

Effects of chemical reaction on mhd free convective oscillatory flow past a porous plate with viscous dissipation and heat sink

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ABSTRACT

An attempt has been made to study the two-dimensional MHD free convective oscillatory flow of an electrically conducting incompressible viscous fluid past an infinite vertical porous plate, through which suction occurs with constant velocity and chemical reaction in the presence of a heat sink. A uniform magnetic field is assumed to be applied transversely to the direction of the free stream taking into account of induced magnetic field. The governing equations involved in the present analysis are solved by using the perturbation method. The velocity, temperature and concentration fields are studied for different parameters such as Grashof number, modified Grashof number, Magnetic field parameter, Schmidt number, Prandtl number, heat sink parameter and Chemical reaction parameter, Eckert number etc.

Key words: Free convective, oscillatory flow, MHD, Chemical reaction, viscous dissipation, Heat sink etc.

INTRODUCTION

The influence of magnetic field on viscous incompressible flow of electrically conducting fluid has its importance in many applications such as extrusion of plastics in the manufacture of rayon and nylon, purification of crude oil, pulp, paper industry, textile industry and in different geophysical cases etc. In many process industries, the cooling of threads or sheets of some polymer materials is of importance in the production line. The rate of cooling can be controlled effectively to achieve final products of desired characteristics by drawing threads, etc. in the presence of an electrically conducting fluid subject to a magnetic field.

MHD plays an important role in agriculture, petroleum industries, geophysics and in astrophysics. Important applications are in the study of geological formations, in exploration and thermal recovery of oil, and in the assessment of aquifers, geothermal reservoirs and underground nuclear waste storage sites. MHD flow has application in metrology, solar physics and in motion of earth's core. Also it has applications in the field of stellar and planetary magnetospheres, aeronautics, chemical engineering and electronics. In the field of power generation, MHD is receiving considerable attention due to the possibilities it offers for much higher thermal efficiencies in power plants.

Jonah Phillip et al. (13) studied the effects of thermal radiation and MHD on the unsteady free convection and mass transform flow past an exponentially accelerated vertical plate with variable temperature. Gireesh Kumar et al. (9) discussed the effects of chemical reaction on transient MHD convection flow past a vertical surface embedded in a porous medium with oscillating temperature. Hemanth Poonia and Chaudhary (12) analyzed the MHD free convection and mass transfer flow over an infinite vertical porous plate with viscous dissipation. Kim (14) investigated unsteady MHD convective flow and heat transfer past a semi-infinite vertical porous moving plate with variable suction. Vijaya kumar et al. (20) studied the thermal diffusion and radiation effects on unsteady MHD flow, through porous medium with variable temperature and mass diffusion in the presence of heat source/sink.

Girish Kumar et al. (10) analyzed the mass transfer effects on MHD flows exponentially accelerated vertical plate in the presence of chemical reaction through porous media.

The study of convective fluid flow with mass transfer along a vertical porous plate in the presence of magnetic field and internal heat generation receiving considerable attention due to its useful applications in different branches of Science and Technology such as cosmical and geophysical science, fire engineering, combustion modeling etc. Soundalgekar (17) investigated unsteady free convection flow along vertical porous plate with different boundary conditions and viscous dissipation effect. Vajravelu (19) studied natural convection flow along a heated semi-infinite vertical plate with internal heat generation. Cookey et al. (5) studied influence of viscous dissipation and radiation on unsteady MHD free convective flow past an infinite heated vertical plate in a porous medium with time dependent suction. Chamkha (4) discussed unsteady MHD convective heat and mass transfer past a semi-infinite vertical permeable moving plate with heat generation. Ahmed (1) studied effects of unsteady free convective MHD flow through a porous medium bounded by an infinite vertical porous plate. Sharma and Singh (16) discussed unsteady MHD free convective flow and heat transfer along a vertical porous plate with variable suction and internal heat generation. Sharma et al. (15) analyzed the heat and mass transfer effects on unsteady MHD free convective flow along a vertical porous plate with internal heat generation and variable suction.

Mass diffusion rates can be changed tremendously with chemical reactions. The chemical reaction effects depend whether the reaction is homogeneous or heterogeneous. This depends on whether they occur at an interface or as a single phase volume reaction. In well-mixed systems, the reaction is heterogeneous, if it takes place at an interface and homogeneous, if it takes place in solution. In majority cases, a chemical reaction depends on the concentration of the species itself. A reaction is said to be first order, if the rate of reaction is directly proportional to the concentration itself (Cussler (6)). A few representative areas of interest in which heat and mass transfer combined along with chemical reaction play an important role in chemical industries like in food processing and polymer production. Chambre and Young (3) have analyzed a first order chemical reaction in the neighborhood of a horizontal plate. Das et al. (7) have studied the effect of homogeneous first order chemical reaction on the flow past an impulsively started vertical plate with uniform heat flux and mass transfer. Again, mass transfer effects on moving isothermal vertical plate in the presence of chemical reaction studied by Das et al. (8). The dimensionless governing equations were solved by the usual Laplace Transform technique. Sudheer Babu et al. (18) have analyzed the radiation and chemical reaction effects on an unsteady MHD convection flow past a vertical moving porous plate embedded in a porous medium with viscous dissipation.

The main objective of the present analysis is to study the unsteady two-dimensional MHD free convective oscillatory flow of an electrically conducting incompressible viscous fluid past an infinite vertical porous plate, through which suction occurs with constant velocity and chemical reaction in the presence of a heat sink. The equations of continuity, momentum, energy and diffusion which govern the flow field are solved to the best possible closed solution.

