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Effect of Wall Absorption on dispersion of a solute in a Herschel –Bulkley Fluid through an annulus

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ABSTRACT

The combined effect of yield stress, irreversible boundary reaction on the dispersion of a tracer in annular region and power index on dispersion process is studied using generalized dispersion model. The study describes the dispersive transport following the injection of a tracer in terms of three effective transport coefficients, viz. the exchange, the convection and the dispersive coefficients. The effect of power index, annular gap, yield stress and wall absorption parameter on the above three effective transport coefficients is discussed. The effect of flow parameters on mean concentration is studied.

Key-Words Generalized dispersion model; Herschel Bulkley fluid; Steady flow

INTRODUCTION

In dispersion of a solute in an annular flow when it is either irreversibly absorbed or undergoes an exchange process at the boundary have several applications. In mass-transfer devices, the formation of concentration gel and polarization layers at transfer surfaces degrades the performance of such devices. The efficiency of such devices, fouling of surfaces inhibits the efficiency. It is established that the fouling in the surfaces can be reduced when the mixing with in the flow conduit is increased.

The study of mass and heat transfer in an annular flow is significant in view of its abundant applications in exothermic chemical reactions, geothermal energy recovery, nuclear reactors. The annular chromatographic method used for separation of metals, sugars and proteins etc. [1, 2] is another example. The indicator dilution technique using catheters to inject dye and to withdraw blood samples is an example in clinical medicine.

Davidson and Schroter [3] studied the pattern of dispersion and uptake an inhaled slug of tissue soluble gas with in a bronchial wall as an assembly of straight rigid tubes with absorbing wall of finite thickness. They observed greater penetration for lower solubility of the gas and the mixing coefficient was shown to decrease with distance into the lung to a value, which may be much smaller than the molecular diffusivity. Phillips et al. [4] investigated the transport of a tracer through a wall layer consisting of a tube containing flowing fluid surrounded by a wall layer in which the tracer was soluble. They observed that the effective convection and dispersion coefficients were of little use in predicting the time - varying concentration at a fixed position as the spatial concentration profile became Gaussian only over the longer time scale. Jayaraman et al. [5] extended the theoretical model of Davidson and Schroter [3] to study the dispersion of solute in a fluid flowing through a curved tube with absorbing walls by using a mathematical model of an infinitely long conduit defined by two concentric curved circular pipes. Their results based on perturbation and spectral methods, confirmed the earlier experimental findings that the influence of secondary flows on the dispersion was reduced if the tracer was very soluble in the wall. Nagarani et al. [6] analyzed the effect of wall absorption on the dispersion of solute in a Casson fluid flowing in an annulus.

Blair and Spanner [7] reported that blood obeys Casson equation only in a limited range, except at very high and very low shear rate and that there is no difference between the Casson plots and the Herschel–Bulkley plots of experimental data over the range where the Casson plot is valid. It is observed that the Casson fluid model can be used for moderate shear rates $\gamma < 10/s$ in smaller diameter tubes whereas the Herschel– Bulkley fluid model can be used at still lower shear rate of flow in very narrow arteries where the yield stress is high [8]. Further, the mathematical model of Herschel–Bulkley fluid also describes the behaviour of Newtonian fluid, Bingham fluid and power law fluid by taking appropriate values of the parameters viz. yield stress and power law index.

In this paper an attempt has been made to study the effect of interphase mass transfer at the outer boundary in the study of dispersion of solute in a Herschel-Bulkley fluid flowing in an annulus. Section 2 presents the mathematical formulation of the problem with appropriate initial and boundary conditions. Section 3 gives the solution of the mathematical model by using the modified derivative expansion method. In section 4 the effect of wall absorption parameters, yield stress and annular gap and the power law index on the three dispersion coefficients viz. absorption, convection and dispersion coefficients and mean concentration is discussed. Section 5 describes the application of the mathematical model to the dispersion in a catheterized artery. The conclusions are presented in section 6

2 Mathematical Formulation

Figure 1 shows the schematic diagram of the annular geometry. The radius of the outer tube is a and that of the inner tube is ka with $k < 1$. For a fully developed laminar flow of a Herschel Bulkley fluid in an annulus, the convective diffusion equation, which describes the local concentration C of a solute as a function of axial distance z, radial distance r and time t in non-dimensional form is given by

$$\frac{\partial C}{\partial t} + w\frac{\partial C}{\partial z} = \left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + \frac{1}{Pe^2}\frac{\partial^2}{\partial z^2}\right)C \tag{1}$$

with the non-dimensional variables

$$C = \frac{\overline{C}}{C_0},\ w = \frac{\overline{w}}{w_0},\ r = \frac{\overline{r}}{a},\ z = \frac{D_m\overline{z}}{a^2 w_0},\ t = \frac{D_m\overline{t}}{a^2},\ Pe = \frac{aw_0}{D_m} \tag{2}$$

where w is non-dimensional axial velocity of the fluid, D_m is coefficient of molecular diffusion, assumed to be constant. C_0 is the reference concentration, Pe is Peclet number, w_0 is the characteristic velocity.

