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Effect of variable viscosity on the peristaltic flow of a Jeffrey fluid in a uniform tube

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ABSTRACT

In this paper, the peristaltic flow of a Jeffrey fluid in a tube with variable viscosity under the assumptions of long wavelength is investigated. The flow is investigated in a wave frame of reference moving with velocity of the wave. The effects of λ_1*, viscosity parameter* α *and amplitude ratio* ϕ *on the pumping characteristics and friction force are discussed in detail through graphs.*

Keywords: amplitude ratio; Friction force; Jeffrey fluid; peristaltic flow; variable viscosity.

INTRODUCTION

Since the first investigation of Latham [11] several theoretical and experimental attempts have been made to understand peristaltic action in both mechanical and physiological situations under various approximations. Based on the experimental work, Burn and Parks [4] have studied the peristaltic flow of a viscous fluid through a pipe and a channel by considering sinusoidal variation at the walls. Shapiro et al. [14] have analyzed peristaltic pumping with long wavelength at low Reynolds number, in a wave frame of reference. Whereas Fung and Yih [5] and Yih and Fung [16] have obtained analytical solutions for peristaltic flow by assuming small amplitude but arbitrary Reynolds number, in a fixed frame of reference. Many of the contributors to the area of peristaltic transport have either followed Shapiro or Fung.

Most of the studies on the peristaltic transport assume the physiological fluids to behave like Newtonian fluids with constant viscosity. However, this approach fails to give and adequate understanding of the peristaltic mechanism involved in small blood vessels, lymphatic vessels, intestine, and ductus efferentus of the male reproductive tracts. In these body organs, the viscosity of the fluid varies across the thickness of the duct (Gold Smith and Skalak [6]). Provost and Schwarz [13] have explained a theoretical study of viscous effects in peristaltic pumping and assumed that the flow is free of inertial effects and that non-Newtonian normal stresses are negligible. Abd El Hakeem et al. [1] have investigated the peristaltic flow of a fluid with variable viscosity under the effect of magnetic field. Abd El Hakeem et al. [2] have investigated the effect of endoscope and fluid with variable viscosity on peristaltic motion. Hayat et al. [7] have investigated the effect of endoscope on the peristaltic flow of a Jeffrey fluid. Ali et al. [3] have investigated peristaltic flow of MHD fluid in a channel with variable viscosity under the effect of slip condition. Hayat and Ali [8] have investigated the effect of variable viscosity on the peristaltic flow of a Newtonian fluid in an asymmetric channel. Peristaltic motion of a Jeffrey fluid under the effect of a magnetic field in a tube was discussed by Hayat and Ali [9]. Recently, Jayarami Reddy et al. [10] have studied the peristaltic flow of a Williamson fluid in an inclined planar channel under the effect of a magnetic field. Subba Reddy et al. [15] have investigated the slip effects on the peristaltic motion of a Jeffrey fluid through a porous medium in an asymmetric channel under the effect magnetic field.

In view of these, we investigated the peristaltic flow of a Jeffrey fluid with variable viscosity under the long wavelength and low Reynolds number assumptions. The velocity components and axial pressure gradient are obtained analytically. Physically, our model corresponds to the transport of chyme in the small intestine (as the radius 1.25 cm of the small intestine is small compared with the long wavelength 8.01 cm). The effects various emerging parameters upon the flow are examined. We make comparison with other studies.

2. Mathematical formulation and solution

We consider the flow of a Jeffrey fluid with variable viscosity through an axismymmetric tube of uniform thickness with a sinusoidal wave traveling down its wall. We choose cylindrical coordinates (R, Z) such that R is the radial coordinate and Z is the axial coordinate. Fig. 1 shows the physical model. The geometry of the wall surface is described mathematically as

$$H(Z,t) = a + b\sin\frac{2\pi}{\lambda}(Z - ct) \tag{2.1}$$

in which a is the radius of the tube at inlet, b is the wave amplitude, λ is the wavelength, c is the wave speed and t is the time.

In the fixed frame of reference (R, Z) the flow is unsteady. However, in a coordinate frame moving with the wave speed c wave framed (r, z) the boundary shape is stationary. The transformation from fixed frame to wave frame is given by

$$z = Z - ct,\ r = R,\ w = W - c,\ u = U \tag{2.2}$$

where (u, w) and (U, W) are velocity components in the wave and fixed frames of reference respectively.

