

Effects of radiation on unsteady MHD free convective flow past an oscillating vertical porous plate embedded in a porous medium with oscillatory heat flux

Sib Sankar Manna¹, Sanatan Das² and Rabindra Nath Jana¹

¹Department of Applied Mathematics, Vidyasagar University, Midnapore 721 102, India

²Department of Mathematics, University of Gour Banga, Malda 732 103, India

ABSTRACT

The effects of radiation on unsteady MHD free convective flow of a viscous incompressible electrically conducting fluid past an oscillating vertical porous plate embedded in a porous medium with an oscillatory heat flux in the presence of a uniform transverse magnetic field have been studied. The governing equations describing the flow are solved analytically. It is observed that the fluid velocity is effected by the magnetic parameter. An increase in radiation parameter leads to decrease the fluid velocity near the plate and to increase away from the plate. The fluid velocity increases near the plate and it decreases away from the plate with an increase in suction parameter. The solution exists for both suction and blowing at the plate. The fluid velocity increases near the plate and it decreases away from the plate with an increase in Darcy number. It is seen that the fluid temperature decreases near the plate and it increases away from the plate with an increase in either radiation parameter or Prandtl number or suction parameter or frequency parameter. Further, it is seen that the amplitude of the shear stress and the tangent of the phase angle at the plate decreases with an increase in either radiation parameter or Prandtl number or suction parameter.

Keywords: MHD free convection, Darcy number, Prandtl number, heat flux, suction/blowing and phase angle.

INTRODUCTION

Magnetohydrodynamic free convection flow through porous media are very important particularly in the fields of petroleum technology for the flow of oil through porous rocks, in chemical engineering for the purification and filtration processes and in the cases like drug permeation through human skin. The principles of this subject are very useful in recovering the water for drinking and irrigation purposes. The knowledge of flows through porous medium is also useful to study the movement of natural gas and water through the oil reservoirs. A number of studies have appeared in the literature where the porous medium is either bounded between parallel plates. At the high temperature attained in some engineering devices, gas, for example, can be ionized and so becomes an electrical conductor. The ionized gas or plasma can be made to interact with the magnetic field and alter heat transfer and friction characteristic. Recently, it is of great interest to study the effect of magnetic field on the temperature distribution and heat transfer when the fluid is not only an electrical conductor but also when it is capable of emitting and absorbing thermal radiation. The heat transfer by thermal radiation is becoming of greater importance when we are concerned with space applications, higher operating temperatures and also power engineering. The radiative free convective flows are encountered in countless industrial and environment processes e.g. heating and cooling chambers, fossil fuel combustion energy processes, evaporation from large open water reservoirs, astrophysical flows, solar power technology and space vehicle re-entry. The radiative heat transfer play an important role in manufacturing industries for the design of reliable equipment. Nuclear power plants, gas turbines and various propulsion device for aircraft,

missiles, satellites and space vehicles are examples of such engineering applications. Such a flow past an infinite vertical plate oscillating in its own plane was first studied by Soundalgekar [1] in case of an isothermal plate. Mansour [2] has studied the interaction of free convection with thermal radiation of the oscillatory flow past a vertical plate. Gholizadeh [3] has presented the MHD oscillatory flow past a vertical porous plate through porous medium in the presence of thermal and mass diffusion with constant heat source. Jha [4] has investigated the role of magnetic field on transient forced and free convection flow past an infinite vertical porous plate through a porous medium with heat source. The flow past an infinite vertical oscillating porous plate embedded in a porous medium have been described by Jaiswal and Soundalgekar [5]. Zhang et al [6] have studied the free convection effects on a heated vertical plate subjected to a periodic oscillation of the plate. The effects of thermal radiation on the flow past an oscillating plate with variable temperature have been studied by Pathak et al. [7]. Sharma et al. [8] have studied the unsteady free convection oscillatory couette flow through a porous medium with periodic wall temperature. The free convection effects on a vertical oscillating porous plate with constant heating have been studied by Toki [9]. Chandrakala [10] has investigated the radiation effects on the flow past an impulsively started vertical oscillating plate with uniform heat flux. In many industrial applications, hydromagnetic flows also occur at very high temperatures in which thermal radiation effects become significant. Radiative MHD convective flows are also important in astrophysical and geophysical regimes. Soundalgekar [11] has studies in hydromagnetic natural convection flow past a vertical surface. Helmy [12] has investigated MHD unsteady free convective flow past a vertical porous plate. Hossain et al. [13] have analyzed the heat transfer response of MHD free convective flow along a vertical plate to surface temperature oscillations. Kim [14] has founded an unsteady MHD convective heat transfer past a semi-infinite vertical porous moving plate with variable suction. Singh et al. [15] have studied the free convection MHD flow of a rotating viscous fluid in a porous medium past a vertical porous plate. The hydrodynamic free convective flow of an optically thin gray gas in the presence of radiation, when the induced magnetic field is taken into account has studied by Raptis et al. [16]. Chandrakala and Raj [17] have studied the effects of thermal radiation on the flow past a semi infinite vertical isothermal plate with uniform heat flux in the presence of transversely applied magnetic field. Chaudhary and Jain [18] have presented the magnetohydrodynamic transient convective flow past a vertical surface embedded in a porous medium with oscillating plate temperature. Chaudhary and Jain [19] have studied the combined heat and mass transfer effects on MHD free convection flow past an oscillating plate embedded in a porous medium. Chandrakala and Bhaskar [20] studied the effects of thermal radiation on the flow past an infinite vertical oscillating isothermal plate in the presence of transversely applied magnetic field. Abd El-Naby et al. [21] investigated magnetohydrodynamic transient natural convection-radiation boundary layer flow with variable surface temperature, showing that velocity, temperature, and skin friction are enhanced with a rise in radiation parameter, whereas Nusselt number is reduced. MHD flow over a moving infinite vertical porous plate with uniform heat flux in the presence of thermal radiation has been investigated by Rani and Murthy [22]. Das [23] has analyzed the exact solution of MHD free convection flow and mass transfer near a moving vertical plate in the presence of thermal radiation. Krishna and Sujatha [24] have studied the MHD free and forced convection flow of Newtonian fluid through a porous medium past an infinite vertical plate in the presence of thermal radiative heat transfer and surface temperature oscillations. Reddy et al. [25] have presented the radiation and chemical reaction effects on free convection MHD flow through a porous medium bounded by vertical surface. The unsteady MHD heat and mass transfer free convection flow of polar fluids past a vertical moving porous plate in a porous medium with heat generation and thermal diffusion has been studied by Saxena and Dubey [26]. The mass transfer effects on MHD mixed convective flow from a vertical surface with Ohmic heating and viscous dissipation has been investigated by Babu and Reddy [27]. Saxena and Dubey [28] have analyzed the effects of MHD free convection heat and mass transfer flow of visco-elastic fluid embedded in a porous medium of variable permeability with radiation effect and heat source in slip flow regime. Devi and Gururaj [29] have studied the effects of variable viscosity and nonlinear radiation on MHD flow with heat transfer over a surface stretching with a power-law velocity. The radiation effects on the unsteady MHD convection flow through a non uniform horizontal channel have been studied by Reddy et al. [30]. The effects of thermal radiation and viscous dissipation on MHD heat and mass diffusion flow past an oscillating vertical plate embedded in a porous medium with variable surface conditions have been analyzed by Kishore et al. [31]. Reddy and Reddy [32] have described the MHD oscillatory flow past a vertical porous plate embedded in a rotating porous medium.