Mathematical Analysis:

We consider the unsteady two-dimensional MHD free convective oscillatory flow of an electrically conducting incompressible viscous fluid past an infinite vertical porous plate, through which suction occurs with constant velocity and chemical reaction in the presence of a heat sink. The x' - axis is along the plate in the upward direction and the y' - axis is normal to it. A uniform magnetic field is applied in the direction perpendicular to the plate. Reynolds number is much less than unity and the induced magnetic field is negligible in comparison with the applied magnetic field. It is also assumed that all the fluid properties are constant except that of the influence of the density variation with temperature and concentration in the body force term (Boussinesq's approximation). Also, there is a chemical reaction between the diffusing species and the fluid. The foreign mass present in the flow is assumed to be a low level and hence Soret and Dufour effects are negligible. Under these assumptions, the governing equations of the flow field are:

Continuity equation

$$\frac{\partial v'}{\partial y'} = 0 \quad (1)$$

Momentum equation

$$\rho \left(\frac{\partial u'}{\partial t'} + v' \frac{\partial u'}{\partial y'} \right) = - \frac{\partial p'}{\partial x'} - \rho g_{x'} + \nu \rho \frac{\partial^2 u'}{\partial y'^2} - (\sigma B_0^2)(u') \quad (2)$$

Energy equation

$$\frac{\partial T'}{\partial t'} + v' \frac{\partial T'}{\partial y'} = \frac{\kappa}{\rho c_p} \frac{\partial^2 T'}{\partial y'^2} + \frac{v}{c_p} \left(\frac{\partial u'}{\partial y'} \right)^2 + \frac{q'}{\rho c_p} (T' - T'_\infty) \quad (3)$$

Diffusion equation

$$\frac{\partial C'}{\partial t'} + v' \frac{\partial C'}{\partial y'} = D \frac{\partial^2 C'}{\partial y'^2} - K_r'(C' - C'_\infty) \quad (4)$$

Where u' and v' are the components of the velocity parallel and perpendicular to the plate, t' - the time, p' - the pressure, ρ - the fluid density, $g_{x'}$ - the acceleration due to gravity, T' - the fluid temperature, v - the kinematic viscosity, C_p - the specific heat at constant pressure, κ - the thermal conductivity, C' - the concentration and D - the chemical diffusivity.

The boundary conditions are:

$$\left. \begin{aligned} u' = 0, v' = -v_0, \frac{\partial T'}{\partial y'} = -\frac{q'}{\kappa}, C' = C'_w \text{ at } y' = 0 \\ u' \rightarrow U' = U_0(1 + \varepsilon e^{i\omega' t'}), T' \rightarrow T'_\infty, C' \rightarrow C'_\infty \text{ as } y' \rightarrow \infty \end{aligned} \right\} \quad (5)$$

Where v_0 is the constant suction velocity and the negative sign indicates that it is towards the plate, q' - the constant heat flux, T'_∞ - the fluid temperature far away from the plate, C'_w - the species concentration at the plate, C'_∞ - the species concentration far away from the plate, U_0 - the mean free stream velocity, ω' - the frequency of vibration of the fluid, and ε ($\varepsilon < 1$) - a constant quantity.

For the free stream, equation (2) becomes:

$$\rho \frac{dU'}{dt'} = -\frac{\partial p'}{\partial x'} - \rho_\infty g_{x'} - \sigma B_0^2 U' \quad (6)$$

On eliminating $\frac{\partial p'}{\partial x'}$ between (2) and (6) we get:

$$\rho \left(\frac{\partial u'}{\partial t'} + v' \frac{\partial u'}{\partial y'} \right) = \rho \frac{dU'}{dt'} + g_{x'}(\rho_\infty - \rho) + \nu \rho \frac{\partial^2 u'}{\partial y'^2} - (\sigma B_0^2)(u' - U'(t')) \quad (7)$$

The state equation is

$$g_{x'}(\rho_\infty - \rho) = g_{x'} \rho \beta (T' - T'_\infty) + g_{x'} \rho \beta^* (C' - C'_\infty) \quad (8)$$

Where β is the coefficient of thermal expansion and β^* is the coefficient of concentration expansion

Equation (1) gives:

$$v' = -v_0 (v_0 > 0) \quad (9)$$

On substituting equations (8), (9) in equations (3), (4) and (7) we take:

$$\frac{\partial u'}{\partial t'} - v_0 \frac{\partial u'}{\partial y'} = \frac{dU'}{dt'} + g_{x'} \beta (T' - T'_\infty) + g_{x'} \beta^* (C' - C'_\infty) + \nu \frac{\partial^2 u'}{\partial y'^2} - \left(\frac{\sigma B_0^2}{\rho} \right) (u' - U'(t')) \quad (10)$$

$$\frac{\partial T'}{\partial t'} - v_0 \frac{\partial T'}{\partial y'} = \frac{\kappa}{\rho c_p} \frac{\partial^2 T'}{\partial y'^2} + \frac{v}{c_p} \left(\frac{\partial u'}{\partial y'} \right)^2 + \frac{q'}{\rho c_p} (T' - T'_\infty) \quad (11)$$

$$\frac{\partial C'}{\partial t'} - v_0 \frac{\partial C'}{\partial y'} = D \frac{\partial^2 C'}{\partial y'^2} - K_r'(C' - C'_\infty) \quad (12)$$