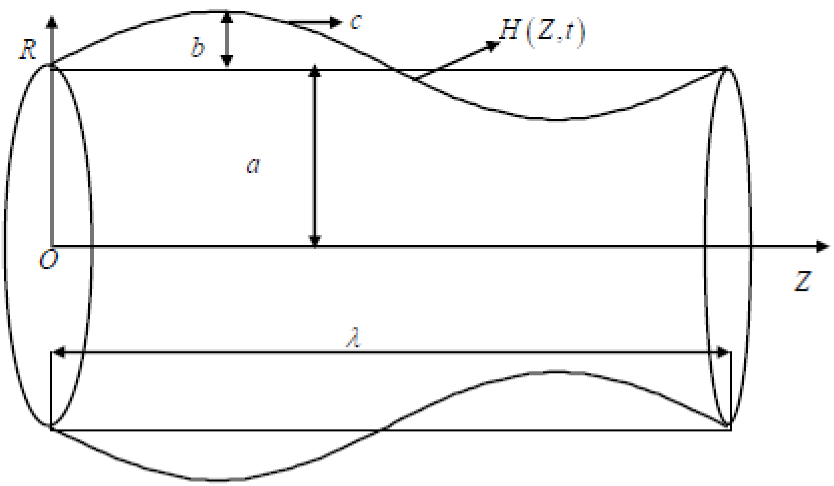

Fig. 1. Physical Model

In the wave frame, the equations governing the flow are

$$\rho\left(u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial z}\right) = -\frac{\partial p}{\partial r} + \frac{1}{r}\frac{\partial}{\partial r}(rS_{rr}) + \frac{\partial}{\partial z}(S_{rz}) - \frac{S_{\theta\theta}}{r} \tag{2.3}$$

$$\rho\left(u\frac{\partial w}{\partial r} + w\frac{\partial w}{\partial z}\right) = -\frac{\partial p}{\partial z} + \frac{1}{r}\frac{\partial}{\partial r}(rS_{rz}) + \frac{\partial}{\partial z}(S_{zz}) \tag{2.4}$$

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} = 0 \tag{2.5}$$

where p is the pressure and ρ is the density of the fluid.
The constitute equation of S for Jeffrey fluid is

$$S=\frac{\mu(r)}{1+\lambda_1}(\dot{\gamma}+\lambda_2\ddot{\gamma}) \tag{2.6}$$

where $\mu(r)$ is the viscosity function, λ_1 is the ratio of relaxation to retardation times, λ_2 is the retardation time, $\dot{\gamma}$ is the shear rate and dots over the quantities denote differentiation with time.

The boundary conditions in the wave frame are

$$u=0,\quad \frac{\partial w}{\partial r}=0 \quad \text{at} \quad r=0 \tag{2.7}$$

$$w=-c \qquad \text{at } r=h \tag{2.8}$$

Introducing the following non-dimensional variables

$$\bar{r}=\frac{r}{a},\bar{z}=\frac{z}{\lambda},\bar{u}=\frac{u}{c\delta},\bar{p}=\frac{pa^2}{\mu_0 c\lambda},\bar{\mu}(r)=\frac{\mu(r)}{\mu_o},\bar{w}=\frac{w}{c},\delta=\frac{a}{\lambda},$$

$$\bar{S}=\frac{aS}{\mu_0 c},\phi=\frac{b}{a},h=1+\phi\sin(2\pi z).$$

where ϕ is the amplitude ratio μ_0 is the viscosity δ is the wave number in the equations (2.3)- (2.8) (dropping bars), we get

$$\mathrm{Re}\,\delta^3\left(u\frac{\partial u}{\partial r}+w\frac{\partial u}{\partial z}\right)=-\frac{\partial p}{\partial r}+\frac{\delta}{r}\frac{\partial}{\partial r}(rS_{rr})+\delta^2\frac{\partial}{\partial z}(S_{rz})-S_{\theta\theta} \tag{2.9}$$

$$\mathrm{Re}\,\delta\left(u\frac{\partial w}{\partial r}+w\frac{\partial w}{\partial z}\right)=-\frac{\partial p}{\partial z}+\frac{1}{r}\frac{\partial}{\partial r}(rS_{rz})+\delta\frac{\partial}{\partial z}(S_{zz}) \tag{2.10}$$

$$\frac{\partial u}{\partial r}+\frac{u}{r}+\frac{\partial w}{\partial z}=0 \tag{2.11}$$

where

$$S_{rr}=\frac{2\delta\mu(r)}{1+\lambda_1}\left[1+\frac{\lambda_2 c\delta}{a}\left(u\frac{\partial}{\partial r}+w\frac{\partial}{\partial z}\right)\right]\frac{\partial u}{\partial r},$$

