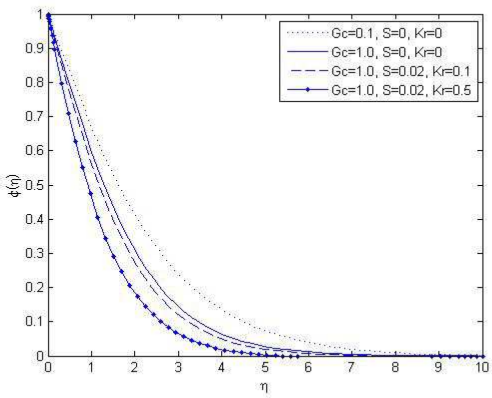

Fig.12: Concentration profiles for different values of Gc, S and Kr for Ha=0.1, Gr=0.1, Sc=0.62, Bi=0.1, Nc=0.01 Pr=0.72 and R=0.1

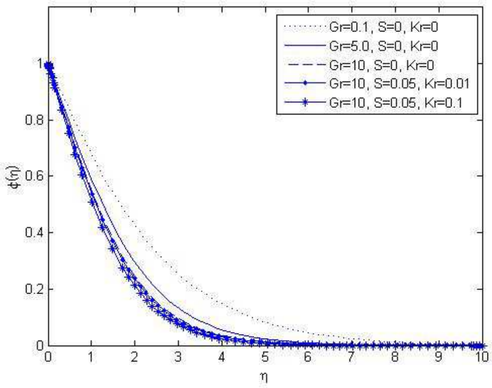

Fig.13: Concentration profiles for different values of Gr, S and Kr for Ha=0.1, Gc=0.1, Sc=0.62, Bi=0.1, Nc=0.01, Pr=0.72 and R=0.1

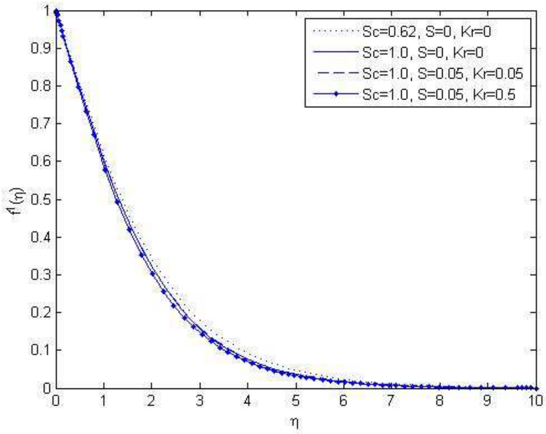

Fig.14: Velocity profiles for different values of Sc, S and Kr for Ha=0.1, Gr=0.1, Gc=0.1, Bi=0.1, Nc=0.01, Pr=0.72 and R=0.1

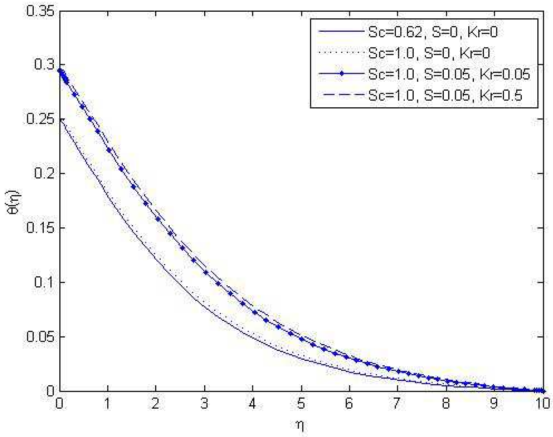

Fig.15: Temperature profiles for different values of Sc, S and Kr for Ha=0.1,Gr=0.1,Gc=0.1, Bi =0.1, Nc=0.01 Pr=0.72 and R=0.1

Fig.21 shows that the velocity distribution for different values of radiation parameter parameter (*R*). It is observed that the velocity increases as the radiation parameter increases in absence of heat source and chemical reaction. It reaches the steady state for higher values of radiation parameter in presence of heat source and chemical reaction. The influence of the thermal radiation parameter *R* on the temperature is shown in the Fig.22. The radiation parameter R defines the relative contribution of conduction heat transfer to thermal radiation transfer. It is observed that as *R* increases, the temperature profiles as well as the thermal boundary layer thickness increases in presence and absence of heat source and chemical reaction.