Using the transformations:

$$y = \frac{y'v_0}{v}, t = \frac{t'v_0^2}{4v}, T = \frac{T' - T'_\infty}{\frac{vq'}{kv_0}}, C = \frac{C' - C'_\infty}{C'_w - C'_\infty}, u = \frac{u'}{U_0}, U = \frac{U'}{U_0}, \omega = \frac{4v\omega'}{v_0^2}, Gr = \frac{g_x'\beta v^2 q'}{kU_0 v_0^3} \text{ (Grashof number)}, Gc = \frac{vg\beta^*(C'_w - C'_\infty)}{U_0 v_0^2} \text{ (modified Grashof number)}, Pr = \frac{\rho v C_p}{k} \text{ (Prandtl number)}, Ec = \frac{kU_0^2 v_0}{C_p v q'} \text{ (Eckert number)}, M = \frac{\sigma B_0^2 v}{\rho v_0^2} \text{ (Magnetic parameter)}, Sc = \frac{v}{D} \text{ (Schmidt number)}, Q = \frac{v^2 Q'}{\kappa v_0^2} \text{ (Heat generation/absorption)}, Kr = \frac{Kr'v}{v_0^2} \text{ (Chemical reaction parameter)} \quad (13)$$

With the help of the non-dimensional quantities (13), equations (10)-(12) reduce to:

$$\frac{1}{4} \frac{\partial u}{\partial t} - \frac{\partial u}{\partial y} = \frac{1}{4} \frac{dU}{dt} + GrT + GcC + \frac{\partial^2 u}{\partial y^2} - M(u - U) \quad (14)$$

$$Pr \left(\frac{1}{4} \frac{\partial T}{\partial t} - \frac{\partial T}{\partial y} \right) = \frac{\partial^2 T}{\partial y^2} + PrEc \left(\frac{\partial u}{\partial y} \right)^2 + QT \quad (15)$$

$$Sc \left(\frac{1}{4} \frac{\partial C}{\partial t} - \frac{\partial C}{\partial y} \right) = \frac{\partial^2 C}{\partial y^2} - KrScC \quad (16)$$

With the boundary conditions:

$$\left. \begin{aligned} u = 0, \quad \frac{\partial T}{\partial y} = -1, \quad C = 1 \quad \text{at } y = 0 \\ u \rightarrow U(t) = 1 + \varepsilon e^{i\omega t}, T \rightarrow 0, C \rightarrow 0 \quad \text{as } y \rightarrow \infty \end{aligned} \right\} \quad (17)$$

In order to solve the system of differential equations (14)-(16) we assume that:

$$u(y, t) = u_0(y) + \varepsilon e^{i\omega t} u_1(y) + \dots \quad (18)$$

$$T(y, t) = T_0(y) + \varepsilon e^{i\omega t} T_1(y) + \dots \quad (19)$$

$$C(y, t) = C_0(y) + \varepsilon e^{i\omega t} C_1(y) + \dots \quad (20)$$

On substituting equations (18)-(19) in equations (14)-(16) we get the following system of differential equations:

$$\frac{d^2 u_0}{dy^2} + \frac{du_0}{dy} - Mu_0 = -[GrT_0 + GcC_0 + M] \quad (21)$$

$$\frac{d^2 u_1}{dy^2} + \frac{du_1}{dy} - \left(\frac{i\omega}{4} - M \right) u_1 = -[GrT_1 + GcC_1 + \left(\frac{i\omega}{4} + M \right)] \quad (22)$$

$$\frac{d^2 T_0}{dy^2} + Pr \frac{dT_0}{dy} + QT_0 = -PrEc \left(\frac{du_0}{dy} \right)^2 \quad (23)$$

$$\frac{d^2 T_1}{dy^2} + Pr \frac{dT_1}{dy} - \frac{i\omega}{4} PrT_1 + QT_1 = -2PrEc \left(\frac{du_0}{dy} \right) \left(\frac{du_1}{dy} \right) \quad (24)$$

$$\frac{d^2 C_0}{dy^2} + Sc \frac{dC_0}{dy} - KrScC_0 = 0 \quad (25)$$

$$\frac{d^2 C_1}{dy^2} + Sc \frac{dC_1}{dy} - Sc \left[\frac{i\omega}{4} - Kr \right] C_1 = 0 \quad (26)$$

The corresponding boundary conditions (17) are:

$$\left. \begin{aligned} u_0 = 0, u_1 = 0, \frac{dT_0}{dy} = -1, \frac{dT_1}{dy} = 0, C_0 = 1, C_1 = 0 \quad \text{at } y = 0 \\ u_0 \rightarrow 1, u_1 \rightarrow 1, T_0 \rightarrow 0, T_1 \rightarrow 0, C_0 \rightarrow 0, C_1 \rightarrow 0 \quad \text{as } y \rightarrow \infty \end{aligned} \right\} \quad (27)$$

In order to solve the system of the differential equations (21)-(26) we put:

$$\left. \begin{aligned} u_0(y) &= u_{01}(y) + Ec u_{02}(y) \\ T_0(y) &= T_{01}(y) + Ec T_{02}(y) \\ C_0(y) &= C_{01}(y) + Ec C_{02}(y) \end{aligned} \right\} \quad (28)$$

$$\left. \begin{aligned} u_1(y) &= u_{11}(y) + Ec u_{12}(y) \\ T_1(y) &= T_{11}(y) + Ec T_{12}(y) \\ C_1(y) &= C_{11}(y) + Ec C_{12}(y) \end{aligned} \right\} \quad (29)$$