In this paper, we study the radiation effects on MHD free convection flow of a viscous incompressible electrically conducting fluid past an oscillating vertical porous plate embedded in a porous medium in the presence of a uniform transverse magnetic field. The plate is oscillating in its own plane with a velocity $u_0 \cos \omega t$, ω being the frequency of the oscillations and u_0 is positive constant. The heat flux at the plate is $-\frac{q}{k} \cos \omega t$, q and k are positive

constants. A uniform magnetic field of strength B_0 is imposed perpendicular to the plate. The governing equations along with the boundary conditions are solved analytically. It is observed that the fluid velocity u_1 decreases near the plate and it increases away from the plate with an increase in either magnetic parameter M^2 or radiation parameter R or Prandtl number Pr or suction parameter S or frequency parameter n or phase angle $n\tau$. The velocity u_1 increases near the plate and it decreases away from the plate with an increase in Darcy number Da . The fluid velocity u_1 increases with an increase in Grashof number Gr . It is also observed that the solution exists for the blowing at the plate. It is seen that the temperature distribution θ decreases near the plate and it increases away from the plate with an increase in either R or Pr or S or n . Further, it is seen that the amplitude of the shear stress R_0 and the tangent of the phase ϕ at the plate decreases with an increase in either R or Pr or S .

Mathematical formulation and its solution

Consider the unsteady flow of a viscous incompressible electrically conducting fluid past an oscillating vertical porous plate embedded in a porous medium with uniform suction or blowing at the plate. The plate oscillates in its own plane with the velocity $u_0 \cos \omega t$ in a given direction. We choose the x -axis along the plate, in the vertical upward direction and y -axis perpendicular to the plate. An external uniform magnetic field of strength B_0 is imposed perpendicular to the plate [See Fig.1] and the plate is taken electrically non-conducting. Thermal radiation acts as a unidirectional flux in the y -direction. The fluid is gray and absorbing-emitting but non-scattering and the magnetic Reynolds number is assumed to be small so that induced magnetic field can be neglected. The velocity components are $(u, v, 0)$ relative to a frame of reference. Since the plate is infinitely long, all the physical quantities will be the function of y and t only. The equation of continuity $\nabla \cdot \vec{q} = 0$ gives $\frac{\partial v}{\partial y} = 0$ which on integration yields $v = -v_0(\text{constant})$, where $\vec{q} \equiv (u, v, 0)$. The constant v_0 denotes the normal velocity at the plate which is positive for suction and negative for blowing. The solenoidal relation $\nabla \cdot \vec{B} = 0$ gives $B_0 = \text{constant}$ everywhere in the fluid where $\vec{B} \equiv (0, B_y, 0)$.

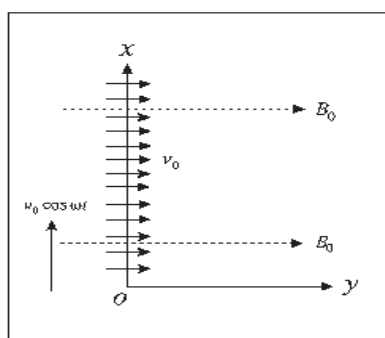


Fig.1: Geometry of the problem

Under usual Boussinesq approximations, the flow of a radiating gas is governed by the following set of equations

$$\frac{\partial u}{\partial t} - v_0 \frac{\partial u}{\partial y} = g\beta(T - T_\infty) + \nu \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho} u - \frac{\mu}{\rho k^*} u, \quad (1)$$

$$\rho c_p \left(\frac{\partial T}{\partial t} - v_0 \frac{\partial T}{\partial y} \right) = k \frac{\partial^2 T}{\partial y^2} - \frac{\partial q_r}{\partial y}, \quad (2)$$

where u is the velocity in the x -direction, g the acceleration due to gravity, β the coefficient of thermal expansion, ν the kinematic viscosity, ρ the fluid density, k the thermal conductivity, c_p the specific heat at

constant pressure and q_r the radiative heat flux.

The boundary conditions of the problem are

$$u = u_0 \cos \omega t, \quad \frac{\partial T}{\partial y} = -\frac{q}{k} \cos \omega t \quad \text{at } y = 0 \quad \text{and} \quad u \rightarrow 0, \quad T \rightarrow T_\infty \quad \text{as } y \rightarrow \infty, \quad (3)$$

where u_0 is a positive constant.

The radiative heat flux can be found from Rosseland approximation [33] and its formula is derived from the diffusion concept of radiative heat transfer in the following way

$$q_r = -\frac{4\sigma}{3k^*} \frac{\partial T^4}{\partial y}, \quad (4)$$

where σ is the Stefan-Boltzman constant and k^* the spectral mean absorption coefficient of the medium. It should be noted that by using the Rosseland approximation, we limit our analysis to optically thick fluids. If the temperature differences within the flow are sufficiently small, then equation (4) can be linearized by expanding T^4 into the Taylor series about T_∞ and neglecting higher order terms to give:

$$T^4 = 4T_\infty^3 T - 3T_\infty^4. \quad (5)$$

It is emphasized here that the equation (5) is widely used in computational fluid dynamics involving radiation absorption problems [34] in expressing the term T^4 as a linear function.

On the use of (4) and (5), equation (2) becomes

$$\rho c_p \left(\frac{\partial T}{\partial t} - v_0 \frac{\partial T}{\partial y} \right) = k \frac{\partial^2 T}{\partial y^2} + \frac{16\sigma^* T_\infty^3}{3k^*} \frac{\partial^2 T}{\partial y^2}. \quad (6)$$

Introducing the non-dimensional variables

$$u_1 = \frac{u}{u_0}, \quad \tau = \frac{u_0^2}{\nu} t, \quad \eta = \frac{u_0}{\nu} y, \quad \theta = \frac{(T - T_\infty) k u_0}{q \nu}, \quad (7)$$

equations (1) and (6) become

$$\frac{\partial u_1}{\partial \tau} - S \frac{\partial u_1}{\partial \eta} = Gr \theta + \frac{\partial^2 u_1}{\partial \eta^2} - \left(M^2 + \frac{1}{Da} \right) u_1, \quad (8)$$

$$\alpha \left(\frac{\partial \theta}{\partial \tau} - S \frac{\partial \theta}{\partial \eta} \right) = \frac{\partial^2 \theta}{\partial \eta^2}, \quad (9)$$

where $M^2 = \frac{\sigma B_0^2 \nu}{\rho u_0^2}$ is the magnetic parameter, $Pr = \frac{\rho c_p \nu}{k}$ the Prandtl number, $Gr = \frac{g \beta q \nu^2}{k u_0^4}$ the Grashof

number, $S = \frac{v_0}{u_0}$ the suction parameter, $Da = \frac{u_0^2 k^*}{\nu^2}$ the Darcy number, $n = \frac{\nu \omega}{u_0^2}$ the frequency parameter and

$$\alpha = \frac{3RPr}{3R+4}.$$

On the use of (7), the boundary conditions (3) become

$$u_1 = \frac{1}{2} (e^{in\tau} + e^{-in\tau}), \quad \frac{\partial \theta}{\partial \eta} = -\frac{1}{2} (e^{in\tau} + e^{-in\tau}) \quad \text{at } \eta = 0. \quad (10)$$

$u_1 \rightarrow 0, \theta \rightarrow 0$ as $\eta \rightarrow \infty$.