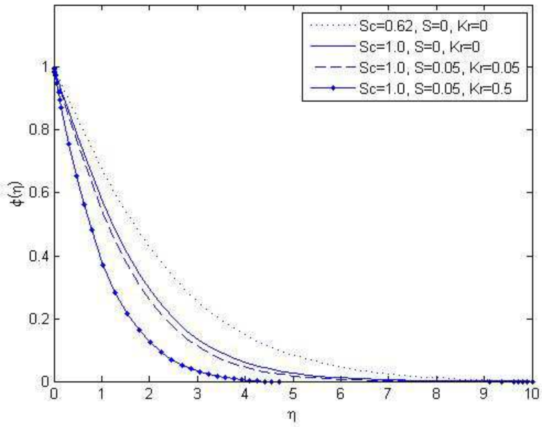

Fig.16: Concentration profiles for different values of Sc, S and Kr for Ha=0.1, Gr=0.1, Gc=0.1, Bi=0.1, Nc=0.01, Pr=0.72 and R=0.1

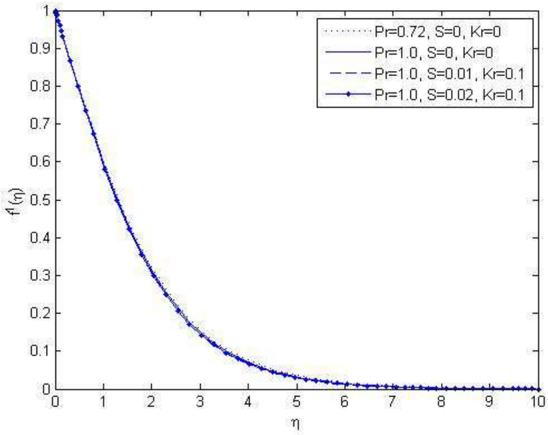

Fig.17: Velocity profiles for different values of Pr, S and Kr for Ha=0.1,Gr=0.1,Gc=0.1, Bi =0.1, Sc =0.62, Nc=0.01 and R=0.1

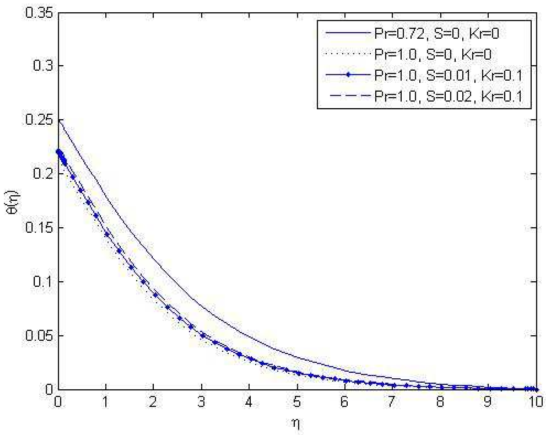

Fig.18: Temperature profiles for different values of S Pr, S and Kr for Ha=0.1, Gr=0.1, Gc=0.1, Bi=0.1, Sc=0.62, Pr=0.72 and R=0.1

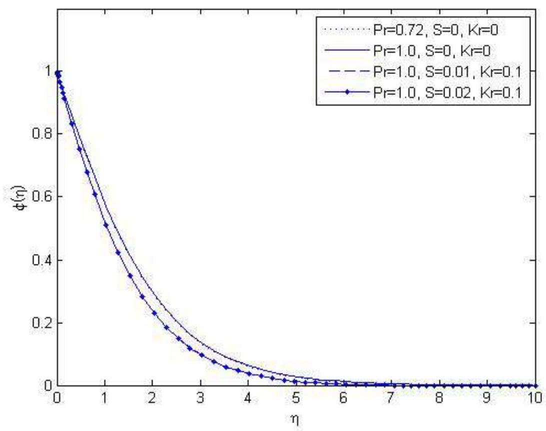

Fig.19: Concentration profiles for different values of Pr, S and Kr for Ha=0.1,Gr=0.1,Gc=0.1, Bi =0.1, Sc =0.62, Nc=0.01 and R=0.1

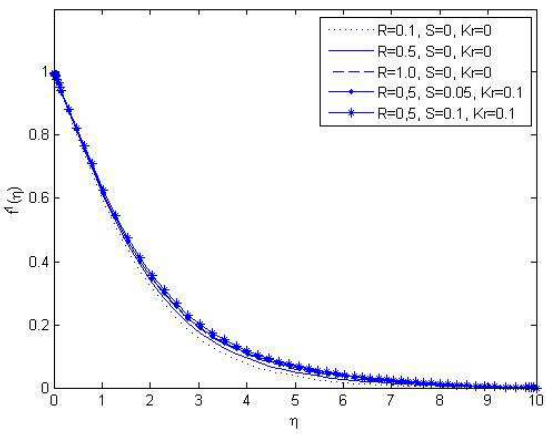

Fig.20: Velocity profiles for different values of R, S and Kr for Ha=0.1, Gr=0.1,Gc=0.1, Sc=0.62, Bi=0.1 Nc=0.01, and Pr=0.72

