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Effects of heat transfer on MHD oscillatory flow of Jeffrey fluid with variable viscosity through porous medium

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ABSTRACT

In this paper, Influence of heat transfer on MHD oscillatory flow Jeffrey fluid with variable viscosity model through porous medium is investigated. The fluid viscosity is assumed to vary as an exponential function of temperature. The effects of various emerging parameters on the velocity field and temperature field are discussed in detail through graphs.

Keywords: Jeffrey fluid with variable viscosity model, MHD, Oscillatory flow, Porous medium.

INTRODUCTION

The flow of an electrically conducting fluid has important applications in many branches of engineering science such as magnetohydrodynamics (MHD) generators, plasma studies, nuclear reactor, geothermal energy extraction, electromagnetic propulsion, the boundary layer control in the field of aerodynamics and so on. Heat transfer effect on laminar flow between parallel plates under the action of transverse magnetic field was studied by Nigam and Singh [1]. Soundalgekar and Bhat [2], have investigated the MHD oscillatory flow of a Newtonian fluid in a channel with heat transfer. MHD flow of viscous fluid between two parallel plates with heat transfer was discussed by Attia and Kotb [3]. Raptis et al. [4], have analyzed the hydromagnetic free convection flow through a porous medium between two parallel plates. Aldoss et al. [5], have studied mixed convection flow from a vertical plate embedded in a porous medium in the presence of a magnetic field. Makinde and Mhone [6], have considered heat transfer to MHD oscillatory flow in a channel filled with porous medium. Mostafa [7], have studied thermal radiation effect on unsteady MHD free convection flow past a vertical plate with temperature dependent viscosity. Unsteady heat transfer to MHD oscillatory flow through a porous medium under slip condition was investigated by Hamza et al. [8].

Moreover the non-Newtonian fluids are more appropriate than Newtonian fluids in many practical applications. Examples of such fluids include certain oils, lubricants, mud, shampoo, ketchup, blood at low shear rate, cosmetic products, polymers and many others. Unlike the viscous fluids, all the non-Newtonian fluids (in terms of their diverse characteristics) cannot be described by a single constitutive relationship. Hence, several models of non-Newtonian fluids are proposed in the literature. Al Khatib and Wilson [9], have studied the Poiseuille flow of a yield stress fluid in a channel. Flow of a visco-plastic fluid in a channel of slowly varying width was studied by Frigaard and Ryan [10]. Ali and Asghar [11], have analyzed by oscillatory channel flow for non-Newtonian fluid. The influence of heat transfer on MHD oscillatory flow of Jeffrey fluid in a channel was discussed by K. Kavita et al. [12].
In view of these we studied the effect of heat transfer on MHD oscillatory flow of a Jeffrey fluid with variable viscosity model through porous medium. The expressions are obtained for velocity and temperature analytically. The effects of various emerging parameters on the velocity and temperature are discussed through graphs in detail.

2. Mathematical Formulation

We consider the flow of a Jeffrey fluid in a channel of width h under the influence of electrically applied magnetic field and radioactive heat transfer as depicted in Fig.1. It is assumed that the fluid has small electrical conductivity and the electromagnetic force produced is very small. We choose the Cartesian coordinate system (x, y), where x - is taken along center of the channel and the y - axis is taken normal to the flow direction.

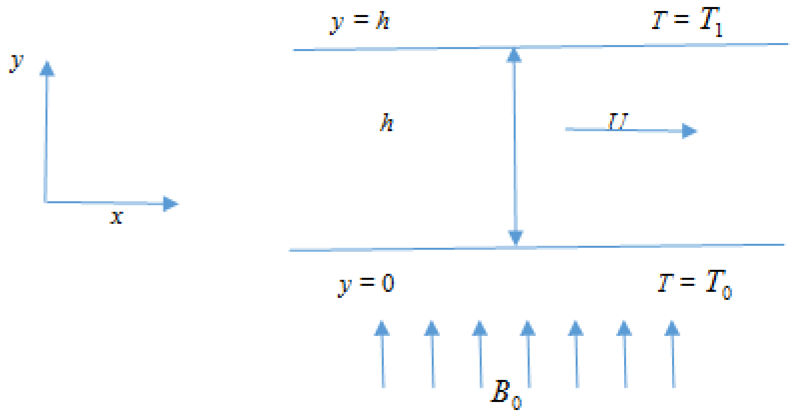

Fig. 1 Geometry of the problem

The constitute equation of S for Jeffrey fluid is

$$S = \frac{\mu}{1+\lambda_1}\left(\dot{\gamma} + \lambda_2 \ddot{\gamma}\right) \tag{1}$$