$$S_{rz}=\frac{\mu(r)}{1+\lambda_1}\left[1+\frac{\lambda_2 c\delta}{a}\left(u\frac{\partial}{\partial r}+w\frac{\partial}{\partial z}\right)\right]\left(\frac{\partial w}{\partial r}+\delta^2\frac{\partial u}{\partial z}\right),$$

$$S_{\theta\theta}=\frac{2\delta\mu(r)}{1+\lambda_1}\left[1+\frac{\lambda_2 c\delta}{a}\left(u\frac{\partial}{\partial r}+w\frac{\partial}{\partial z}\right)\right]\frac{u}{r},$$

$$S_{zz}=\frac{2\delta\mu(r)}{1+\lambda_1}\left[1+\frac{\lambda_2 c\delta}{a}\left(u\frac{\partial}{\partial r}+w\frac{\partial}{\partial z}\right)\right]\frac{\partial w}{\partial z},$$

and $\mathrm{Re}=\dfrac{\rho ac}{\mu_0}$ is the Reynolds number.

The corresponding non-dimensional boundary conditions are

$$\frac{dw}{dr}=0,\quad u=0 \qquad \text{at} \qquad r=0, \tag{2.12}$$

$$w=-1 \qquad \text{at} \qquad r=h=1+\phi\sin 2\pi z, \tag{2.13}$$

Using the long wavelength approximation ($\delta<<1$) and low Reynolds number ($\mathrm{Re}\rightarrow 0$), assumption the equations (2.9) and (2.10) become

$$\frac{\partial p}{\partial r} = 0, \tag{2.14}$$

$$\frac{\partial p}{\partial z} = \frac{1}{(1+\lambda_1)r}\frac{\partial}{\partial r}\left(r\mu(r)\frac{\partial w}{\partial r}\right). \tag{2.15}$$

From Eqs. (2.14) and (2.15), we have

$$\frac{dp}{dz} = \frac{1}{(1+\lambda_1)r}\frac{\partial}{\partial r}\left(r\mu(r)\frac{\partial w}{\partial r}\right). \tag{2.16}$$

The effect of viscosity variation on peristaltic flow can be investigated for any given function $\mu(r)$. For the present investigation, we assume the viscosity variation in the dimensionless form as

$$\mu(r) = e^{-\alpha r} \quad \text{or} \quad \mu(r) = 1-\alpha r \quad \text{for} \quad \alpha << 1 \tag{2.17}$$

3. Solution

Solving Eq. (2.16) using the boundary conditions Eqs. (2.12) and (2.13) we get

$$w = -1 + \left(\frac{1+\lambda_1}{2}\right)\frac{dp}{dz}\int_h^r \left[\frac{r}{\mu(r)}\right]dr \tag{3.1}$$

The dimensionless volume flow rate in the wave frame is given by

$$q = 2\int_0^h wrdr = -h^2 + (1+\lambda_1)\frac{dp}{dz}\int_0^h\left[\int_h^r \frac{r}{\mu(r)}dr\right]rdr \tag{3.2}$$

Eq. (3.2) can be rewritten as

$$\frac{dp}{dz} = \frac{(q+h^2)}{(1+\lambda_1)\times I_1(h)} \tag{3.3}$$

where $I_1 = (h) = \int_0^h\left[\int_h^r \frac{r}{\mu(r)}dr\right]rdr$.

Substituting Eq. (2.17) in Eq. (3.3), we get

$$\frac{dp}{dz} = \frac{-8}{(1+\lambda_1)}\frac{(q+h^2)}{h^4}\left[1-\frac{4}{5}\alpha h\right] \tag{3.4}$$

The dimensionless instantaneous volume flow rate in the fixed frame of reference is given by

$$Q(x,t) = 2\int_0^h WRdr = 2\int_0^h (w+1)rdr = q+h^2 \tag{3.5}$$

The dimensionless time mean flow over a period T(=λ/c) of the peristaltic wave, is defined as

$$\overline{Q} = \frac{1}{T}\int_0^T Q(x,t)dt = q + \int_0^1 h^2 dx = q + 1 + \frac{\phi^2}{2} \tag{3.6}$$

From Eq. (3.4) and Eq. (3.6), we have

$$\frac{dp}{dz} = \frac{-8}{(1+\lambda_1)}\frac{\left(\overline{Q}-1-\frac{\phi^2}{2}+h^2\right)\left(1-\frac{4}{5}\alpha h\right)}{h^4} \tag{3.7}$$

The pressure rise Δp per one wave length and friction force F (on the wall) are respectively given as

$$\Delta p = \int_0^1 \frac{dp}{dz}dz \tag{3.8}$$

and $F = \int_0^1 h^2 \left(-\frac{dp}{dz} \right) dz$ (3.9)

DISCUSSION OF THE RESULTS

In order to study the effects of viscosity parameter α, λ_1 and amplitude ratio ϕ on the pressure rise Δp and friction force per wavelength, the integrals Eqs. (3.8) and (3.9) are solved numerically. Numerical simulation here is performed using the computational software Matlab.