In this system, equating the coefficients of Ec^0 and Ec^1 we get:

$$\frac{d^2 u_{01}}{dy^2} + \frac{du_{01}}{dy} - Mu_{01} = -(GrT_{01} + GcC_{01} + M) \quad (30)$$

$$\frac{d^2 u_{02}}{dy^2} + \frac{du_{02}}{dy} - Mu_{02} = -(GrT_{02} + GcC_{02}) \quad (31)$$

$$\frac{d^2 T_{01}}{dy^2} + Pr \frac{dT_{01}}{dy} + QT_{01} = 0 \quad (32)$$

$$\frac{d^2 T_{02}}{dy^2} + Pr \frac{dT_{02}}{dy} + QT_{02} = -2PrEc \left(\frac{du_{01}}{dy} \right)^2 \quad (33)$$

$$\frac{d^2 C_{01}}{dy^2} + Sc \frac{dC_{01}}{dy} - KrScC_{01} = 0 \quad (34)$$

$$\frac{d^2 C_{02}}{dy^2} + Sc \frac{dC_{02}}{dy} - KrScC_{02} = 0 \quad (35)$$

$$\frac{d^2 u_{11}}{dy^2} + \frac{du_{11}}{dy} - \left(\frac{i\omega}{4} + M \right) u_{11} = - \left(\frac{i\omega}{4} + GrT_{11} + GcC_{11} + M \right) \quad (36)$$

$$\frac{d^2 u_{12}}{dy^2} + \frac{du_{12}}{dy} - \left(\frac{i\omega}{4} + M \right) u_{11} = -(GrT_{12} + GcC_{12}) \quad (37)$$

$$\frac{d^2 T_{11}}{dy^2} + Pr \frac{dT_{11}}{dy} - \left(\frac{i\omega}{4} Pr - Q \right) T_{11} = 0 \quad (38)$$

$$\frac{d^2 T_{12}}{dy^2} + Pr \frac{dT_{12}}{dy} - \left(\frac{i\omega}{4} Pr - Q \right) T_{12} = -2PrEc \left(\frac{du_{01}}{dy} \right) \left(\frac{du_{11}}{dy} \right) \quad (39)$$

$$\frac{d^2 C_{11}}{dy^2} + Sc \frac{dC_{11}}{dy} - Sc \left(\frac{i\omega}{4} - Kr \right) C_{11} = 0 \quad (40)$$

$$\frac{d^2 C_{12}}{dy^2} + Sc \frac{dC_{12}}{dy} - Sc \left(\frac{i\omega}{4} - Kr \right) C_{12} = 0 \quad (41)$$

The corresponding boundary conditions (27) become:

$$\left. \begin{aligned} u_{00} = 0, \quad u_{01} = 0, \quad u_{11} = 0, \quad u_{12} = 0 \\ \frac{dT_{00}}{dy} = -1, \quad \frac{dT_{01}}{dy} = 0, \quad \frac{dT_{11}}{dy} = 0, \quad \frac{dT_{12}}{dy} = 0 \\ C_{00} = 1, \quad C_{01} = 0, \quad C_{11} = 0, \quad C_{12} = 0 \end{aligned} \right\} \text{at } y = 0 \quad (42)$$

$$\left. \begin{aligned} u_{00} \rightarrow 1, \quad u_{01} \rightarrow 0, \quad u_{11} \rightarrow 1, \quad u_{12} \rightarrow 0 \\ T_{00} \rightarrow 0, \quad T_{01} \rightarrow 0, \quad T_{11} \rightarrow 0, \quad T_{12} \rightarrow 0 \\ C_{00} \rightarrow 0, \quad C_{01} \rightarrow 0, \quad C_{11} \rightarrow 0, \quad C_{12} \rightarrow 0 \end{aligned} \right\} \text{as } y \rightarrow \infty$$

Solution of the Problem:

Solving these differential equations from (30) – (41), using boundary conditions (42), and then making use of equations (28) – (29), finally with the help of equations (18), (19) and (20) we obtain the velocity, temperature and concentration fields are as follows:

$$u_{10} = \alpha_5 e^{\alpha_2 y} + \alpha_3 e^{\beta_2 y} + \alpha_4 e^{\gamma_2 y} + 1$$

$$u_{02} = \alpha_{13} e^{\alpha_2 y} + \alpha_6 e^{\beta_2 y} + \alpha_7 e^{2\alpha_2 y} + \alpha_8 e^{2\beta_2 y} + \alpha_9 e^{2\gamma_2 y} + \alpha_{10} e^{(\alpha_2 + \beta_2)y} + \alpha_{11} e^{(\beta_2 + \gamma_2)y} + \alpha_{12} e^{(\gamma_2 + \alpha_2)y}$$

$$u_0 = (\alpha_5 e^{\alpha_2 y} + \alpha_3 e^{\beta_2 y} + \alpha_4 e^{\gamma_2 y} + 1) \\ + Ec(\alpha_{13} e^{\alpha_2 y} + \alpha_6 e^{\beta_2 y} + \alpha_7 e^{2\alpha_2 y} + \alpha_8 e^{2\beta_2 y} + \alpha_9 e^{2\gamma_2 y} + \alpha_{10} e^{(\alpha_2 + \beta_2)y} + \alpha_{11} e^{(\beta_2 + \gamma_2)y} \\ + \alpha_{12} e^{(\gamma_2 + \alpha_2)y})$$

$$u_{11} = -e^{\alpha_{15} y} + 1$$

$$u_{12} = \alpha_{20} e^{\alpha_{15} y} + \alpha_{16} e^{\beta_{11} y} + \alpha_{17} e^{(\alpha_2 + \alpha_{15})y} + \alpha_{18} e^{(\beta_2 + \alpha_{15})y} + \alpha_{19} e^{(\gamma_2 + \alpha_{15})y}$$

$$u_1 = (-e^{\alpha_{15} y} + 1) + Ec(\alpha_{20} e^{\alpha_{15} y} + \alpha_{16} e^{\beta_{11} y} + \alpha_{17} e^{(\alpha_2 + \alpha_{15})y} + \alpha_{18} e^{(\beta_2 + \alpha_{15})y} + \alpha_{19} e^{(\gamma_2 + \alpha_{15})y})$$

$$T_{01} = \frac{-1}{\beta_2} e^{\beta_2 y}$$

$$T_{02} = \beta_9 e^{\beta_2 y} + \beta_3 e^{2\alpha_2 y} + \beta_4 e^{2\beta_2 y} + \beta_5 e^{2\gamma_2 y} + \beta_6 e^{(\alpha_2 + \beta_2)y} + \beta_7 e^{(\beta_2 + \gamma_2)y} + \beta_8 e^{(\alpha_2 + \gamma_2)y}$$