where μ is the dynamic viscosity, λ_1 is the ratio of relaxation to retardation times, λ_2 is the retardation time, $\dot{\gamma}$ is the shear rate and dots over the quantities denote differentiation with time.

The basic equations of momentum and energy governing such a flow, subject to the Boussinesq approximation, are:

$$\rho \frac{\partial \bar{u}}{\partial \bar{t}} = -\frac{\partial \bar{p}}{\partial \bar{x}} + \frac{\mu(T)}{1+\lambda_1}\frac{\partial^2 \bar{u}}{\partial \bar{y}^2} + \rho g \beta (T - T_0) - \sigma B_0^2 \bar{u} - \frac{\mu(T)}{k}\bar{u} \tag{2}$$

$$\rho \frac{\partial T}{\partial \bar{t}} = \frac{K}{c_p}\frac{\partial^2 T}{\partial \bar{y}^2} - \frac{1}{c_p}\frac{\partial q}{\partial \bar{y}} \tag{3}$$

The boundary conditions are given by

$$\bar{u} = 0 \quad \text{at} \quad \bar{y} = 0, \quad \text{and} \quad \bar{u} = 0 \quad \text{at} \quad \bar{y} = h \tag{4}$$

$$T = T_0 \quad \text{at} \quad \bar{y} = 0, \quad \text{and} \quad T = T_1 \quad \text{at} \quad \bar{y} = h \tag{5}$$

where $\bar{u}$ is the axial velocity, T is the fluid temperature, $\bar{p}$ is the pressure, ρ is the fluid density, B_0 is the magnetic field strength, σ is the conductivity of the fluid, g is the acceleration due to gravity, β is the coefficient of volume expansion due to temperature, c_p is the specific heat at constant pressure, k is the thermal conductivity and q is the radioactive heat flux.

Following Cogley et al. [13], it is assumed that the fluid is optically thin with a relatively low density and the radioactive heat flux is given by

$$\frac{\partial q}{\partial y} = 4\alpha^2 (T_0 - T) \tag{6}$$

here α is the mean radiation absorption coefficient.

Introducing the following non-dimensional variables

$$\left.\begin{aligned} & x=\frac{\bar{x}}{h}, y=\frac{\bar{y}}{h}, u=\frac{\bar{u}}{U}, \theta=\frac{T-T_0}{T_1-T_0}, t=\frac{\bar{t}U}{h}, p=\frac{\bar{p}h}{\mu U}, M^2=\frac{\sigma h^2 B_0^2}{\mu}, Da=\frac{k}{a^2}, \\ & \mu(\theta)=\frac{\mu(T)}{\mu_0}, Gr=\frac{pg\beta h^2(T_1-T_0)}{\mu U}, \mathrm{Re}=\frac{\rho hU}{\mu}, Pe=\frac{\rho hUc_p}{K}, N^2=\frac{4\alpha^2h^2}{K} \end{aligned}\right\} \tag{7}$$

where U is the mean flow velocity, Da Darcy number, Re Reynolds number, Gr Grashof number, M magnetic parameter, Pe is the Peclet number and N is the radiation parameter.