To solve equations (8) and (9) subject to the boundary conditions (10), we assume the solution in the following form

$$\theta(\eta, \tau) = g_1(\eta)e^{i\tau} + \bar{g}_1(\eta)e^{-i\tau}, \quad (11)$$

$$u_1(\eta, \tau) = f_1(\eta)e^{i\tau} + \bar{f}_1(\eta)e^{-i\tau}. \quad (12)$$

Substituting (11) and (12) in the equations (8) and (9) we find that $f_1(\eta)$, $\bar{f}_1(\eta)$, $g_1(\eta)$ and $\bar{g}_1(\eta)$ satisfy the following equations

$$\alpha[ig_1(\eta) - Sg_1'(\eta)] = g_1''(\eta), \quad (13)$$

$$\alpha[-in\bar{g}_1(\eta) - S\bar{g}_1'(\eta)] = \bar{g}_1''(\eta), \quad (14)$$

$$in f_1(\eta) - S f_1'(\eta) = Gr g_1(\eta) + f_1''(\eta) - \left(M^2 + \frac{1}{Da}\right) f_1(\eta), \quad (15)$$

$$-in \bar{f}_1(\eta) - S \bar{f}_1'(\eta) = Gr \bar{g}_1(\eta) + \bar{f}_1''(\eta) - \left(M^2 + \frac{1}{Da}\right) \bar{f}_1(\eta), \quad (16)$$

where prime denotes differentiation with respect to η .

The corresponding boundary conditions for $f_1(\eta)$, $\bar{f}_1(\eta)$, $g_1(\eta)$ and $\bar{g}_1(\eta)$ are

$$\begin{aligned} f_1(0) = \bar{f}_1(0) &= \frac{1}{2}, \quad g_1'(0) = \bar{g}_1'(0) = -\frac{1}{2}, \\ f_1 \rightarrow 0, \quad \bar{f}_1 \rightarrow 0 \text{ and } g_1 \rightarrow 0, \quad \bar{g}_1 \rightarrow 0 \text{ as } \eta \rightarrow \infty. \end{aligned} \quad (17)$$

The solution of the equations (13) to (16) subject to the boundary conditions (17) are

$$g_1(\eta) = \frac{1}{2(\alpha_1 + i\beta_1)} e^{-(\alpha_1 + i\beta_1)\eta}, \quad \bar{g}_1(\eta) = \frac{1}{2(\alpha_1 - i\beta_1)} e^{-(\alpha_1 - i\beta_1)\eta}, \quad (18)$$

$$f_1(\eta) = \frac{1}{2} e^{-(\alpha_2 + i\beta_2)\eta} + \frac{Gr}{2} (A - iB) \left[e^{-(\alpha_2 + i\beta_2)\eta} - e^{-(\alpha_1 + i\beta_1)\eta} \right], \quad (19)$$

$$\bar{f}_1(\eta) = \frac{1}{2} e^{-(\alpha_2 - i\beta_2)\eta} + \frac{Gr}{2} (A + iB) \left[e^{-(\alpha_2 - i\beta_2)\eta} - e^{-(\alpha_1 - i\beta_1)\eta} \right], \quad (20)$$

where

$$\begin{aligned} \alpha_1 &= -\frac{S\alpha}{2} + \frac{1}{\sqrt{2}} \left[\left\{ \left(\frac{S^2\alpha^2}{4} \right)^2 + n^2\alpha^2 \right\}^{\frac{1}{2}} + \frac{S^2\alpha^2}{4} \right]^{\frac{1}{2}}, \\ \beta_1 &= \frac{1}{\sqrt{2}} \left[\left\{ \left(\frac{S^2\alpha^2}{4} \right)^2 + n^2\alpha^2 \right\}^{\frac{1}{2}} - \frac{S^2\alpha^2}{4} \right]^{\frac{1}{2}}, \end{aligned} \quad (21)$$

$$\alpha_2 = -\frac{S}{2} + \frac{1}{\sqrt{2}} \left[\left\{ \left(\frac{S^2}{4} + M^2 + \frac{1}{Da} \right)^2 + n^2 \right\}^{\frac{1}{2}} + \left(\frac{S^2}{4} + M^2 + \frac{1}{Da} \right) \right]^{\frac{1}{2}},$$

$$\beta_2 = \frac{1}{\sqrt{2}} \left[\left\{ \left(\frac{S^2}{4} + M^2 + \frac{1}{Da} \right)^2 + n^2 \right\}^{\frac{1}{2}} - \left(\frac{S^2}{4} + M^2 + \frac{1}{Da} \right) \right]^{\frac{1}{2}},$$

$$A_1 = \alpha_1^2 - \beta_1^2 - S\alpha_1 - \left(M^2 + \frac{1}{Da}\right), \quad B_1 = 2\alpha_1\beta_1 - S\beta_1 - n,$$

$$A_2 = \alpha_1 A_1 - \beta_1 B_1, \quad B_2 = \alpha_1 B_1 + \beta_1 A_1, \quad A = \frac{A_2}{A_2^2 + B_2^2}, \quad B = \frac{B_2}{A_2^2 + B_2^2}.$$

On the use of (18)-(20), equations (12) and (11) yield

$$\theta(\eta, \tau) = \frac{e^{-\alpha_1 \eta}}{\alpha_1^2 + \beta_1^2} [\alpha_1 \cos(n\tau - \beta_1 \eta) + \beta_1 \sin(n\tau - \beta_1 \eta)], \quad (22)$$

$$u_1(\eta, \tau) = e^{-\alpha_2 \eta} \left[\cos(n\tau - \beta_2 \eta) + Gr \left[A \cos(n\tau - \beta_2 \eta) + B \sin(n\tau - \beta_2 \eta) \right] \right] - Gr \left[A \cos(n\tau - \beta_1 \eta) - B \sin(n\tau - \beta_1 \eta) \right] e^{-\alpha_1 \eta}. \quad (23)$$

The solutions (22) and (23) are valid for both suction and blowing at the plate. Equations (14) and (15) of Rani and Murty [22] are incorrect as they are independent of time t . This is due to the error in the boundary condition (4) of Rani and Murty [22]. The correct boundary condition is given by equation (3) and hence the equations (22) and (23) are not identical with the equations (14) and (15) of Rani and Murty [22] in the absence of suction/ blowing ($S = 0$), frequency of oscillations ($n = 0$) and porosity of the medium ($Da \rightarrow \infty$).

RESULTS AND DISCUSSION

The equation (22) shows that there exists a thin thermal boundary layer of thickness of the order $O(\alpha_1^{-1})$ near the plate ($\eta = 0$) where α_1 is given by (21). The thickness of this thermal boundary layer decreases with an increase in either suction parameter S or frequency parameter n or Prandtl number Pr . It is seen from (23) that there exists two-deck boundary layer near the plate ($\eta = 0$) having the thicknesses of the order $O(\alpha_1^{-1})$ and $O(\alpha_2^{-1})$ respectively where α_1 and α_2 are given by (21). The first one is the modified Stokes' boundary layer and the second boundary layer is due to convection of the flow. The thickness of the modified Stokes' boundary layer decreases with an increase in either magnetic parameter M^2 or suction parameter S or frequency parameter n while it increases with an increase in Darcy number Da , because α_2 increases with an increase in either M^2 or S or n while it decreases with an increase in Da .