$$T_0 = \left(\frac{-1}{\beta_2} e^{\beta_2 y} \right) + Ec(\beta_9 e^{\beta_2 y} + \beta_3 e^{2\alpha_2 y} + \beta_4 e^{2\beta_2 y} + \beta_5 e^{2\gamma_2 y} + \beta_6 e^{(\alpha_2 + \beta_2)y} + \beta_7 e^{(\beta_2 + \gamma_2)y} + \beta_8 e^{(\alpha_2 + \gamma_2)y})$$

$$T_{11} = 0$$

$$T_{12} = \beta_{15} e^{\beta_{11} y} + \beta_{12} e^{(\alpha_2 + \alpha_{15})y} + \beta_{13} e^{(\beta_2 + \alpha_{15})y} + \beta_{14} e^{(\gamma_2 + \alpha_{15})y}$$

$$T_1 = Ec(\beta_{15} e^{\beta_{11} y} + \beta_{12} e^{(\alpha_2 + \alpha_{15})y} + \beta_{13} e^{(\beta_2 + \alpha_{15})y} + \beta_{14} e^{(\gamma_2 + \alpha_{15})y})$$

$$C_{01} = e^{\gamma_2 y},$$

$$C_{02} = 0$$

$$C_0 = e^{\gamma_2 y}$$

$$C_{11} = 0$$

$$C_{12} = 0$$

$$C_1 = 0$$

$$u(y) = u_0 + \varepsilon(\cos(wt) + i\sin(wt))u_1 \\ = ((\alpha_5 e^{\alpha_2 y} + \alpha_3 e^{\beta_2 y} + \alpha_4 e^{\gamma_2 y} + 1) \\ + Ec(\alpha_{13} e^{\alpha_2 y} + \alpha_6 e^{\beta_2 y} + \alpha_7 e^{2\alpha_2 y} + \alpha_8 e^{2\beta_2 y} + \alpha_9 e^{2\gamma_2 y} + \alpha_{10} e^{(\alpha_2 + \beta_2)y} + \alpha_{11} e^{(\beta_2 + \gamma_2)y} \\ + \alpha_{12} e^{(\gamma_2 + \alpha_2)y})) + \varepsilon(\cos(wt) + i\sin(wt))(-e^{\alpha_{15} y} + 1) \\ + Ec(\alpha_{20} e^{\alpha_{15} y} + \alpha_{16} e^{\beta_{11} y} + \alpha_{17} e^{(\alpha_2 + \alpha_{15})y} + \alpha_{18} e^{(\beta_2 + \alpha_{15})y} + \alpha_{19} e^{(\gamma_2 + \alpha_{15})y})$$

$$T(y) = T_0 + \varepsilon(\cos(wt) + i\sin(wt))T_1 \\ = \left(\left(\frac{-1}{\beta_2} e^{\beta_2 y} \right) \right. \\ \left. + Ec(\beta_9 e^{\beta_2 y} + \beta_3 e^{2\alpha_2 y} + \beta_4 e^{2\beta_2 y} + \beta_5 e^{2\gamma_2 y} + \beta_6 e^{(\alpha_2 + \beta_2)y} + \beta_7 e^{(\beta_2 + \gamma_2)y} + \beta_8 e^{(\alpha_2 + \gamma_2)y}) \right) \\ + \varepsilon(\cos(wt) + i\sin(wt)) \left(Ec(\beta_{15} e^{\beta_{11} y} + \beta_{12} e^{(\alpha_2 + \alpha_{15})y} + \beta_{13} e^{(\beta_2 + \alpha_{15})y} + \beta_{14} e^{(\gamma_2 + \alpha_{15})y}) \right)$$

$$C(y) = C_0 + \varepsilon(\cos(wt) + i\sin(wt))C_1 = e^{\gamma_2 y}$$

RESULTS AND DISCUSSION

The chemical reaction effects on MHD free convective oscillatory flow past a porous plate in the presence of heat sink have been studied. The governing equations are solved by using perturbation method and approximate solutions are obtained for velocity, temperature and concentration fields. The effects of the flow parameters such as magnetic parameter (M), suction parameter (S), Grashof number for heat and mass transfer (Gr , Gc), Schmidt number (Sc), Chemical reaction parameter (Kr), Prandtl number (Pr) and Eckert number (Ec) on the velocity, temperature and concentration profiles of the flow field are presented with help of velocity profiles (Figures 1-8), temperature profiles (Figures 9-11) and concentration profile (Figures 12-13).

Figures 1(a) and 1(b) display the influence of chemical reaction parameter (Kr) on the transient velocity (u) and concentration (C). It is clear that increasing the chemical reaction parameter tends to decrease the velocity as well as species concentration of the fluid. This means that in the case of suction, the chemical reaction decelerates the fluid motion. In turn, this causes the concentration buoyancy effects to decrease as k increases. Consequently, less flow is induced along the plate resulting in decrease in the fluid velocity in the boundary layer.

Figs. 2(a) and 2(b) depict the velocity (u) and temperature (T) profiles for different values of the heat sink parameter Q . It is noticed that an increase in the heat sink parameter Q results in an increase in the velocity and temperature within the boundary layer.