Substituting (7) into equations (2)-(6), we obtain

$$\frac{\rho hU}{\mu_0}\frac{\partial u}{\partial t}=-\frac{\partial p}{\partial x}+\frac{\mu(\theta)}{1+\lambda_1}\frac{\partial^2 u}{\partial y^2}+\frac{\rho g\alpha(T_1-T_0)}{\mu_0 U}\theta-\frac{\sigma B_0^2h^2}{\mu_0}u-\frac{h^2\mu(\theta)}{k}u \tag{8}$$

$$\frac{\rho hUc_p}{K}\frac{\partial}{\partial t}(\theta(T_1-T_0)+T_0)=\frac{\partial^2}{\partial y^2}(\theta(T_1-T_0)+T_0)-\frac{4h^2\alpha^2(T_0-T)}{K} \tag{9}$$

After simplify, we obtain the following non-dimensional equations:

$$\mathrm{Re}\frac{\partial u}{\partial t}=-\frac{\partial p}{\partial x}+\frac{\mu(\theta)}{1+\lambda_1}\frac{\partial^2 u}{\partial y^2}-\left(M^2+\frac{\mu(\theta)}{Da}\right)u+Gr\theta \tag{10}$$

$$Pe\frac{\partial\theta}{\partial t}=\frac{\partial^2\theta}{\partial y^2}+N^2\theta \tag{11}$$

The corresponding non-dimensional boundary conditions are

$$u=0 \text{ at } y=0, \text{ and } u=0 \text{ at } y=1 \tag{12}$$

$$\theta=0 \text{ at } y=0, \text{ and } \theta=1 \text{ at } y=1 \tag{13}$$

3. Solution of the Problem

To solve the temperature equation (11) with boundary conditions (13), let

$$\theta(y,t)=\theta_0(y)e^{i\omega t} \tag{14}$$

where ω is the frequency of the oscillation.

Substituting the equations (14) into the equations (11), we have

$$\frac{\partial^2\theta_0}{\partial y^2}+(N^2-i\omega Pe)\theta_0=0 \tag{15}$$

The solution of equation (15) with boundary conditions θ_0(0) = 0 and θ_0 (1) = 1 is

$$\theta_0(y)=\csc(\phi)\sin(\phi y) \tag{16}$$

where $\phi=\sqrt{N^2-i\omega Pe}$, hence

$$\theta(y,t)=\csc(\phi)\sin(\phi y)e^{i\omega t} \tag{17}$$

To solve the momentum equation (10) for purely oscillatory flow, let

$$-\frac{\partial p}{\partial x}=\lambda e^{i\omega t} \tag{18}$$

$$u(y,t) = u_0(y)e^{i\omega t} \tag{19}$$

where λ is a real constant.

Substituting the equations (18) and (19) into the equations (10), we have

$$\mathrm{Re}\, i\omega u_0(y) = \lambda + \frac{\mu(\theta)}{1+\lambda_1}\frac{\partial^2 u_0(y)}{\partial y^2} - \left(M^2 + \frac{\mu(\theta)}{Da}\right)u_0(y) + Gr\theta_0 \tag{20}$$

The Reynold model of viscosity is used to describe the variation of viscosity with temperature. The Reynold model of viscosity is defined as Person [14]

$$\mu(\theta) = e^{-\varepsilon\theta} \tag{21}$$

Using the Maclaurin series expansion the above expression can be written as

$$\mu(\theta) = 1 - \varepsilon\theta + \frac{\varepsilon^2\theta^2}{2}, \quad \varepsilon << 1 \tag{22}$$

Here $\varepsilon = 0$ corresponds to the constant viscosity case.

Compensating equation (22) into equation (20), we have

$$\mathrm{Re}\, i\omega u_0(y) = \lambda + \frac{1}{1+\lambda_1}\left(1 - \varepsilon\theta + \frac{\varepsilon^2\theta^2}{2}\right)\frac{\partial^2 u_0(y)}{\partial y^2} - \left(M^2 + \frac{1}{Da}\left(1 - \varepsilon\theta + \frac{\varepsilon^2\theta^2}{2}\right)\right)u_0(y) + Gr\theta_0 \tag{23}$$

Small ε suggests the use of perturbation technique to solve equation (22). Accordingly, we write:

$$u_0(y) = u_{00}(y) + \varepsilon u_{01}(y) + \varepsilon^2 u_{02}(y) + O(\varepsilon^3) \tag{24}$$

Substituting equation (24) into equation (23) with boundary conditions (12), then equating the like powers of ε, we obtain