Fig. 2. shows the relation between pressure rise Δp and time averaged flux $\overline{Q}$ for different values of for different values of λ_1 with $\phi = 0.6$ and $\alpha = 0.1$. In the pumping region ($\Delta p > 0$), the time averaged flux $\overline{Q}$ increases as λ_1 decreases. Whereas in the co-pumping region ($\Delta p < 0$), $\overline{Q}$ increases by increasing the λ_1. Also, it is noticed that the pumping is more for Newtonian fluid ($\lambda_1 \to 0$) than the Jeffrey fluid.

The variation of pressure rise Δp with time averaged volume flux $\overline{Q}$ for different values of viscosity parameter α with $\phi = 0.6$ and $\lambda_1 = 0.3$ as shown in Fig.3. It is observed that, increasing α decreases the pumping ($\Delta p > 0$) but free pumping ($\Delta p = 0$) and co-pumping ($\Delta p < 0$) are increases. When $\alpha \to 0$, our results coincide with those results obtained by Nagendra [12]. Furthermore, as $\alpha \to 0$ and $\lambda_1 \to 0$ our results agree with the results of Shapiro et al. [14].

The effect of amplitude ratio ϕ on the pumping characteristics is plotted in Fig. 4. for $\lambda_1 = 0.3$ and $\alpha = 0.1$. We observed that the larger the amplitude ratio, greater the pressure rise against which the pump works. In the co-pumping region $\overline{Q}$ decreases as amplitude ratio ϕ increases for appropriately chosen $\Delta p < 0$.

In order to illustrate the effect of λ_1, viscosity parameter α and amplitude ratio ϕ on the friction force on the tube wall, figures 5-7 are plotted. From Fig. 5, it is observed that the friction force first increase and then decrease with an increase in λ_1. From Fig. 6, it is concluded that the magnitude of the friction force decreases with increasing viscosity parameter α. The friction force first decrease and then increase with the increase in amplitude ratio as shown in Fig. 7. In general, figures 2-7 show that the friction force has an opposite character in comparison to the pressure rise.

CONCLUSION

In this paper, we studied the peristaltic motion of a Jeffrey fluid with variable viscosity through an axisymmetric tube is studied under the assumptions of long wavelength and negligible inertia. The effect of λ_1, viscosity parameter α and amplitude ratio ϕ on peristaltic pumping and friction force are examined. With the increase in λ_1, the pressure rise decreases first and then increases. The behaviour of viscosity parameter α on the pressure rise is similar to that of λ_1. The pressure rise first increases and then decreases with increase in amplitude ratio. The friction force has an opposite character in comparison to the pressure rise. Furthermore, our results agree with the results of Shapiro et al. (1969) when $\alpha \to 0$ and $\lambda_1 \to 0$.

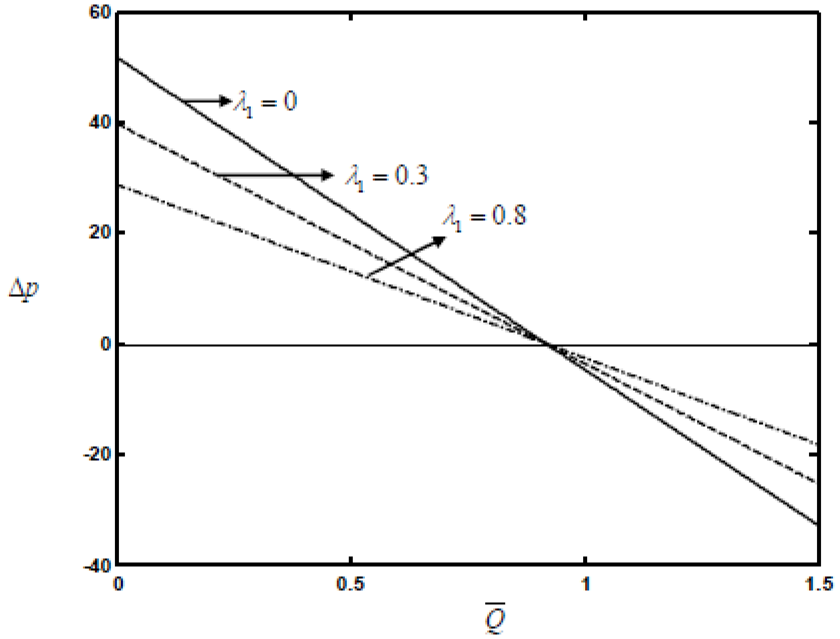

Fig. 2. The variation of pressure rise Δp with time averaged volume *flow rate $\bar{Q}$ for different values of λ_1 with $\phi = 0.6$ and $\alpha = 0.1$*