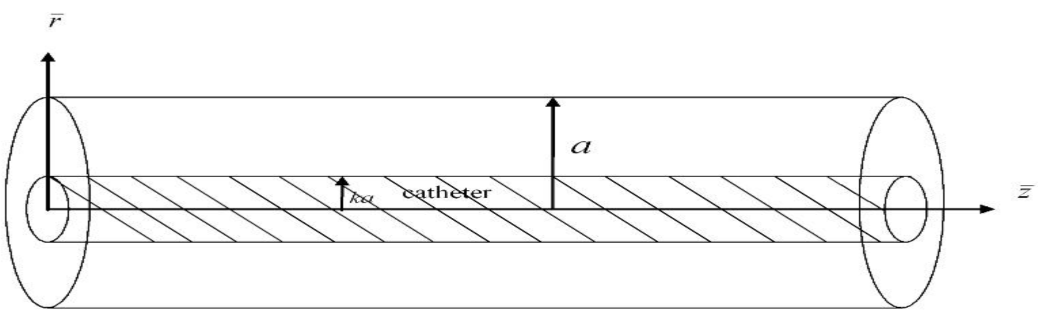

Fig 1 Schematic diagram of a catheterized artery

Initial and Boundary conditions

At an instant of time, the amount of tracer left in the system, its convective velocity and the extent of shear distribution depend upon the initial discharge. The initial distribution is taken at $t = 0$ as the case when the solute of mass m is introduced instantaneously at the plane z = 0, uniformly over the cross-section of a circle of radius, da (where $k < d < 1$) concentric, in the annular region, with the tube. Thus, the initial distribution of the solute assumed in a variable separable form in terms of the non-dimensional quantities is given by

$$C(0,z,r) = \psi_1(z)B_1(r) \qquad \text{for } k < r < 1 \tag{3}$$

with $\psi_1(z) = \delta(z)/((d^2 - k^2)Pe)$ (4.1)

where $\delta(z)$ denotes the Dirac delta function,

and $$B_1(r) = \begin{cases} 1 & k < r \le d \\ 0 & d < r \le 1 \end{cases} \tag{4.2}$$

In the current model we consider the heterogeneous reaction mechanism occurring at the outer wall (r = 1) of the tube so that

$$\frac{\partial C}{\partial r}(t, z, 1) = -\beta C(t, z, 1) \tag{5}$$

where β is the non-dimensional wall reaction parameter. In addition we have the boundary condition at $r = k$

$$\frac{\partial C}{\partial r}(t, z, k) = 0 \tag{6}$$

describing the impermeable wall of the inner tube.

and $$C(t, \infty, r) = \frac{\partial C}{\partial z}(t, \infty, r) = 0 \tag{7}$$

The constitutive equation for a Herschel Bulkley fluid relating the stress (τ) and shear rate $\left(\frac{dw}{dr}\right)$ in non-dimensional form is given by

$$\tau = \tau_y + \left(-\frac{dw}{dr}\right)^{1/n} \qquad \text{if } \tau \ge \tau_y \tag{8.1}$$

and $$\frac{dw}{dr} = 0, \qquad \text{if } \tau \le \tau_y \tag{8.2}$$

where $$\tau = \frac{\bar{\tau}}{2\mu_\infty (w_0/a)}, \qquad \tau_y = \frac{\bar{\tau}_y}{2\mu_\infty (w_0/a)} \tag{8.3}$$

$\bar{\tau}$ and $\bar{\tau}_y$ are the dimensional shear stress and yield stress respectively and τ_y is dimensionless yield stress. The velocity equations are given by

$$w(r) = w^+(r) = 2\left[\int_k^r \left(\frac{\lambda^2 - r^2}{r}\right)^n dr - n\Lambda \int_k^r \left(\frac{\lambda^2 - r^2}{r}\right)^{n-1} dr\right] \qquad \text{if } k \le r \le \lambda_1 \tag{9.1}$$

$$w = w^-(r) = w_p = w^+(\lambda_1) = w^{++}(\lambda_2) = \text{constant} \qquad \text{if } \lambda_1 \le r \le \lambda_2 \tag{9.2}$$

$$w(r) = w^{++}(r) = 2\left[\int_r^1 \left(\frac{r^2 - \lambda^2}{r}\right)^n dr - n\Lambda \int_r^1 \left(\frac{r^2 - \lambda^2}{r}\right)^{n-1} dr\right] \qquad \text{if } \lambda_2 \le r \le 1 \tag{9.3}$$

where $\Lambda = \lambda_2 - \lambda_1 = \tau_y$ (9.4)

is the width of the plug flow region and $\lambda^2 = \lambda_1 \lambda_2$, (9.5)

The superscripts '+' and '+ +' represents the shear flow region $k \le r \le \lambda_1$ and $\lambda_2 \le r \le 1$, respectively, and the superscript ' – ' represents the plug flow region $\lambda_1 \le r \le \lambda_2$. From eq (9.4) and from (9.2) and using (9.5), we obtain

$$\int_k^{\lambda_1} \left((\lambda_1(\Lambda+\lambda_1)-r^2)/r\right)^n dr - \int_{\Lambda+\lambda_1}^{1} \left((r^2-\lambda_1(\Lambda+\lambda_1))/r\right)^n dr - n\Lambda \int_k^{\lambda_1} \left((\lambda_1(\Lambda+\lambda_1)-r^2)/r\right)^{n-1} dr + n\Lambda \int_{\Lambda+\lambda_1}^{1} \left((r^2-\lambda_1(\Lambda+\lambda_1))/r\right)^{n-1} dr = 0$$

which is an integral equation to be solved for λ_1 numerically by using Regula falsi method, and λ_2 can be obtained from equation (9.4), once λ_1 is known