3.1 Zeros-order system (ε^0)

$$\frac{\partial^2 u_{00}}{\partial y^2} - (1+\lambda_1)\left(M^2 + i\omega \mathrm{Re} + \frac{1}{Da}\right)u_{00} = -(1+\lambda_1)(\lambda + Gr\theta_0) \tag{25}$$

The associated boundary conditions are

$$u_{00}(0) = u_{00}(1) = 0 \tag{26}$$

3.2 First-order system (ε)

$$\frac{\partial^2 u_{01}}{\partial y^2} - (1+\lambda_1)\left(M^2 + i\omega \mathrm{Re} + \frac{1}{Da}\right)u_{01} = \left(\frac{\partial^2 u_0}{\partial y^2} - \frac{(1+\lambda_1)}{Da}u_0\right)\theta_0 \tag{27}$$

The associated boundary conditions are

$$u_{01}(0) = u_{01}(1) = 0 \tag{28}$$

3.3 Second-order system (ε^2)

$$\frac{\partial^2 u_{02}}{\partial y^2} - (1+\lambda_1)\left(M^2 + i\omega \mathrm{Re} + \frac{1}{Da}\right)u_{02} = \left\{\frac{\partial^2 u_1}{\partial y^2} - \frac{(1+\lambda_1)}{Da}u_1 - \frac{\theta_0 e^{iwt}}{2}\left(\frac{\partial^2 u_0}{\partial y^2} - \frac{(1+\lambda_1)}{Da}u_0\right)\right\}\theta_0 \tag{29}$$

The associated boundary conditions are

$$u_{02}(0) = u_{02}(1) = 0 \tag{30}$$

3.4 Zeros-order solution
The solution of equation (25) subset to the associates boundary conditions (26) is;

$$u_{00} = \frac{B}{A\left(1+e^{\sqrt{A}}\right)}\left(1+e^{\sqrt{A}} - e^{\sqrt{A}y} - e^{\sqrt{A}(1-y)}\right) \tag{31}$$

3.4 First-order solution
The solution of equation (27) subset to the associates boundary conditions (28) is;

$$u_{01} = \frac{Be^{iwt}\theta_0}{4A^2\left(1+e^{\sqrt{A}}\right)}\left\{\begin{array}{l} -e^{-\sqrt{A}y}\left(1+e^{\sqrt{A}}\right)\left(\begin{array}{l} e^{1+\sqrt{A}(1-2y)+2\sqrt{A}y}(1-A) - 4e^{1+\sqrt{A}y}(1+e^{\sqrt{A}}) \\ 2\sqrt{A}e^{\sqrt{A}}y(e-A)[1-e^{\sqrt{A}}] + e^{2\sqrt{A}y}(e-A) \end{array}\right) \\ +\left(-A-3e-e^{1+\sqrt{A}}(3+2\sqrt{A}) - Ae^{\sqrt{A}}(1-2\sqrt{A})\right)e^{\sqrt{A}y} \\ -e^{\sqrt{A}(1-y)}\left(A(1+e^{\sqrt{A}})(A+3e)+2\sqrt{A}(A-e)\right) \end{array}\right\} \tag{32}$$

where $A = (1+\lambda_1)\left(M^2 + i\omega\text{Re} + \dfrac{1}{Da}\right)$, and $B = (1+\lambda_1)(\lambda + Gr\theta_0)$.

The formula of u_{02} is a long.

Finally, the perturbation solutions up to second order for u_0 is given by

$$u_0 = u_{00} + \varepsilon u_{01} + \varepsilon^2 u_{02}$$

Therefore, the fluid velocity is given as

$$u(y,t) = u_0 e^{i\omega t} \tag{33}$$

RESULTS AND DISCUSSION

In this section, the numerical and computational results are discussed for the problem of an incompressible non-Newtonian Jeffrey fluid with variable viscosity through a porous medium channel in detail through the graphical illustrations. The numerical evaluations of the analytical results and some important results are displayed graphically in figure (2) - (9). MATHEMATICA program is used to find out numerical results and illustrations. The analytical solutions of the momentum equation is obtained by using perturbation technique. All the obtained solutions are discussed graphically under the variations of various pertinent parameters in the present section.

Based on equation (33), figures (2) - (6), illustrates the effects of the parameters N, ω, λ, λ_1, *Da*, *M*, Re and Pe on the velocity.