We have presented the non-dimensional velocity u_1 and temperature θ for several values of magnetic parameter M^2 , radiation parameter R , Prandtl number Pr , Grashof number Gr , suction parameter S , frequency parameter n and phase angle $n\tau$ in Figs.2-14. Fig.2 shows that the fluid velocity u_1 decreases for $\eta \leq 1.75$ and it increases for $\eta > 1.75$ with an increase in magnetic parameter M^2 . The presence of a magnetic field in an electrically conducting fluid introduces a force called Lorentz force which acts against the flow if the magnetic field is applied in the normal direction as considered in the present problem. This type of resistive force tends to slow down the flow field. Since the magnetic field has a stabilizing influence, the maximum velocity overshoot is observed for the conducting air while minimum overshoot takes place for the water. It is observed from Fig.3 that the velocity u_1 decreases for $\eta \leq 2.5$ and it increases for $\eta > 2.5$ with an increase in radiation parameter R . The radiation parameter arises only in the energy equation in the thermal diffusion term and via coupling of the temperature field with the buoyancy terms in the momentum equation, the velocity is indirectly influenced by thermal radiation effects. An increase in R clearly reduces substantially the velocity in the boundary layer i.e. decelerates the flow. Fig.4 shows that the velocity u_1 decreases near the plate and it increases away from the plate with an increase in Prandtl number Pr . Prandtl number Pr encapsulates the ratio of momentum diffusivity to thermal diffusivity for a given fluid. It is also the product of dynamic viscosity and specific heat capacity divided by thermal conductivity. Higher Pr fluids will therefore possess higher viscosities (and lower thermal conductivities) implying that such fluids will flow slower than lower Pr fluids. It is seen from Fig.5 that the velocity u_1 increases for $\eta \leq 1.75$ and it decreases for $\eta > 1.75$ with an increase in Grashof number Gr . An increase in Grashof number leads to an increase in velocity, this is because, increase in Grashof number means more heating and less density. Fig.6 displays that the fluid velocity u_1 increases for $\eta \leq 1.45$ and it decreases for $\eta > 1.45$ with an increase in suction parameter S . This means that the suction at the plate have a retarding influence on the flow field. It is seen from Figs.7 and 8 that the velocity u_1 decreases near the plate and it increases away from the plate with an increase in either frequency parameter n or phase angle $n\tau$. This means that the frequency parameter or the phase angle have a retarding influence on the flow

field near the plate. Fig.9 illustrated that the fluid velocity u_1 increases near the plate and it decreases away from the plate with an increase in Darcy number Da . It is observed from Fig.10 that the temperature θ decreases near the plate and it increases away from the plate with an increase in radiation parameter R . Increasing radiation parameter clearly depressed the fluid temperature in presence of conducting air ($Pr = 0.71$) and magnetohydrodynamic flow. Fig.11 reveals that the fluid temperature θ decreases near the plate and it increases away from the plate with an increase in Prandtl number Pr . Prandtl number controls the relative thickness of the momentum and thermal boundary layers. When Pr is of low value, heat diffusion exceeds momentum diffusion. For $Pr < 1$, the thickness of the thermal boundary layer therefore exceeds the thickness of the velocity boundary layer that is, temperatures will be greater. In Fig.11, temperatures are seen to decrease considerably with an increase in the value of Pr as we progress into the boundary layer regime; profiles also decay much more sharply for higher Pr values since momentum diffusion exceeds energy diffusion for $Pr > 1$. It is found from Fig.12 that the fluid temperature θ decreases for $\eta \leq 1.75$ and it increases for $\eta > 1.75$ with an increase in suction parameter S . It is found from Fig.13 that the fluid temperature θ decreases near the plate and it increases away from the plate with an increase in frequency parameter n . Fig.14 shows that the fluid temperature θ increases near the plate and it decreases away from the plate with an increase in phase angle $n\tau$.

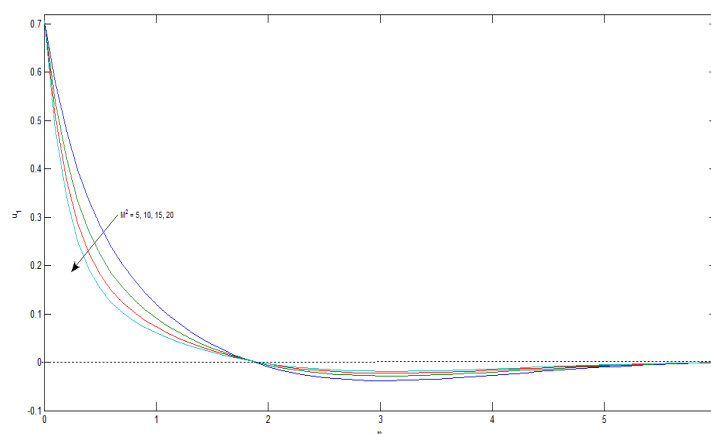


Fig.2: Velocity profiles for different M^2 when $R = 4$, $S = 0.5$, $Pr = 0.71$, $Gr = 5$, $n = 2$, $\tau = 0.5$, $Da = 0.1$ and

$$n\tau = \frac{\pi}{4}$$

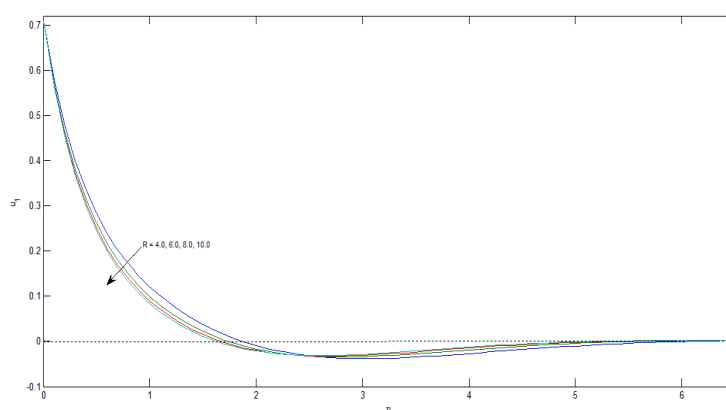


Fig.3: Velocity profiles for different R when $M^2 = 5$, $S = 0.5$, $Pr = 0.71$, $Gr = 5$, $n = 2$, $\tau = 0.5$, $Da = 0.1$ and

$$n\tau = \frac{\pi}{4}$$

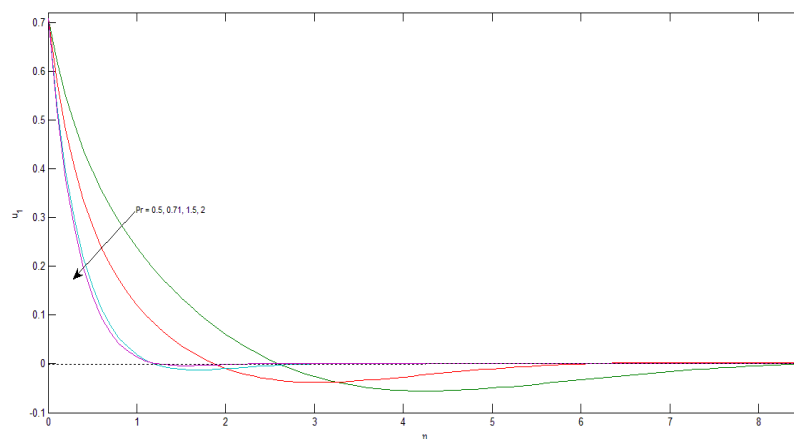


Fig.4: Velocity profiles for Pr when $M^2 = 5$, $S = 0.5$, $R = 4$, $Gr = 5$, $n = 2$, $\tau = 0.5$, $Da = 0.1$ and $n\tau = \frac{\pi}{4}$.