Method of solution

In order to solve the convection-diffusion equation (1) along with the associated sets of initial and boundary conditions (3-7), we express the concentration following [9] as

$$C(t, r, z) = \sum_{j=0}^{\infty} f_j(t,r) \frac{\partial^j C_m}{\partial z^j} \tag{10}$$

where the average concentration, C_m is expressed as

$$C_m = D_1 \int_k^1 C\, r\, dr \tag{11}$$

Multiplying equation (1) by $\frac{2r}{1-k^2}$ and integrating with respect to *r* from *k* to 1 yields

$$\frac{\partial C_m}{\partial t} = \frac{1}{Pe^2} \frac{\partial^2 C_m}{\partial z^2} + D_1 \frac{\partial C}{\partial r}(t,z,1) - D_1 \frac{\partial}{\partial z} \int_0^1 w(t,r)\, C(t,z,r)\, r\, dr \tag{12}$$

Introducing (10) into (12), the following dispersion model for C_m is obtained as

$$\frac{\partial C_j}{\partial t} = \sum_{j=0}^{\infty} K_j(t) \frac{\partial^j C_m}{\partial z^j} \tag{13}$$

$$\text{where } K_j(t) = \frac{\delta_{j,2}}{Pe^2} + D_1 \frac{\partial f_j}{\partial r}(t,1) - D_1 \int_k^1 f_{j-1}(t,r)\, w(t,r)\, r\, dr \quad j = 0, 1, \ldots \tag{14}$$

where δ denotes the Kronecker delta. The exchange coefficient K_0 (t) accounts for the non zero solute flux at the boundary and will be zero if there is no wall absorption at the wall. K_1 and K_2 correspond to the convective and dispersion coefficients respectively. Sankarasubramanian and Gill [9] showed that equation (13) can be truncated by neglecting higher order terms involved K_3, K_4 …… etc and then generalized dispersion model takes the form

$$\frac{\partial C_m}{\partial t} = K_0(t) C_m + K_1(t) \frac{\partial C_m}{\partial z} + K_2(t) \frac{\partial^2 C_m}{\partial z^2} \tag{15}$$

Using equations (10), (13) in equation (1) and equating coefficients of $\frac{\partial^j C_m}{\partial z^j}$, *j* = 0,1, 2,----- the following set of differential equations for f_j is obtained as

$$\frac{\partial f_j}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r}\left(r \frac{\partial f_j}{\partial r}\right) - w(t,r) f_{j-1} + \frac{1}{Pe^2} f_{j-2} - \sum_{i=0}^{j} K_i f_{j-i} \qquad j = 0, 1, 2\ldots \tag{16}$$

where $f_{-1} = 0 = f_{-2}$.

From eq (3), (4) and (11) we get

$$C_m(0,z) = D_1 \int_k^1 r\, C(0, r, z)\, dr = D_1 \psi_1(z) I_1 \tag{17.1}$$

which gives $f_0(0,\ r) = \dfrac{B_1(r)}{D_1\ I_1}$ (17.2)

and $f_j(0,\ r) = 0 \quad j = 1,\ 2.......$ (17.3)

$$\frac{\partial f_j}{\partial r}(t,1) = -\beta f_j(t,1) \qquad j = 0,1,\ 2....... \tag{17.4}$$

$$\frac{\partial f_j}{\partial r}(t,\ k) = 0 \quad \text{for} \quad j = 0,1,2..... \tag{17.5}$$

$$\int_k^1 f_j(t,\ r)\, r\ dr = \delta_{j,0} / D_1 \tag{17.6}$$

The coupled equations (14) and (16) are solved one after the other to obtain the solution of (f_j, K_j) j = 0, 1, 2 .

Evaluation of f_0(t, r) and $K_0(t)$

The functions f_0 and exchange coefficient K_0 are independent of velocity field, and hence can be solved directly.

From equation (16) for j = 0, we obtain

$$\frac{\partial f_0}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial f_0}{\partial r}\right) - f_0 K_0 \tag{18}$$

Introducing the transformation

$$f_0(t,\ r,) = e^{-\int_0^t K_0(\eta)\,d\eta}\ g_0(t,r) \tag{19}$$

equation (18, takes the form

$$\frac{\partial g_0}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial g_0}{\partial r}\right) \tag{20}$$

The initial and boundary conditions on g_0 are

$$g_0(0,\ r) = \frac{B_1(r)}{D_1\ I_1} \tag{21}$$

$$\frac{\partial g_0}{\partial r}(t,1) = -\beta g_0(t,1) \tag{22}$$

$$\frac{\partial g_0}{\partial r}(t,k) = 0 \tag{23}$$

The solution of g_0 satisfying the initial and boundary conditions (21) - (23) is given by

$$g_0(t,r) = \sum_{j=0}^{\infty} \frac{A_j}{J_1(u_j k)} E_0(\mu_j r)\ e^{-\mu_j^2 t} \tag{24}$$

where $E_0(\mu_j r) = [J_1(\mu_j k) Y_0(\mu_j r) - Y_1(\mu_j k) J_0(\mu_j r)]$ (25)

and μ_j are the roots of the transcendental equation

$$\mu_j \left(Y_1(\mu_j k) J_1(\mu_j) - Y_1(\mu_j)\ J_1(\mu_j k)\right) + \beta\left(Y_0(\mu_j) J_1(\mu_j k) - Y_1(\mu_j k)\ J_0(\mu_j)\right) = 0 \tag{26}$$

and $A_n = \dfrac{\mu_j^2 (1-k^2) J_1(\mu_j k)}{(\mu_j^2 + \beta^2) E_0^2(\mu_j) - k^2 \mu_j^2 E_0^2(\mu_j k)} \dfrac{\int_k^1 r B_1(r) E_0(\mu_n r)\ dr}{I_1}$ (27)

and J_0, J_1, Y_0, Y_1 are the Bessel functions of first kind, second kind of order zero and one respectively.