Figure (2) illustrates the effects of the parameters N and ω on the velocity distribution function *u* vs. *y*. It is found that the velocity profile *u* decreases with increasing N, while *u* increases with increasing ω, and attains its maximum height at the center line of the channel. The fluid velocity starts increasing and tends to be constant at the walls, as specified by the boundary conditions.

From figure (3) one can depict here that the velocity profile *u* rising up with the increasing effects of both the parameters λ and λ_1. Figure (4) contains the behavior of *u* under the variation of *Da* and *M*, one can depict here that *u* go down with the increasing effects of both the parameters *Da* and *M*.

Figure (5) illustrates the effects of the parameters Re and Pe on the velocity distribution function *u* vs. *y*. It is found that the velocity profile *u* increases with increasing Re, while *u* decreases with increasing Pe. Figure (6) show that velocity distribution increases with the increasing of the parameters ε.

Based on equation (17), figures (7) - (9), illustrates the effects of the parameters N, Pe and ω on the temperature. Figure (7) illustrates the effects of the parameters N on the temperature θ vs. y. The temperature increases with increasing N. Figure (8) show that temperature increases with the increasing of the parameters Pe. Figure (9) contains the behavior of θ under the variation of ω, one can depict here that the temperature go down with the increasing effects of ω.

5. Concluding Remarks

The present study deals with the effect of heat transfer on MHD oscillatory flow of Jeffrey fluid with variable viscosity model through porous medium channel. The perturbation series in the viscosity parameter (ε<<1) was used to obtain explicit forms for velocity field . We obtained the analytical solution of the problem.

The results are analyzed for different values of pertinent parameters namely radiation parameter N, frequency of the oscillation ω, Darcy number *Da*, Reynolds number Re, constant viscosity case ε, Jeffrey parameter λ_1, magnetic parameter *M* and Peclet number *Pe*. The main findings can be summarized as follows:

1. The axial velocity increases with the increase in ω, λ, λ_1, Re, and ε. Further, the axial velocity increases with increase in ϕ, when $0 < y < 0.77$.
2. The axial velocity decreases with the increase in N, *Da*, *M*, and Pe.
3. The coefficient of temperature increases with increasing values of N and Pe, while the temperature decreases with increasing ω.
4. The velocity for Jeffrey fluid with variable viscosity is less than velocity of Jeffrey fluid with constant viscosity, see Kavita et al. (2012).

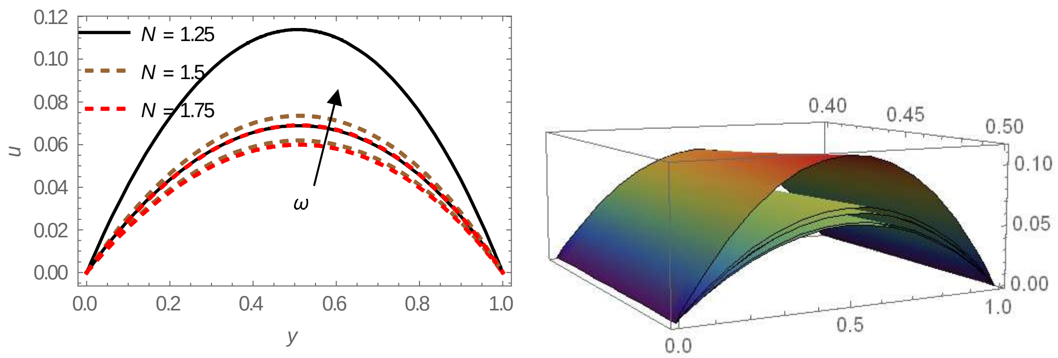

Fig. 2 Velocity profile for various values of N, and ω with

$$t=0.5, \mathrm{Re}=1, Pe=0.7, Gr=1, Da=0.8, M=1, \lambda_1=0.3, \lambda=1, \varepsilon=0.2$$