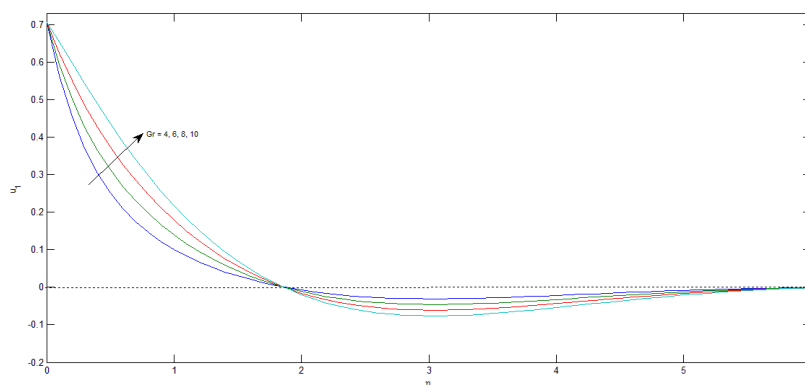


Fig.5: Velocity profiles for Gr when $M^2 = 5$, $S = 0.5$, $R = 4$, $Pr = 0.71$, $n = 2$, $\tau = 0.5$, $Da = 0.1$ and $n\tau = \frac{\pi}{4}$.

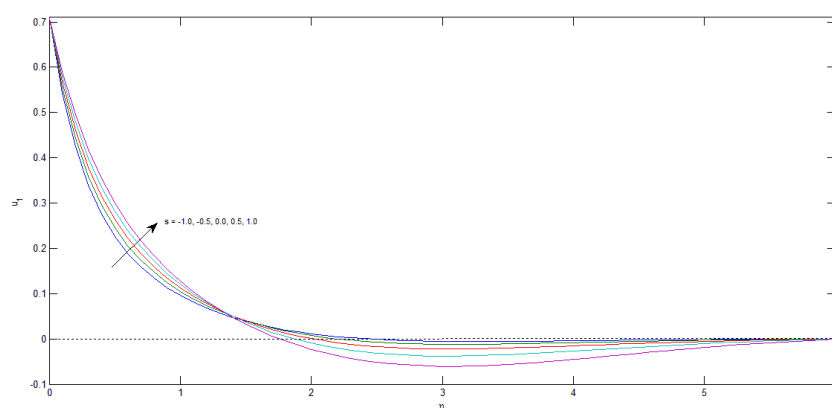


Fig.6: Velocity profiles for S when $M^2 = 5$, $Gr = 5$, $R = 4$, $Pr = 0.71$, $n = 2$, $\tau = 0.5$, $Da = 0.1$ and $n\tau = \frac{\pi}{4}$.

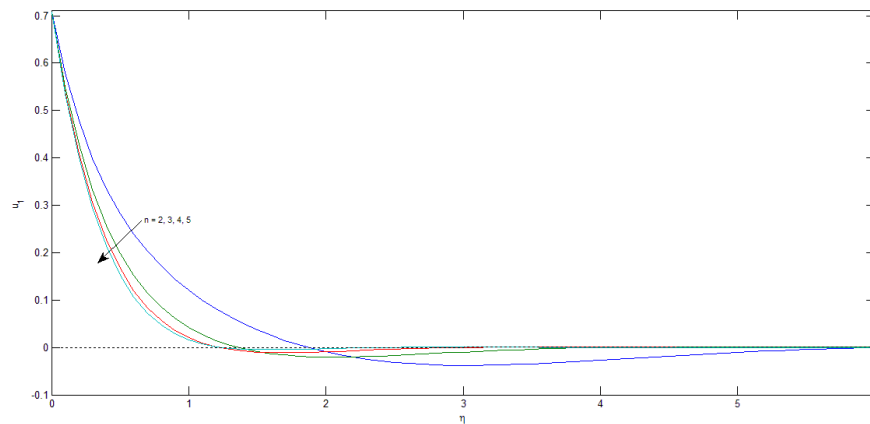


Fig.7: Velocity profiles for the variation of n when $M^2 = 5, S = 0.5, Gr = 5, R = 4, Pr = 0.71, \tau = 0.5, Da = 0.1$ and $n\tau = \frac{\pi}{4}$.

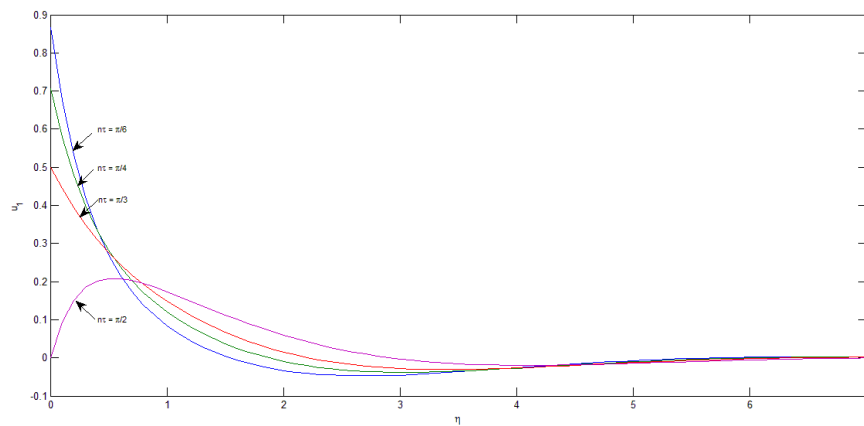


Fig.8: Velocity profiles for $n\tau$ when $M^2 = 5, S = 0.5, Gr = 5, R = 4, Pr = 0.71, \tau = 0.5$ and $Da = 0.1$.

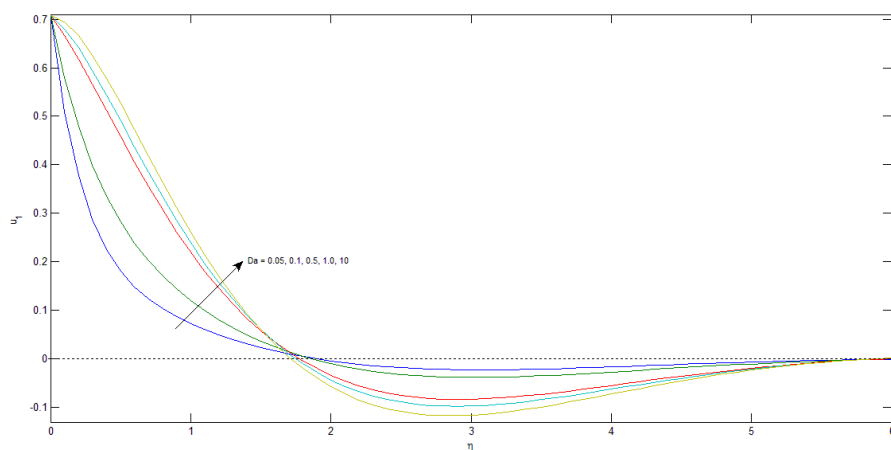


Fig.9: Velocity profiles for Da when $M^2 = 5, S = 0.5, Gr = 5, R = 4, Pr = 0.71, \tau = 0.5, n\tau = \frac{\pi}{4}$ and $n = 2$.