From (17.6), we have $\int_k^1 r f_0(t,r)\ dr = 1/D_1$ (28)

Using the transformation (19) and equation (28), we obtain

$$e^{-\int_0^t K_0(\eta)\,d\eta} = 1/D_1 I_2 \tag{29}$$

From (29) and (19), the expression $f_0(t,r)$ can be obtained as

$$f_0(t,\, r) = g_0(t,r)/D_1\, I_2 \tag{30}$$

which can be written as

$$f_0(t,\, r) = \sum_0^\infty D_2\, E_0(\mu_j r) \Big/ D_1 \sum_0^\infty \frac{D_2}{\mu_j} D_3 \tag{31}$$

Using equations (14) and (31), the exchange coefficient $K_0(t)$ is obtained as

$$K_0(t) = -\left(\sum_0^\infty \mu_j D_2\; D_3\right) \Big/ \left(\sum_0^\infty \frac{D_2}{\mu_j} D_3\right) \tag{32}$$

The expressions for $f_0(t, r)$ and $K_0(t)$ reduce to those derived by Sankarasubramanian and Gill (1973) in the limiting case $k\to 0$.

Asymptotic Expansions for f_j's and K_j's for j = 0, 1, 2, …… for steady flow

The coupling between $f_j(t, r)$ and K_j (t) and solving the transformed problem for, say $f_1(t, r)$ would result in complicated integral equation in K_1(t,. The higher order functions are even more difficult to solve. Hence, we restrict ourselves to the asymptotic steady-state representations of $f_j(t, r)$ and K_j (t) for the case of steady flow, we will obtain solutions (f_j, K_j), j = 0, 1, 2 for large times, so that the dispersion model defined in (15) is a representation of the asymptotic results under steady state conditions.

As $t\to\infty$, the asymptotic representation for f_0, K_0, K_1 and K_2 are

$$f_0(\infty,\, r) = \mu_0\; E_0(\mu_0 r)/D_1\, D_4 \tag{33}$$

$$K_0(\infty) = -\mu_0^2 \tag{34}$$

$$K_1 = \frac{-\;2\mu_0^2}{(\mu_0^2+\beta^2)E_0^2(\mu_0) - k^2\mu_0^2\, E_0^2(\mu_0 k)} \int_k^1 w(r)\; r\; E_0^2(\mu_0 r)\; dr \tag{35}$$

$$f_1 = \sum_{n=1}^\infty \frac{B_n}{J_1(\mu_n k)}\left[E_0(\mu_n r) - \frac{I_3}{I_4}\; E_0(\mu_n r)\right] \tag{36}$$

where B_n's are given by

$$B_n = \int_k^1 [w(r)+K_1]\; f_0(r)\; r\, E_0(\mu_n r)\; dr \Big/ (\mu_0^2-\mu_n^2)\, J_1(\mu_n k)$$

$$K_2 = \frac{1}{Pe^2} - \frac{\int_k^1 [w(r)+K_1]\, f_1\; E_0(\mu_0 r)\, r\, dr}{\int_k^1 f_0\, E_0(\mu_0 r)\; r\, dr} \tag{37}$$

where $D_1 = \dfrac{2}{(1-k^2)}$; $D_2 = \dfrac{A_n}{J_1(u_n\, k)}\; e^{-\mu_n{}^2 t}$;

$D_3 = J_1(\mu_n k)\, Y_1(\mu_n) - Y_1(\mu_n k)\, J_1(\mu_n)$; $D_4 = J_1(\mu_0 k)\, Y_1(\mu_0) - Y_1(\mu_0 k)\, J_1(\mu_0)$; $I_1 = \int_k^1 rB_1(r)\,dr$;

$I_2 = \int_k^1 g_0(t,r)\, r\, dr$; $I_3 = \int_k^1 rE_0(\mu_n r)\, dr$; $I_4 = \int_k^1 rE_0(\mu_0 r)\, dr$

The mean concentration C_m is obtained from equation (12) with initial and boundary conditions given by (3) and (7) is

$$C_m(t,z) = \frac{1}{2(Pe)\sqrt{\pi\xi}} \exp\left(\zeta - \frac{z_1^2}{4\xi}\right) \tag{38}$$

where $\zeta(t) = \int_0^t K_0(\eta)\,d\eta$, $z_1(t,z) = z + \int_0^t K_1(\eta)\,d\eta$, $\xi(t) = \int_0^t K_2(\eta)\,d\eta$

RESULTS AND DISCUSSION

In the present analysis, the development of the dispersive transport following the injection of a chemically active tracer in a solvent flowing through an annulus with reactive outer wall has been investigated. The fluid flowing through the annulus is taken as Herschel- Bulkley fluid. The analysis uses the derivative expansion method proposed by [9]. The effect of moderate wall absorption, in the presence of a coaxial inner tube, on the three effective transport coefficients, viz. the exchange, the convection and the dispersion coefficients is studied. The value of the wall absorption parameter β is taken in the range of 0.01 to 100. The ratio of the radius of inner cylinder to that of the outer cylinder k is varied from 0.1 to 0.4. The yield stress τ_y is taken to range from 0.05 to 0.15. The results are discussed for different fluids Newtonian ($\tau_y = 0$, n = 1), Power law ($\tau_y = 0$, n = 2), Bingham ($\tau_y \neq 0$, n = 1) and Herschel-Bulkley fluids ($\tau_y \neq 0$, n = 2).