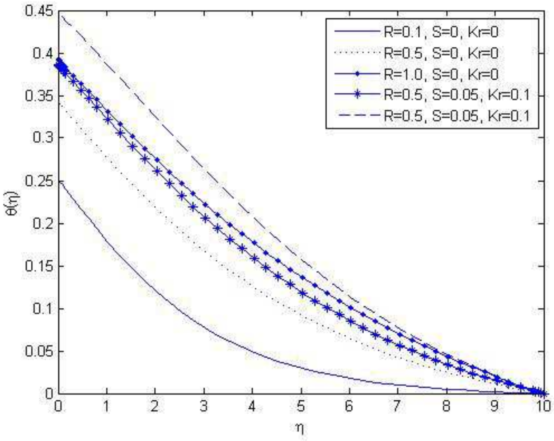

Fig.21: Temperature profiles for different values of R, S, and Kr for Ha=0.1,Gr=0.1,Gc=0.1,Sc=0.62, Bi=0.1, Nc=0.01 and Pr=0.72

Numerical results for the skin-friction coefficient, the Nusselt number, the Sherwood number and the plate surface temperature for various values of physical parameters are reported in Tables. It is observed that in absence of radiation parameter that is, for R=0; (10), (11) and (12) together with boundary condition (13) are the same as those obtained by Rout [32]. The present results are compared with that of Rout *et al* [32] for the local skin friction coefficient, the Nusselt numbers, the plate surface temperature and the local Sherwood number in Table-1 for reduced case R=0 and found that there is an excellent agreement. It is important to note that the momentum equation is coupled with heat and mass transfer equations and hence the Prandtl number, Schmidt number, chemical reaction parameter, and source term have an influence on skin friction in our present problem.

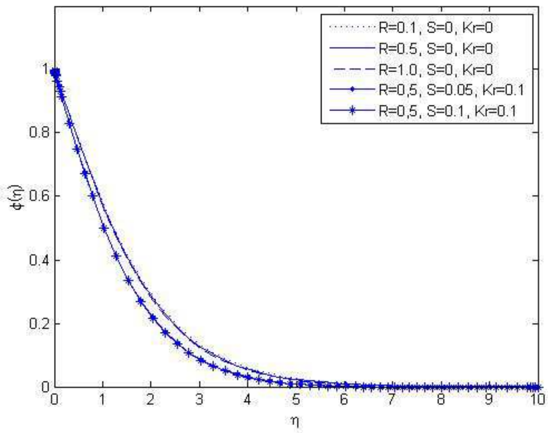

Fig.22: Concentration profiles for different values of R, S and Kr for Ha=0.1, Gr=0.1,Gc=0.1, Sc=0.62, Bi=0.1, Nc=0.01, and Pr=0.72

Table 2 shows the analysis of the local skin friction coefficient, the local Nusselt number, the plate surface temperature, and the Sherwood number for various values of the physical parameters. It is seen that the values of $f''(0)$ are always negative. Physically, negative sign of $f''(0)$ implies that plate exerts a drag force on the fluid that causes the movement of the fluid on the surface. It is observed that as the local convective heat transfer parameter increases the magnitude of the local skin friction coefficient, the Nusselt number, the Sherwood number and the plate surface temperature increase. It is observed that as the Prandtl number increases the Nusselt number increase, while the skin friction coefficient, the plate surface temperature and Sherwood number decreases. As the heat source parameter increases, the magnitude of the skin friction coefficient, the Sherwood number and the plate surface temperature increase, while the Nusselt Number decreases. It can be seen that the magnitude of the local skin friction coefficient, the Sherwood number and the plate surface temperature increase where as the Nusselt number decreases with an increase in the radiation parameter.

With the data in the first two rows of Table 3, it is noticed that as the chemical reaction parameter increases the skin friction coefficient, the Nusselt number, the plate surface temperature and the Sherwood number decrease. With the data in the last three rows of the Table 3 as Schmidt number increases the Nusselt number and the Sherwood number increase while the skin friction coefficient and the plate surface temperature decreases.