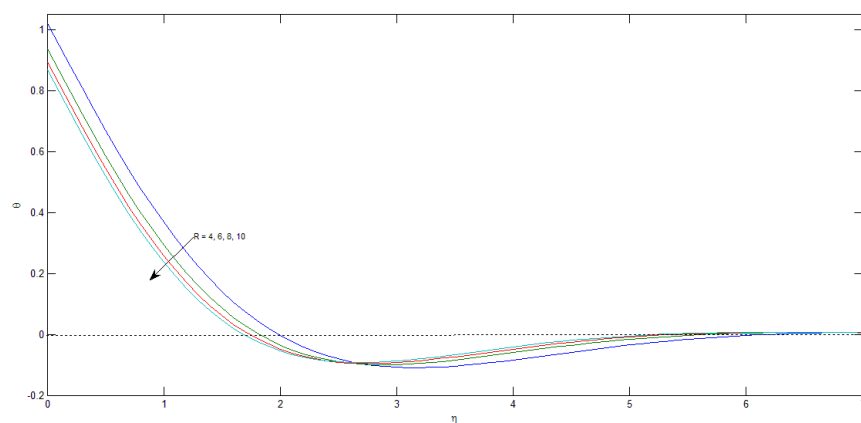


Fig.10: Temperature profiles for R when $Pr = 0.71$, $S = 0.5$, $n = 2$ and $n\tau = \frac{\pi}{4}$

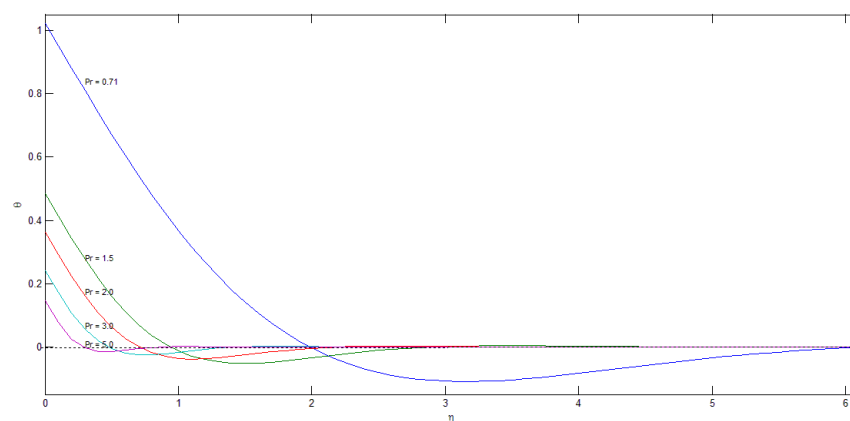


Fig.11: Temperature profiles for Pr when $R = 4$, $S = 0.5$, $n = 2$ and $n\tau = \frac{\pi}{4}$

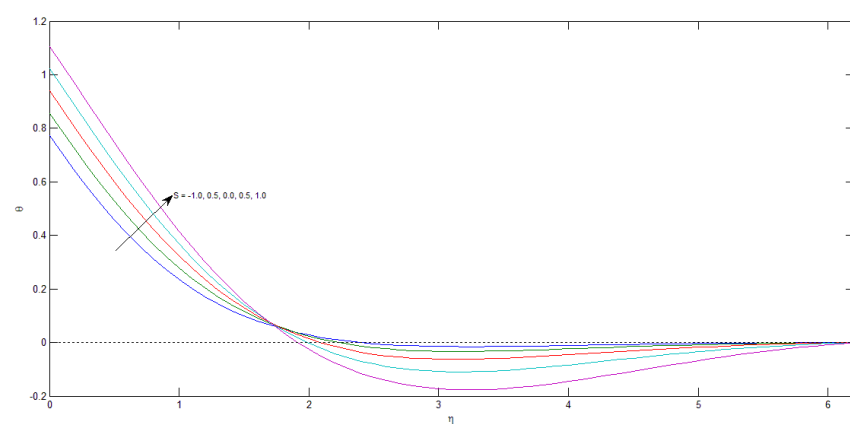


Fig.12: Temperature profiles for S when $Pr = 0.71$, $R = 4$, $n = 2$ and $n\tau = \frac{\pi}{4}$

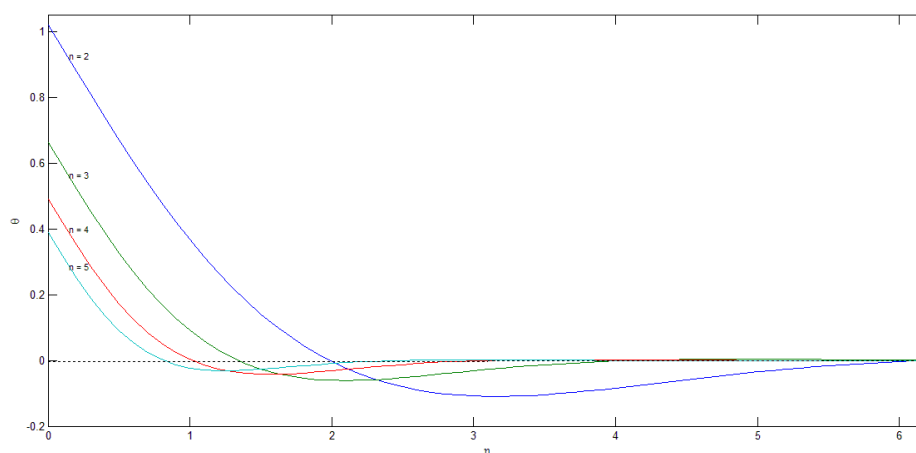


Fig.13: Temperature profiles for n when $Pr = 0.71$, $R = 4$, $S = 0.5$ and $n\tau = \frac{\pi}{4}$

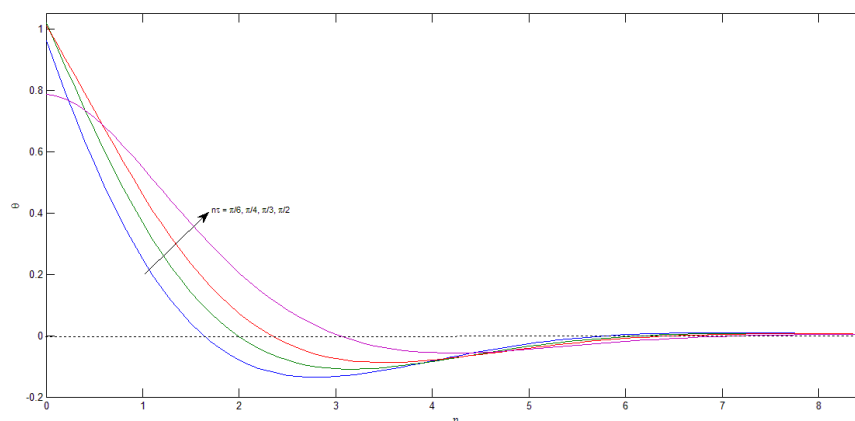


Fig.14: Temperature profiles for $n\tau$ when $R = 4$, $Pr = 0.71$, $S = 0.5$ and $n = 2$