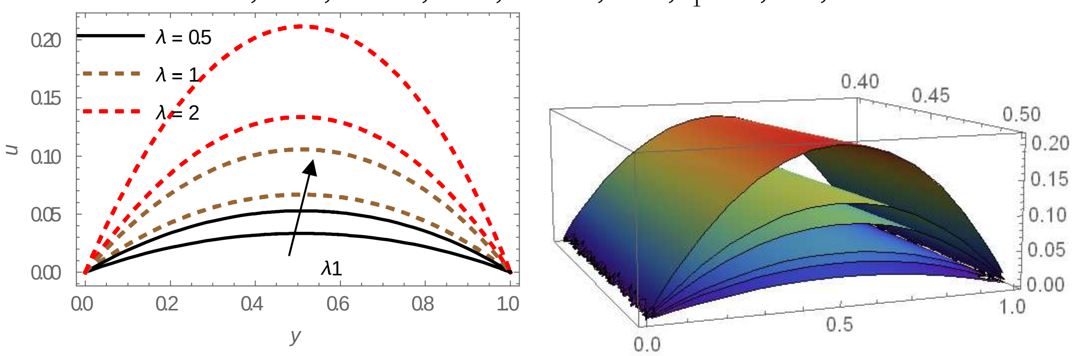

Fig. 3 Velocity profile for various values of λ, and λ_1 with

$$t=0.5, \mathrm{Re}=1, Pe=0.7, Gr=1, Da=0.8, M=1, N=1, \omega=1, \varepsilon=0.2$$

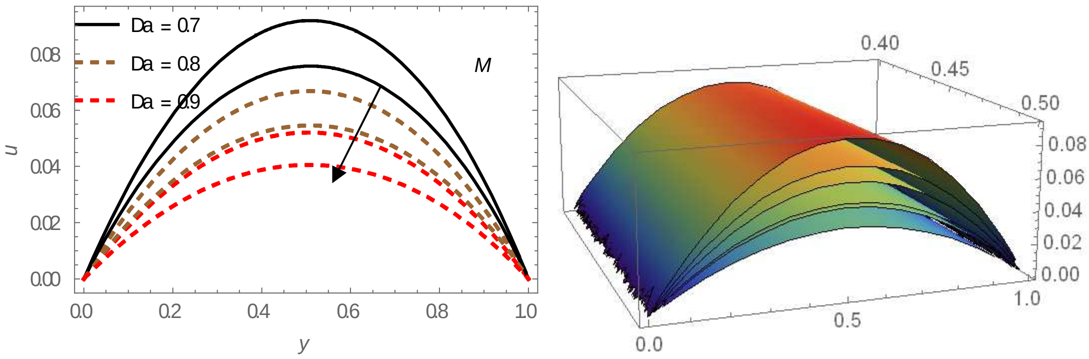

Fig. 4 Velocity profile for various values of Da **, and** M **with**
$t=0.5, \mathrm{Re}=1, Pe=0.7, Gr=1, N=1, \omega=1, \lambda_1=0.3, \lambda=1, \varepsilon=0.2$

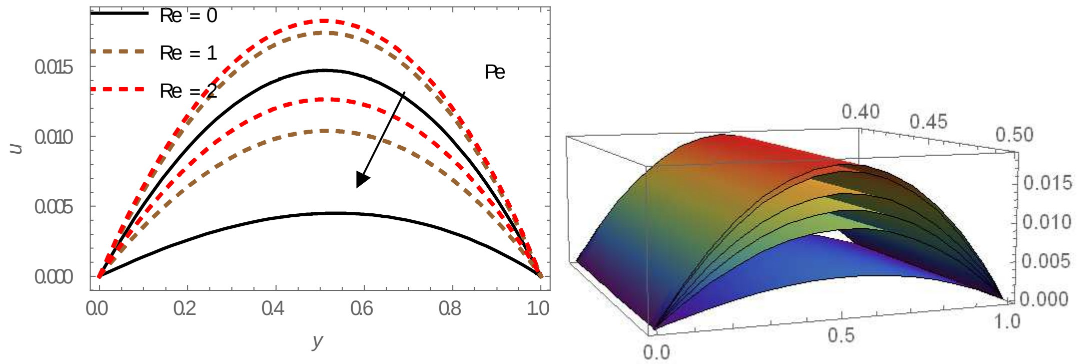

Fig. 5 Velocity profile for various values of Re **, and** Pe **with**
$t=0.5, Da=0.8, M=1, Gr=1, N=1, \omega=1, \lambda_1=0.3, \lambda=1, \varepsilon=0.2$