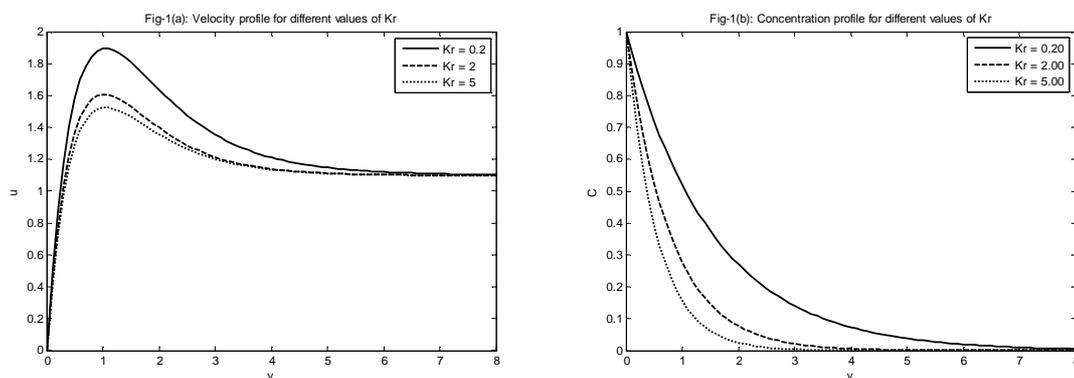
Figs. 3(a) and 3(b) are shown that the behavior of the velocity (u) and temperature (T) for different values of the Prandtl number Pr . The numerical results show that the effect of increasing values of Pr results in a decreasing velocity. From Fig. 3(b), it is observed that an increase in Pr results in a decrease of the thermal boundary layer thickness and in general lower average temperature within the boundary layer. The reason is that smaller values of Pr are equivalent to increase in the thermal conductivity of the fluid and therefore heat is able to diffuse away from the heated surface more rapidly for higher values of Pr .

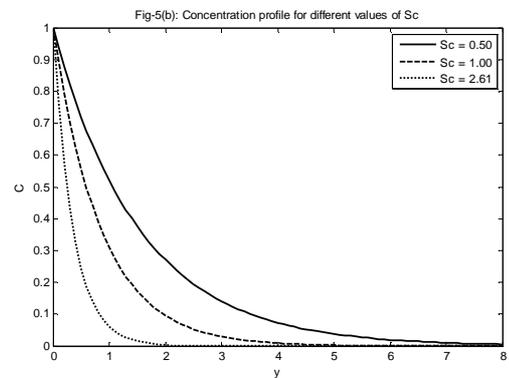
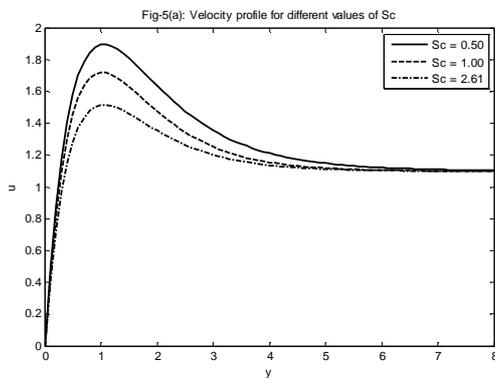
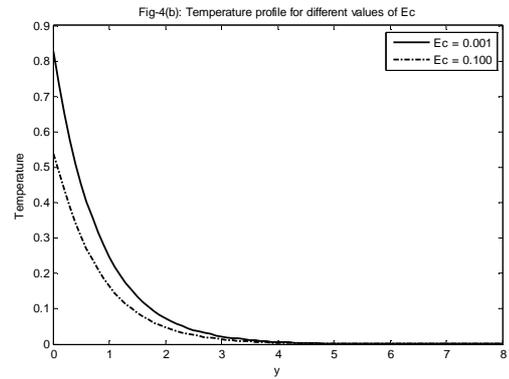
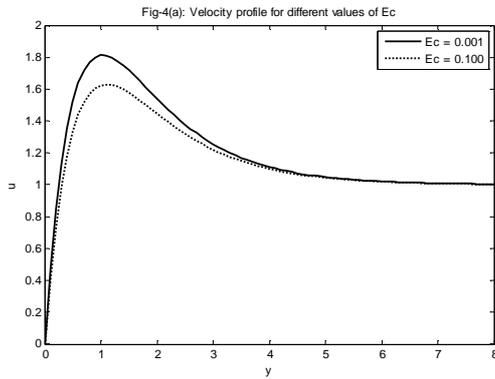
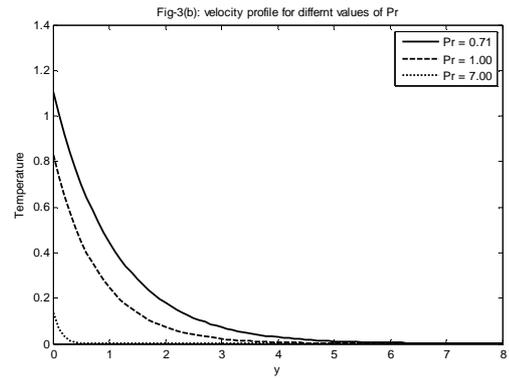
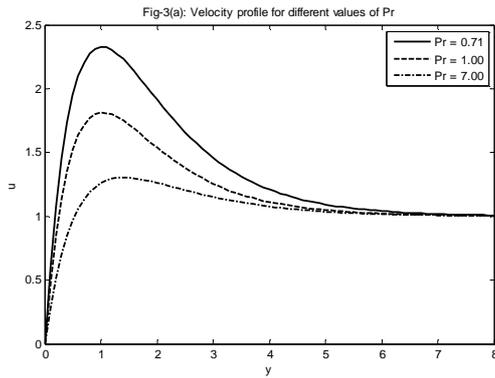
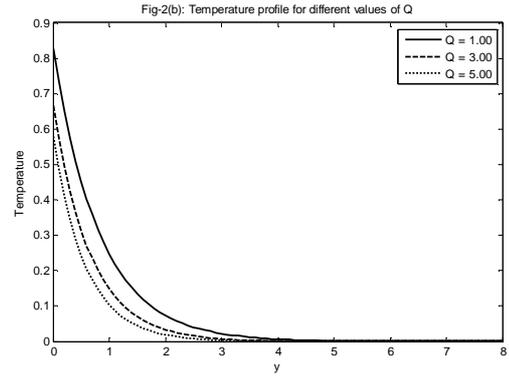
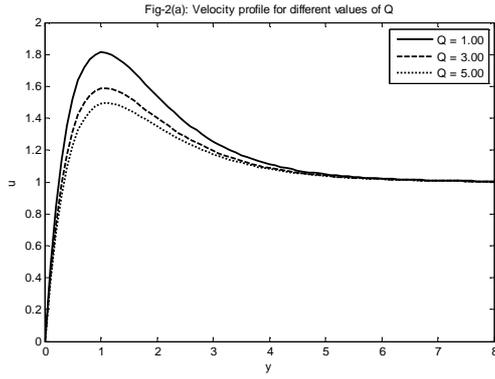
The effects of viscous dissipative heat (Ec) on the transient velocity (u) as well as temperature (T) have been plotted in Fig: 4(a) and 4(b). It is noticed that an increase in viscous dissipative heat leads to increase in both the velocity as well as the temperature.