The non-dimensional shear stress at the plate ($\eta = 0$) is given by

$$\tau_x = \left(\frac{\partial u_1}{\partial \eta} \right)_{\eta=0} = -R_0 \cos(n\tau + \phi), \quad (24)$$

where

$$R_0 = \left[\left\{ \alpha_2 + Gr \{ A(\alpha_2 - \alpha_1) + B(\beta_2 - \beta_1) \} \right\}^2 + \left\{ \beta_2 + Gr \{ A(\beta_2 - \beta_1) - B(\alpha_2 - \alpha_1) \} \right\}^2 \right]^{\frac{1}{2}}, \quad (25)$$

$$\tan \phi = \frac{\beta_2 + Gr \{ A(\beta_2 - \beta_1) - B(\alpha_2 - \alpha_1) \}}{\alpha_2 + Gr \{ A(\alpha_2 - \alpha_1) + B(\beta_2 - \beta_1) \}}, \quad (26)$$

where α , β , α_1 , β_1 , A and B are given by (21).

The variations of amplitude of shear stress R_0 and the tangent of the phase angle of shear stress $\tan \phi$ respectively drawn against M^2 for different values of R , Pr , S and n with $n\tau = \frac{\pi}{4}$ and are shown in Figs.15-21. It is observed from Figs.15 and 16 that the amplitude R_0 increases with an increase in either radiation parameter R or

Prandtl number Pr Fig. 17 shows that the amplitude R_0 decreases due to the increase of suction parameter S . Fig.18 shows that R_0 increases with an increase in frequency parameter n . It is observed from Figs.19 and 20 that the tangent of the phase angle $\tan \phi$ decreases with an increase in either radiation parameter R or Prandtl number Pr . Fig. 21 shows that the tangent of the phase angle $\tan \phi$ increases with an increase in suction parameter S .

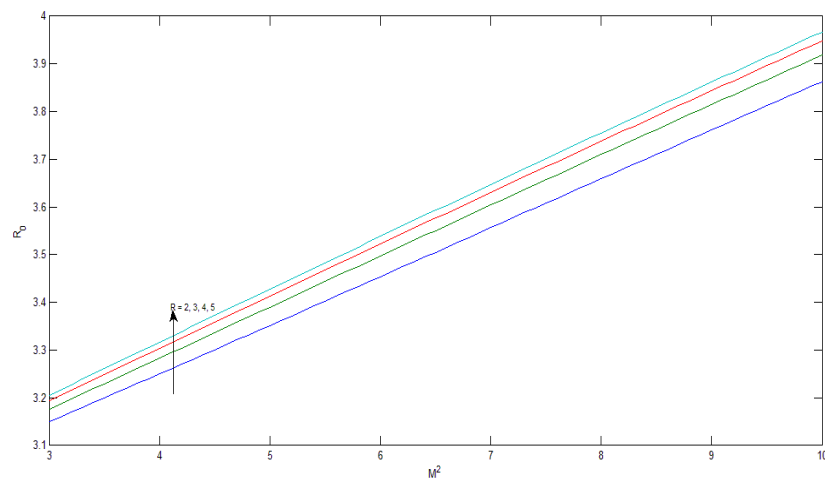


Fig.15: Amplitude R_0 for R when $S = 0.5$, $Pr = 0.71$, $S = 0.5$ and $n = 2$.