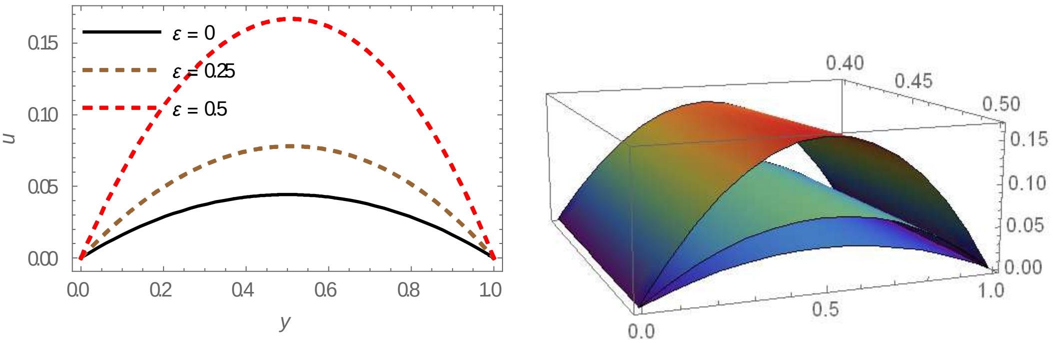

Fig. 6 Velocity profile for various values of ε **with**
$t=0.5, \mathrm{Re}=1, Pe=1, Da=0.8, M=1, Gr=1, N=1, \omega=1, \lambda_1=0.3, \lambda=1$

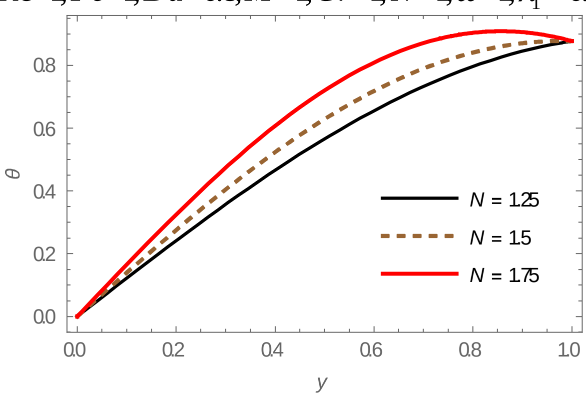

Fig. 7 Temperature distribution for various values of N **and** $t=0.5, Pe=0.7, \omega=1.$

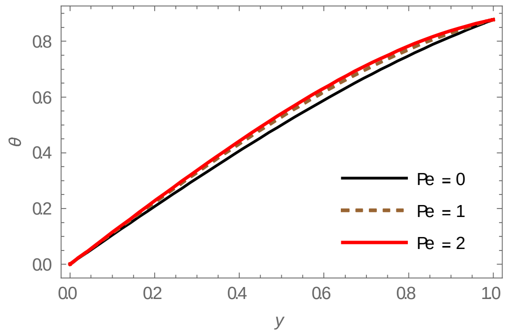

Fig. 8 Temperature distribution for various values of Pe **and** $t=0.5, N=1, \omega=1$

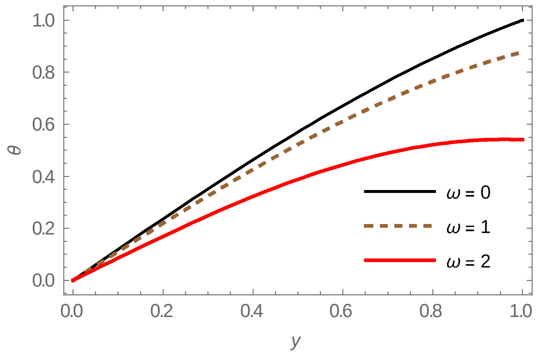

Fig. 9 Temperature distribution for various values of ω **and** $t=0.5, Pe=0.7, N=1$

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