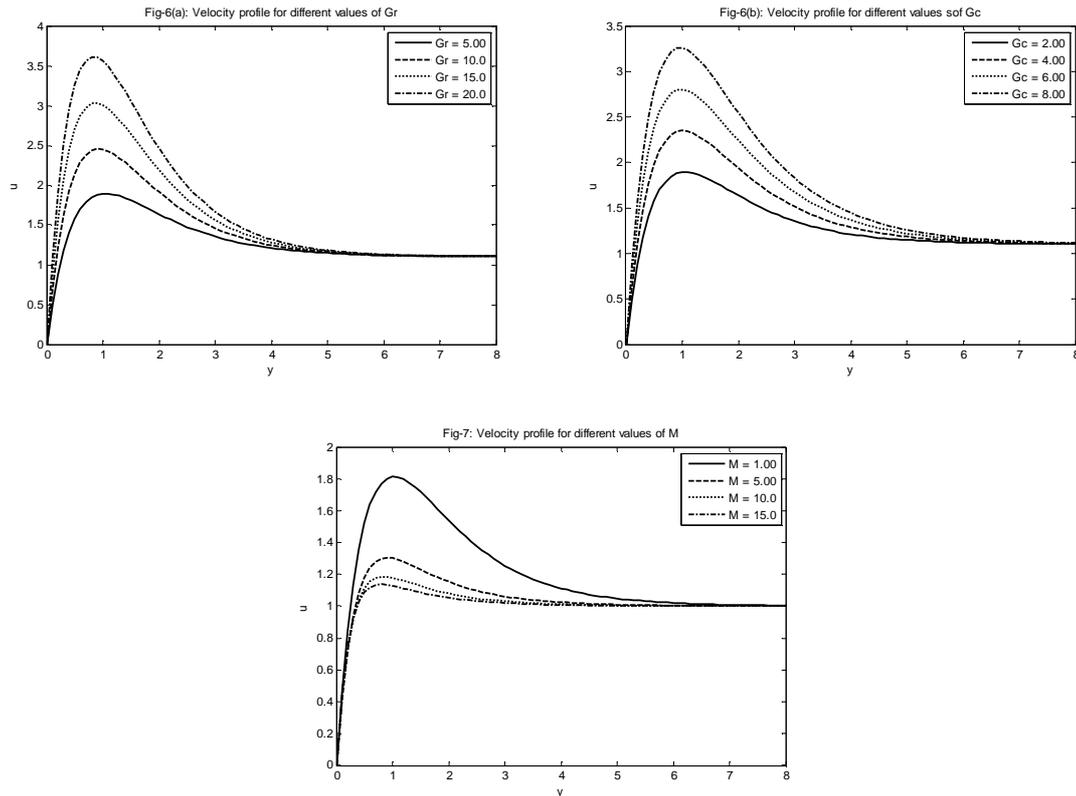
Fig: 5(a) and 5(b) illustrates the velocity (u) and concentration profiles (C) for Schmidt number (Sc). It is noticed that effect of increasing value of Sc is to decrease transient velocity as well as concentration profiles. This is consistent with the fact that, increase in Sc means decrease of molecular diffusivity D those results in decrease of fluid motion and concentration boundary layer. Hence velocity and concentration of species is higher for small values of Sc and lower for larger values of Sc .

The velocity (u) profiles for different values of the thermal Grashof number Gr are described in Figure 6(a). It is observed that an increase in Gr leads to rise in the values of velocity. For the case of different values of the modified Grashof number Gc , the velocity profiles are shown in the Figure 6(b). It is observed that an increase in Gr leads to a rise in the values of velocity.

The influence of magnetic parameter M , on the velocity (u) is shown in Fig.7. An increase in M reduces the velocity. The application of a transverse magnetic field to an electrically conducting field gives rise to a resistive type of force called Lorentz force. This force has the tendency to slow down the fluid. This trend is evident from Fig.7.