Asymptotic absorption coefficient K_0

Figure 2 shows the variation of the absorption coefficient (- K_0) with β and k for large times. When the wall absorption parameter is 100, the value of - K_0 increases from 5.65 in the case of a tube to 9.37 in an annular flow (k = 0.4). Thus, there is more absorption of solutes at the wall in an annulus compared to the tubular flow.

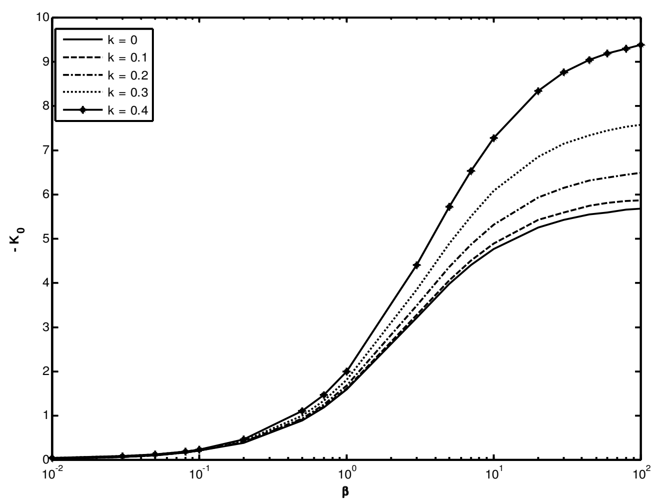

Fig 2 Variation of negative asymptotic absorption coefficient $-K_0$ with absorption parameter β for different values of k

Asymptotic convection coefficient K_1

Figure 3 (a) presents the asymptotic convection coefficient $-K_1$ versus the wall absorption parameter β for variation of *k*. In the absence of solute flux across the wall(β = 0), when the fluid flowing is Newtonian, $-K_1 = 1/2$ [10] indicates that the solute is being convected along the annulus with an average velocity of the flow. In the case of pipe flow of a Newtonian fluid flow [9], when wall reaction is present, the solute distribution moves with a velocity greater than the average velocity. But in an annular flow, the solute is observed to be convected along with a velocity lower than the average velocity of the flow when the yield stress is as small as 0.05. In the presence of catheter (k = 0.1) the - K_1 reduces to half of its value corresponding to that in a tubular flow when $\beta = 0.01$, $\tau_y = 0.05$

and n = 2. $-K_1$ is noticed to decrease with decrease in annular gap. For β = 100, convection coefficient decreases from 0.2392 (k =0.1) to 0. 048 (k = 0.4)