Table I: Comparison of the present results with that of Rout et al.[33]

Bi	*Gr*	*Gc*	*Ha*	*Pr*	*Sc*	$f''(0)$	$-\theta'(0)$	$\theta(0)$	$-\varphi'(0)$	$f''(0)$	$-\theta'(0)$	$\theta(0)$	$-\varphi'(0)$
0.1	0.1	0.1	0.1	0.72	0.62	-0.402271	0.078635	0.213643	0.3337425	-0.402271	0.078635	0.213643	0.333742
1.0	0.1	0.1	0.1	0.72	0.62	-0.352136	0.273153	0.726846	0.3410294	-0.352136	0.273153	0.726846	0.341029
0.1	0.5	0.1	0.1	0.72	0.62	-0.322212	0.079173	0.208264	0.3451301	-0.322212	0.079173	0.208264	0.345130
0.1	1.0	0.1	0.1	0.72	0.62	-0.231251	0.079691	0.203088	0.3566654	-0.231251	0.079691	0.203088	0.356665
0.1	0.1	0.5	0.1	0.72	0.62	-0.026410	0.080711	0.192889	0.3813954	-0.026410	0.080711	0.192889	0.381395
0.1	0.1	1.0	0.1	0.72	0.62	-0.379918	0.082040	0.179592	0.4176699	-0.379918	0.082040	0.179592	0.417669
0.1	0.1	0.1	0.1	1.0	0.62	-0.407908	0.081935	0.180640	0.3325180	-0.407908	0.081935	0.180640	0.332518
0.1	0.1	0.1	0.1	0.72	0.78	-0.411704	0.078484	0.215159	0.3844559	-0.411704	0.078484	0.215159	0.384455

Table II: Numerical values of the skin friction $f''(0)$, the Nusselt number $\theta'(0)$, the plate surface temperature $\theta(0)$, and the Sherwood number $-\varphi'(0)$ for Ha=0.1, Gr=0.1, Gc=0.1, Nc=0.01, Sc=0.62 and Kr=0.1

Bi	*Pr*	*S*	*R*	$f''(0)$	$-\theta'(0)$	$\theta(0)$	$-\varphi'(0)$
0.1	0.72	0.1	0.1	-0.341462	0.141510	0.716980	0.431615
1.0	0.72	0.1	0.1	-0.327392	0.167453	0.832547	0.433803
1.0	1.0	0.1	0.1	-0.552561	0.527388	0.472612	0.363068
1.0	0.72	0.3	0.1	-0.445453	0.335706	0.664294	0.402446
1.0	0.72	0.5	0.1	-0.356520	0.068822	0.931178	0.421469
1.0	0.72	0.1	0.1	-0.552561	0.527388	0.472612	0.363068
1.0	0.72	0.1	0.5	-0.309078	0.134049	0.865951	0.437040

Table.III: Numerical values of the skin friction $f''(0)$, the Nusselt number $\theta'(0)$, the plate surface temperature $\theta(0)$, and the Sherwood number $-\varphi'(0)$ for Ha=0.1, Gr=0.1, Gc=0.1, Bi=1.0, Nc=0.01, Pr=0.62 and R=0.1

Sc	*Kr*	*f''(0)*	*-θ'(0)*	*θ(0)*	*-φ'(0)*
0.62	0.1	-0.280727	0.096062	0.903938	0.443107
0.62	0.5	-0.298034	0.095509	0.904491	0.672872
0.62	0.1	-0.280727	0.096062	0.903938	0.443107
0.78	0.1	-0.334601	0.165182	0.834818	0.493644
1.0	0.1	-0.342029	0.836929	0.163071	0.567942

CONCLUSION

A steady two-dimensional boundary layer model has been developed for the flow of a hydromagnetic, viscous, incompressible fluid past a moving vertical flat plate in presence of heat source, thermal radiation and chemical reaction with convective surface boundary condition. The governing boundary layer equations are reduced to nonlinear ordinary differential equations using similarity transformations and the resulting equations are then solved numerically using Runge-Kutta fourth order method along with shooting technique. A parametric study is conducted to illustrate the behavior of various physical quantities for different values of the governing parameters and the results are summarized as follows.

- The Skin friction coefficient, the mass transfer rate and the plate surface temperature increase, while heat transfer rate decrease for the increasing values of the heat source parameter or radiation parameter.
- An increase in the strength of chemical reacting substances causes an increase in the magnitude of the plate surface temperature and Sherwood number but opposite behavior is seen for the case of the Skin friction coefficient and local Nusselt number.
- With an increase in the magnetic parameter, the velocity decreases in the absence of heat source and chemical reaction parameter and in presence of source and chemical reaction parameter no such appreciable change is observed. While temperature and concentration increases in absence of source and chemical reaction parameter. Further it can be seen that temperature increases, concentration decrease due to increase of heat source as well as chemical reaction parameter.
- The thermal boundary layer thickness increases with an increase in the heat source parameter, chemical reaction parameter, plate surface convective heat parameter Schmidt number and Prandtl number, hence temperature increases while the mass flux boundary layer thickness decreases.
- As the Prandtl number increases, Nusselt number increase, while the skin friction coefficient, the plate surface temperature and Sherwood number decrease.
- An increase in the radiation parameter leads to a little increase in the velocity, but in the presence of heat source and chemical reaction parameter no change is observed. It is also observed that the temperature increases in presence and absence of source and chemical reaction parameter.

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