Appendix:

$$\alpha_1 = \frac{-1+\sqrt{1+4M}}{2}, \alpha_2 = \frac{-1-\sqrt{1+4M}}{2}, \alpha_3 = \frac{Gr}{\beta_2(\beta_2^2 + \beta_2 - M)}, \alpha_4 = \frac{Gc}{\gamma_2^2 + \gamma_2 - M}, \alpha_5 = -[1 + \alpha_3 + \alpha_4]$$

$$\alpha_6 = \frac{-Gr\beta_9}{\beta_2^2 + \beta_2 - M}, \alpha_7 = \frac{-Gr\beta_3}{(2\alpha_2)^2 + (2\alpha_2) - M}, \alpha_8 = \frac{-Gr\beta_4}{(2\beta_2)^2 + (2\beta_2) - M},$$

$$\alpha_9 = \frac{-Gr\beta_5}{(2\gamma_2)^2 + (2\gamma_2) - M}, \alpha_{10} = \frac{-Gr\beta_6}{(\alpha_2 + \beta_2)^2 + (\alpha_2 + \beta_2) - M}, \alpha_{11} = \frac{-Gr\beta_7}{(\gamma_2 + \beta_2)^2 + (\gamma_2 + \beta_2) - M},$$

$$\alpha_{12} = \frac{-Gr\beta_8}{(\gamma_2 + \alpha_2)^2 + (\gamma_2 + \alpha_2) - M}, \alpha_{13} = -[\alpha_6 + \alpha_7 + \alpha_8 + \alpha_9 + \alpha_{10} + \alpha_{11} + \alpha_{12}],$$

$$\alpha_{14} = \frac{-1+\sqrt{1+4(\frac{i\omega}{4}+M)}}{2}, \alpha_{15} = \frac{-1-\sqrt{1+4(\frac{i\omega}{4}+M)}}{2},$$

$$\alpha_{16} = \frac{-Gr\beta_{15}}{\beta_{11}^2 + \beta_{11} - (\frac{i\omega}{4} + M)}, \alpha_{17} = \frac{-Gr\beta_{12}}{(\alpha_2 + \alpha_{15})^2 + (\alpha_2 + \alpha_{15}) - (\frac{i\omega}{4} + M)},$$

$$\alpha_{18} = \frac{-Gr\beta_{13}}{(\beta_2 + \alpha_{15})^2 + (\beta_2 + \alpha_{15}) - (\frac{i\omega}{4} + M)}, \alpha_{19} = \frac{-Gr\beta_{14}}{(\gamma_2 + \alpha_{15})^2 + (\gamma_2 + \alpha_{15}) - (\frac{i\omega}{4} + M)},$$

$$\alpha_{20} = -[\alpha_{16} + \alpha_{17} + \alpha_{18} + \alpha_{19}],$$

$$\beta_1 = \frac{-Pr + \sqrt{Pr^2 + 4(-Q)}}{2}, \beta_2 = \frac{-Pr - \sqrt{Pr^2 + 4(-Q)}}{2}, \beta_3 = \frac{-2PrEc(\alpha_2\alpha_5)^2}{(2\alpha_2)^2 + Pr(2\alpha_2) + Q},$$

$$\beta_4 = \frac{-2PrEc(\beta_2\alpha_3)^2}{(2\beta_2)^2 + Pr(2\beta_2) + Q}, \beta_5 = \frac{-2PrEc(\gamma_2\alpha_4)^2}{(2\gamma_2)^2 + Pr(2\gamma_2) + Q}, \beta_6 = \frac{-4PrEc\alpha_2\alpha_3\alpha_5\beta_2}{(\alpha_2 + \beta_2)^2 + Pr(\alpha_2 + \beta_2) + Q},$$

$$\beta_7 = \frac{-4PrEc\beta_2\gamma_2\alpha_3\alpha_4}{(\beta_2 + \gamma_2)^2 + Pr(\beta_2 + \gamma_2) + Q}, \beta_8 = \frac{-4PrEc\alpha_2\gamma_2\alpha_4\alpha_5}{(\alpha_2 + \gamma_2)^2 + Pr(\alpha_2 + \gamma_2) + Q},$$

$$\beta_9 = \frac{-1}{\beta_2} [2\alpha_2\beta_3 + 2\beta_2\beta_5 + 2\gamma_2\beta_5 + (\alpha_2 + \beta_2)\beta_6 + (\beta_2 + \gamma_2)\beta_7 + (\alpha_2 + \gamma_2)\beta_8]$$

$$\beta_{10} = \frac{-Pr + \sqrt{Pr^2 + 4\left(\frac{i\omega}{4}Pr - Q\right)}}{2}, \beta_{11} = \frac{-Pr - \sqrt{Pr^2 + 4\left(\frac{i\omega}{4}Pr - Q\right)}}{2},$$

$$\beta_{12} = \frac{2PrEc\alpha_2\alpha_5\alpha_{15}}{(\alpha_2 + \alpha_{15})^2 + Pr(\alpha_2 + \alpha_{15}) - \left(\frac{i\omega}{4}Pr - Q\right)}, \beta_{13} = \frac{2PrEc\beta_2\alpha_3\alpha_{15}}{(\beta_2 + \alpha_{15})^2 + Pr(\beta_2 + \alpha_{15}) - \left(\frac{i\omega}{4}Pr - Q\right)}$$

$$\beta_{14} = \frac{2PrEc\gamma_2\alpha_4\alpha_{15}}{(\gamma_2 + \alpha_{15})^2 + Pr(\gamma_2 + \alpha_{15}) - \left(\frac{i\omega}{4}Pr - Q\right)},$$

$$\beta_{15} = \frac{-1}{\beta_{11}} [\beta_{12}(\alpha_2 + \alpha_{15}) + \beta_{13}(\beta_2 + \alpha_{15}) + \beta_{14}(\gamma_2 + \alpha_{15})]$$

$$\gamma_1 = \frac{-Sc + \sqrt{Sc^2 + 4KrSc}}{2}, \gamma_2 = \frac{-Sc - \sqrt{Sc^2 + 4KrSc}}{2}, \gamma_3 = \frac{-Sc + \sqrt{Sc^2 + 4\left(\frac{i\omega}{4} - Kr\right)Sc}}{2},$$

$$\gamma_4 = \frac{-Sc - \sqrt{Sc^2 + 4\left(\frac{i\omega}{4} - Kr\right)Sc}}{2}$$

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