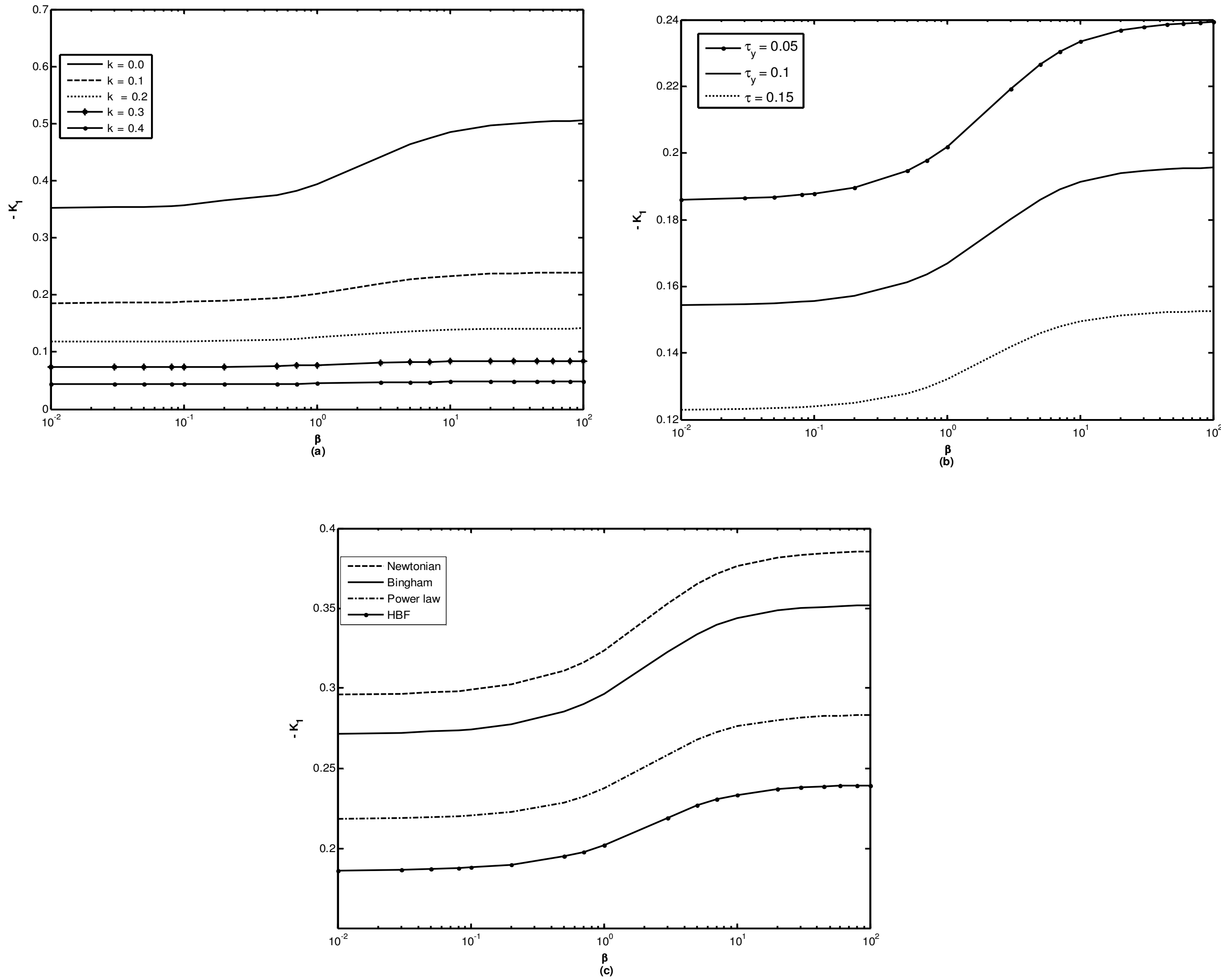

Fig 3 Variation of negative asymptotic convection coefficient $-K_1$ with absorption parameter β for (a) different values of k for τ_y = 0.05, n =2 (b) for different values of τ_y when n = 2 (c) for different fluids

The asymptotic convection coefficient $-K_1$ reduces Fig 3 (b) from 0.2393 (τ_y = 0.05) to 0.1526 (τ_y = 0.15) with increase of yield stress when $\beta = 100$. Fig 3c gives the reduction in the asymptotic convective coefficient ($-K_1$) due to non-Newtonian rheology. The drop in the $-K_1$ is much more rapid in case of Power law and Herschel- Bulkley fluids when compared to Bingham fluids. When the fluid is Casson $-K_1$ assumes less value than the corresponding values in Herschel-Bulkley fluid [6].

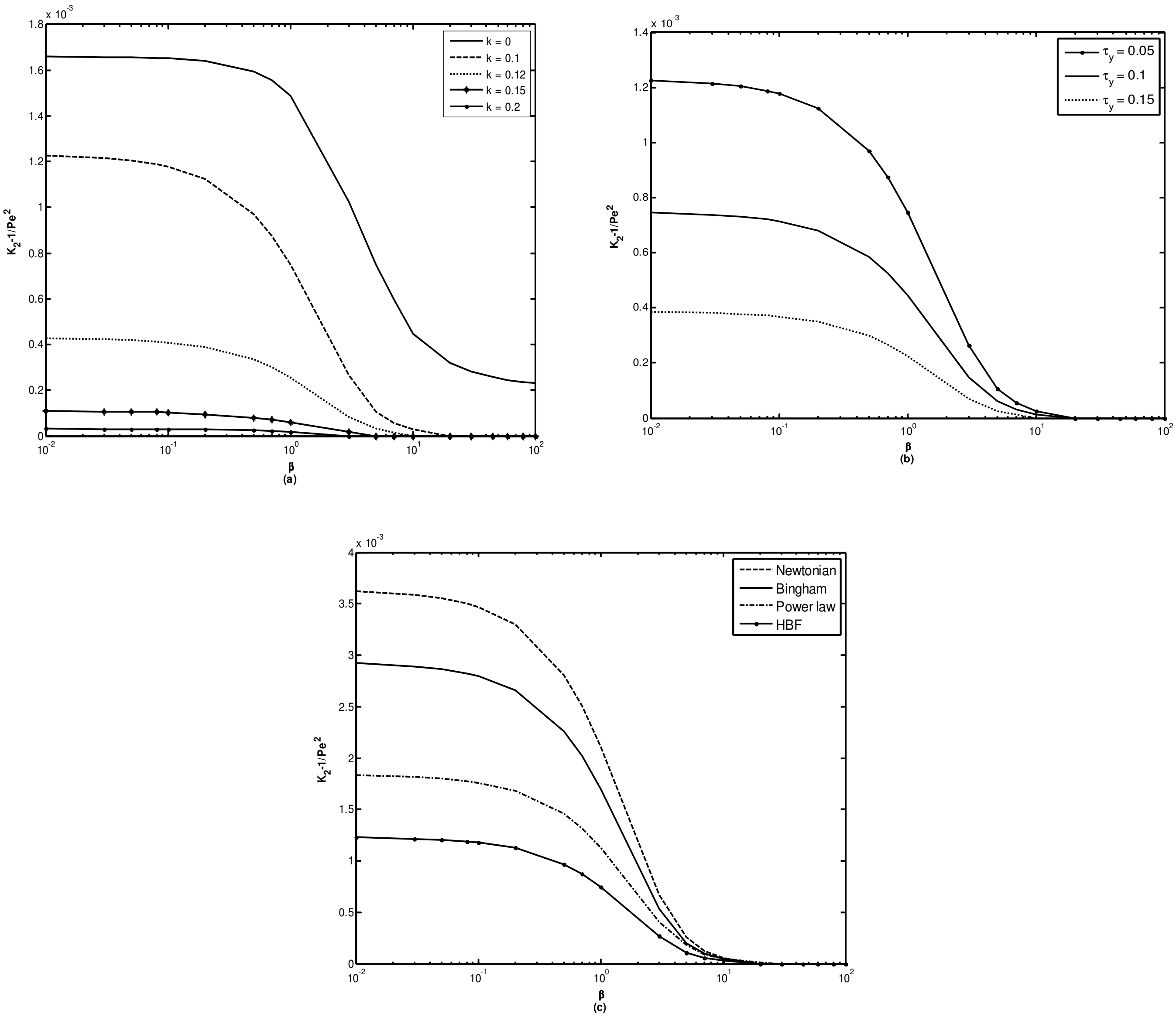

Fig 4 Variation of asymptotic dispersion coefficient $K_2 - 1/Pe^2$ with absorption parameter β (a) for different values of k when τ_y = 0.05, n = 2 (b) for different values of τ_y when k = 0.1, n = 2 (c) for different fluids when k = 0.1