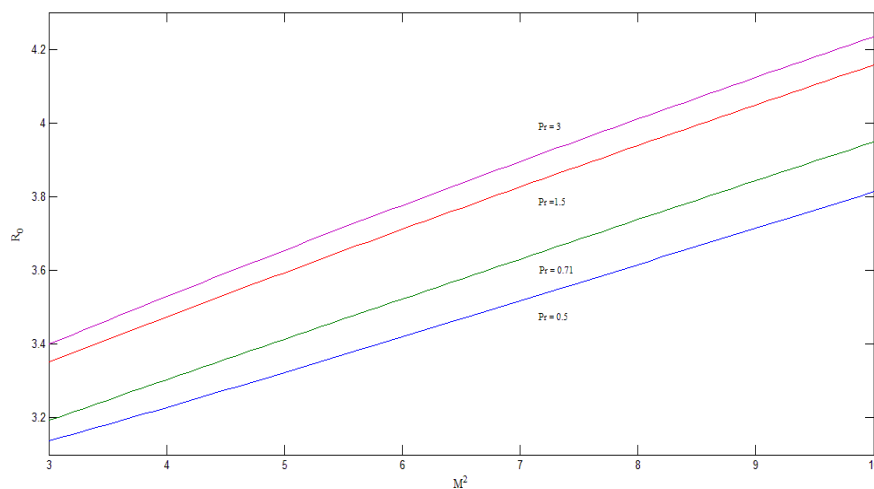


Fig.16: Amplitude R_0 for Pr when $R = 4$, $S = 0.5$ and $n = 2$

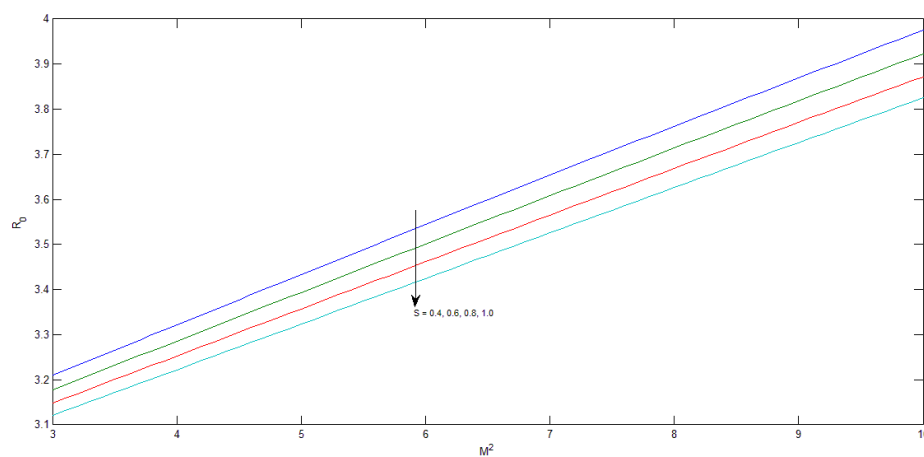


Fig.17: Amplitude R_0 for S when $R = 4$, $Pr = 0.71$ and $n = 2$

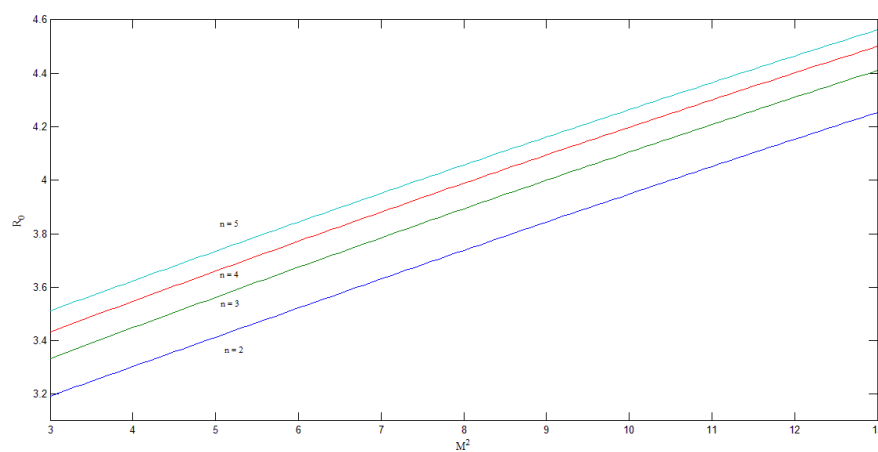


Fig.18: Amplitude R_0 for n when $R = 4$, $S = 0.5$ and $Pr = 0.71$

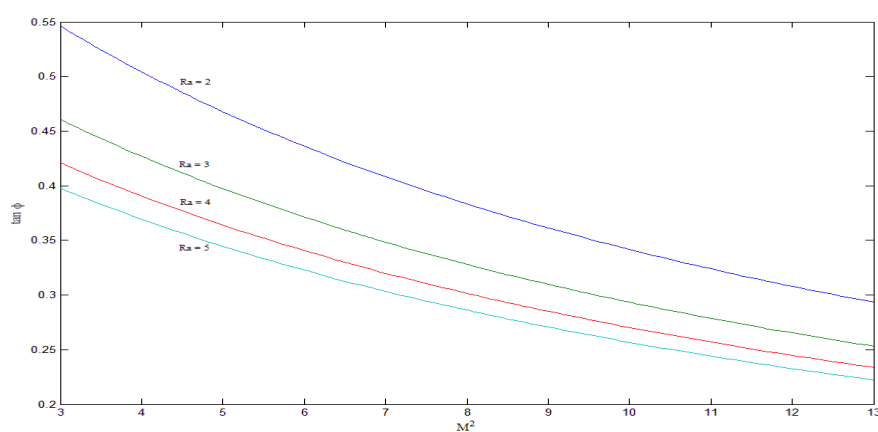


Fig.19: Tangent of the phase $\tan \phi$ for R when $S = 0.5$, $Pr = 0.71$ and $n = 2$

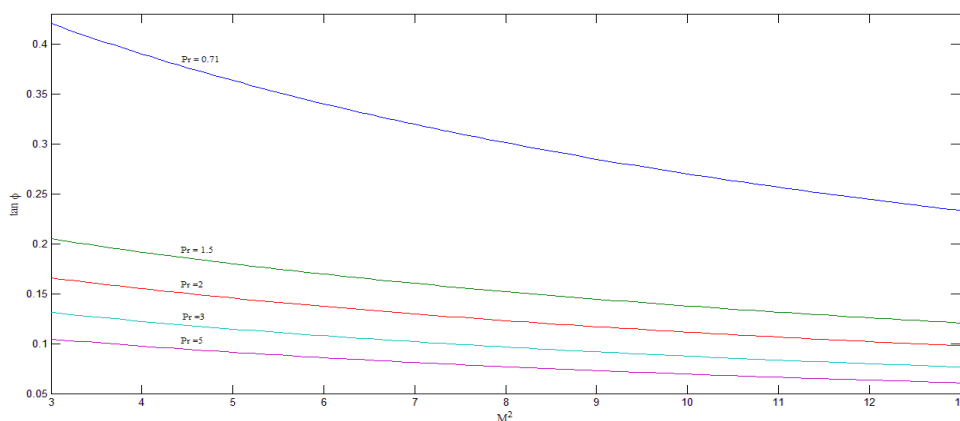


Fig.20: Tangent of the phase $\tan \phi$ for Pr when $R = 4$, $S = 0.5$ and $n = 2$

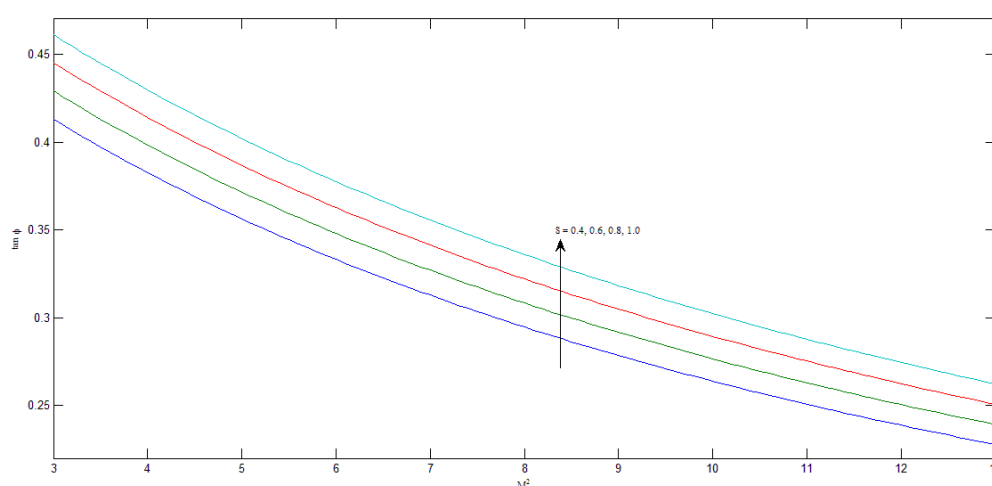


Fig.21: Tangent of the phase $\tan \phi$ for S when $R = 4$, $Pr = 0.71$ and $n = 2$

The variations of the plate temperature $\theta(0, \tau)$ are presented in Table 1. It is observed from Table 1 that the plate temperature $\theta(0, \tau)$ decreases with an increase in either radiation parameter R or Prandtl number Pr . Further, it is seen that the plate temperature $\theta(0, \tau)$ increases with an increase in suction parameter S .

Table 1. Variation of plate temperature $\theta(0, \tau)$ when $n\tau = \frac{\pi}{4}$

	S				Pr			
Ra	0.0	0.5	1.0	1.5	0.25	0.50	0.71	0.80
2	1.17371	1.27749	1.38177	1.48782	3.62806	1.81403	1.27749	1.13377
4	0.93897	1.02199	1.10541	1.19026	2.90245	1.45123	1.02199	0.90702
6	0.86072	0.93682	1.01329	1.09107	2.66058	1.33029	0.93682	0.83143
8	0.82160	0.92633	0.96724	2.53964	1.26982	1.04147	0.89424	0.79364

CONCLUSION

The radiation effects on MHD free convection flow of a viscous incompressible electrically conducting fluid past an oscillating vertical porous plate embedded in a porous medium in the presence of a uniform transverse magnetic field. It is found that the fluid velocity influences in the presence of magnetic field. An increase in radiation parameter leads to reduce the fluid velocity for $\eta \leq 2.5$ and accelerate it for $\eta > 2.5$. The fluid velocity increases near the plate and it

decreases away from the plate with an increase in suction parameter. The fluid velocity increases near the plate and it decreases away from the plate with an increase in Darcy number. The fluid temperature decreases near the plate and it increases away from the plate with an increase in either radiation parameter or Prandtl number or suction parameter or frequency parameter. Further, it is seen that the amplitude of the shear stress and the tangent of the phase angle at the plate decreases with an increase in either radiation parameter or Prandtl number or suction parameter.

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