Asymptotic dispersion coefficient K_2

Fig. 4 shows the variation of asymptotic dispersion coefficient K_2 (from which the additive contribution of the axial diffusion $1/Pe^2$ has been deducted) against the wall absorption parameter β for different values of k, τ_y and n. The effect of β on K_2 is significant unlike in the case of $-K_0$ and $-K_1$. As $\beta \to 100$ the dispersion coefficient $K_2 \to 0$ which is a multi fold decrease from its value for no wall reaction, which is 1.23×10^{-3} when k = 0.1 , n = 2 and τ_y = 0.05. $(K_2 - 1/Pe^2)$ decreases from 0.01653 (in the case of tubular flow) to 0.001177 (in an annular flow) k = 0.1, when $\beta = 0.1$, $\tau_y = 0.05$ and n = 2.

The absorption of solutes is more as the annular gap decreases, which is also observed by [6]. Consequent to the boundary absorption, the axial dispersion inhibits by the transverse diffusion and thus the axial dispersion coefficient decreases. It is seen that from fig 4 (a), K_2 decreases from 0.74×10^{-3} when k is as small as 0.1 to 1.05×10^{-5} when k = 0 .2 for $\beta = 1.0$ and $\tau_y = 0.05$. This also noticed by [10] in the case of solute dispersion in an annulus with no wall absorption when the fluid flowing in Newtonian. From fig 4(b), it is observed that the effect of yield stress is to reduce the dispersion coefficient when $\tau_y = 0.15$ the value of K_2 (0.386×10^{-3}) is reduced 3 times of the value (1.226×10^{-3}) corresponds to the value when $\tau_y = 0.05$ for $\beta = 0.01$

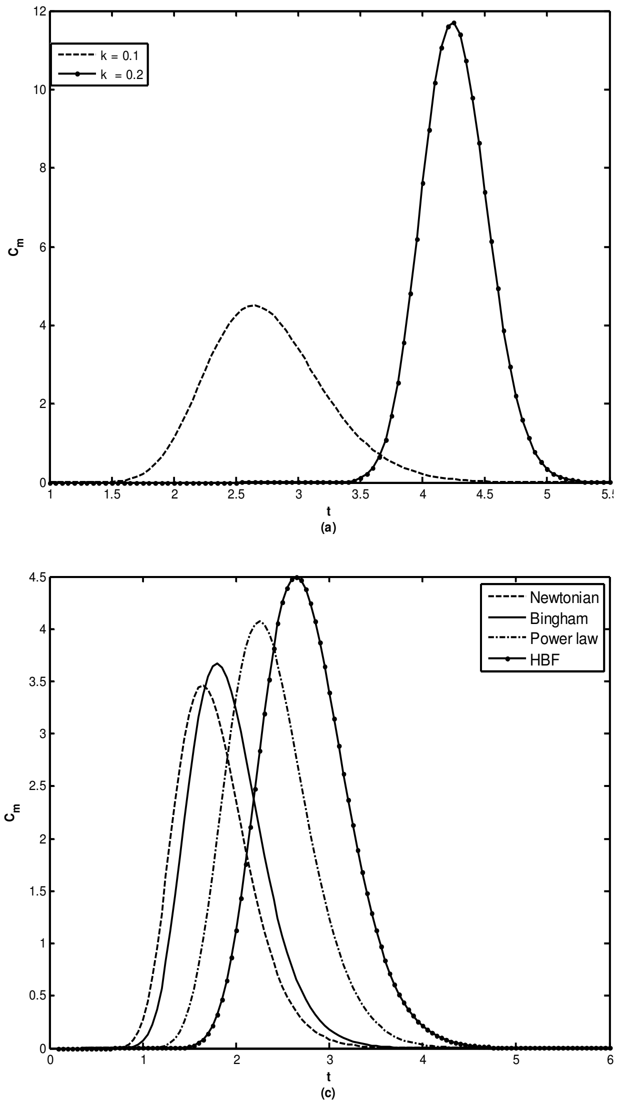

Fig 5 Variation of mean concentration C_m with time t for

(a) different values of k when $\beta = 0.01$, $\tau_y = 0.05$, z = 0.5, n =2 (b) different values of k when $\beta = 1$, $\tau_y = 0.05$, z = 0.5, n =2

(c) different fluids when $\beta = 0.01$, k = 0.1, z = 0.5 (d) different fluids when $\beta = 1$, k = 0.1, z = 0.5

Fig 4 (c) presents the dispersion coefficient for variation of n and τ_y. It is observed that when $\beta = 0.1$, there is a threefold reduction in the dispersion coefficient in the case of Herschel- Bulkley fluid corresponding to that of a Newtonian case when k = 0.1

Mean Concentration

The mean concentration C_m as a function of time is plotted in fig.5 (a, b, c, d) for variation in k and n. The peak mean concentration increases from (4.4970) to (11.7061) when k changes from 0.1 to 0.2 when $\beta = 0.01$, $\tau_y = 0.05$ and n = 2, when $\beta = 1$, there is a seven fold reduction in the mean concentration when k increases from 0.1 to 0.2

The variation of non- Newtonian rheology on peak mean concentration when k = 0.1, $\beta = 0.01$, z = 0.5 is described in fig 5a. The peak mean concentration in the case of Herschel- Bulkley fluid is higher compared to its corresponding value in the cases of power and Newtonian fluids. The behavior is totally reversed when $\beta = 1$

5 Application to catheterized artery

The investigations of the present mathematical model are relevant in understanding the catheter insertion related artifacts in flow measurements in the cardiovascular system. Catheters are used to inject the dye and to take blood samples for the purpose of measurements [11, 12].The insertion of a catheter into the blood vessels can be one of the significant causes for the formation eddies and mixing of blood. This method of taking blood samples invariably introduces some distortion in the time-concentration curve and thus the recorded curve does not represent the *in situ* concentration – time relation at the withdrawal site [13]. The experimental studies of [14, 15], report the calibrations based on mathematical models for the removal of the catheter distortion. The results obtained from the mathematical model for removal of catheters introduced distortion of indicator dilution curves of [16] showed a sharper and higher peak and less smooth than the measured curves.

The analysis of dispersion process with boundary absorption is important to understand the indicator dilution technique as lung and blood vessels involve conductive walls. The objective is to provide a correction for the catheter induced errors in the measured values based on the concept of longitudinal diffusion of substances due to the combined action of convection and diffusion. In this mathematical model, the outer tube represents the artery wall with conductance. The inner coaxial tube can correspond to a catheter. The annular gap k , or equivalent to the ratio of the radius of the catheter to the artery, is varied in the range 0.1 to 0.4. The wall absorption parameter β quantifies the permeability of the arterial wall and its values are varied from 0.01 to 1, so that the results can be related to physiological situations.

Table 1 Variation of K_0 , K_1 and K_2 with β and k for $\tau_y = 0.05$, n = 2

k	$\beta = 0.01$			$\beta = 0.1$			$\beta = 1$		
	$-K_0$	$-K_1$	$K_2 - 1/Pe^2$	$-K_0$	$-K_1$	$K_2 - 1/Pe^2$	$-K_0$	$-K_1$	$K_2 - 1/Pe^2$
0.0	0.0199	0.3522	0.0017	0.1951	0.3565	0.0015	1.5770	0.3936	0.0015
0.1	0.0201	0.1861	0.0012	0.1971	0.1879	0.0009	1.6003	0.2020	0.002
0.2	0.0207	0.1177	0.0005	0.2036	0.1185	0.0001	1.6710	0.1252	0
0.3	0.0219	0.0735	0	0.2153	0.0739	0	1.7953	0.0772	0
0.4	0.0237	0.0434	0	0.2337	0.0436	0	1.9894	0.0451	0

Table 1 represents the variation of the asymmetric absorption coefficient $-K_0$, convection coefficient $-K_1$ and the dispersion coefficient $K_2 - 1/Pe^2$ with the wall absorption parameter β and annular gap. It is observed that the presence of the catheter inhibits the convection coefficient [decreases from 0.3565 (tube) to 0.1879 (k = 0.1)] and dispersion coefficient [decreases from 0.0015 (k = 0.1) to 0.0001 (k = 0.3)] when $\beta = 0.1$, $\tau_y = 0.05$ and n = 2

Table 2 gives the values of the convective and dispersion coefficients for different values of yield stress and annular gap when n = 2 and $\beta = 1$. There is a double fold reduction in the convective coefficient and dispersion coefficient due to the presence of a catheter (k = 0.1). As the catheter size increases (k = 0.4) there is a fivefold diminution in $-K_1$ when (k = 0.1) when $\tau_y = 0.1$ while the reduction factor in the dispersion coefficient is 1000. With increase in the yield stress the asymptotic convection coefficient decreases from 0.1667 ($\tau_y = 0.1$) to 0.1320 ($\tau_y = 0.15$) and the corresponding reduction in $K_2 - 1/Pe^2$ is double fold. The combined impact of the presence or increase in radius of the catheter and yield stress is to diminutive the convection and dispersion coefficients.

Table 2 Variation of K_1 and K_2 with k and τ_y for β=1, n = 2

k	$\tau_y = 0.1$		$\tau_y = 0.15$	
	$-K_1$	$K_2 - 1/Pe^2$	$-K_1$	$K_2 - 1/Pe^2$
0.0	0.342226	0.001053	0.294841	0.000726
0.1	0.166785	0.000447	0.132047	0.000224
0.2	0.100115	0	0.075340	0
0.3	0.058887	0	0.04101	0
0.4	0.032138	0	0.01957	0

CONCLUSION

The dispersion of a solute in a Herschel- Bulkley fluid in an annular region is analyzed using the generalized dispersion model. The effect of wall conductance at the outer boundary is considered. Following the injection of a tracer the entire dispersion process is described by the three effective transport coefficients, viz. the exchange coefficient, the convection and the dispersion coefficients. It is observed that the analytic absorption coefficient $-K_0$, increases from 5.67, in the case of a tubular flow (k = 0) to 9.37 for the annular flow (k = 0.4) for large values of wall absorption parameter (β = 100). Thus the absorption of solutes is favoured at the walls in an annulus. The absorption and convection coefficients are found to increase with β whereas the dispersion coefficient is seen to decrease withβ. The absorption coefficient is independent of yield stress and power law rheology. The yield stress and power law rheology reduce the convection and dispersion coefficients. The results of the mathematical model are discussed to understand the dispersion process in a catheterized artery. The presence of a catheter and increase in the size of the catheter favours the dyes or other solutes to get out of the blood vessels for all values of the absorption parameter. The non- Newtonian and power law rheology has a significant effect on the convection and dispersion coefficients.

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