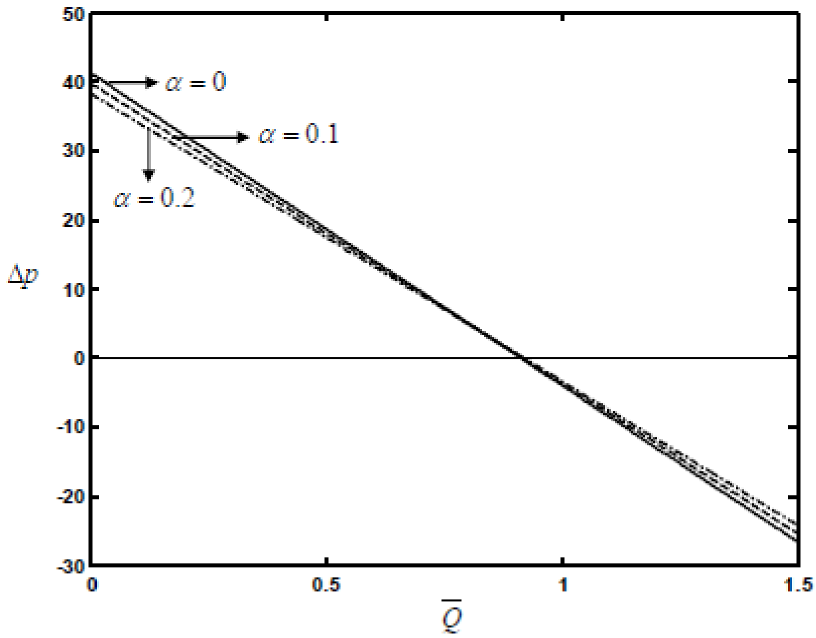

Fig. 3. The variation of pressure rise Δp with time averaged volume *flow rate $\bar{Q}$ for different values of α with $\phi = 0.6$ and $\lambda_1 = 0.3$.*

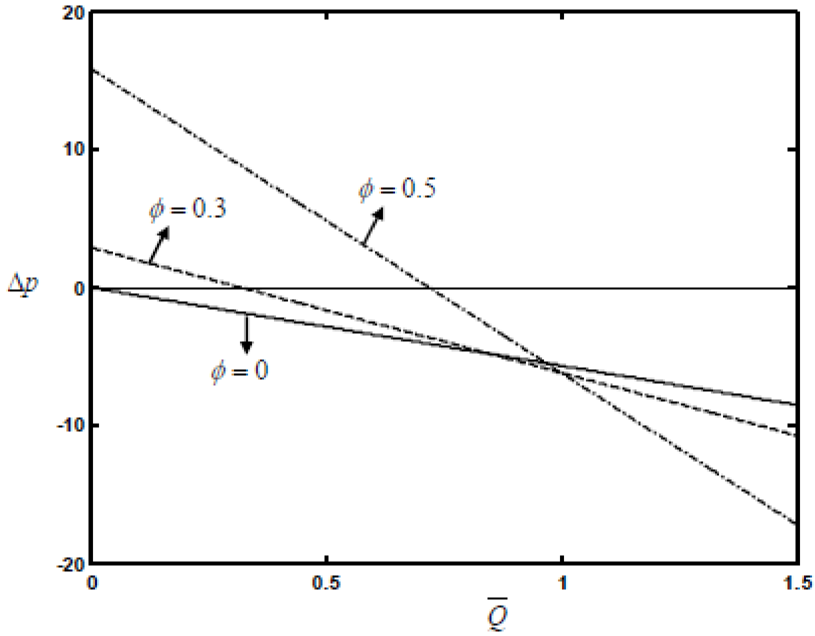

Fig. 4. The variation of pressure rise Δp with time averaged volume *flow rate* $\overline{Q}$ *for different values of* ϕ *with* $\alpha = 0.1$ *and* $\lambda_1 = 0.3$.

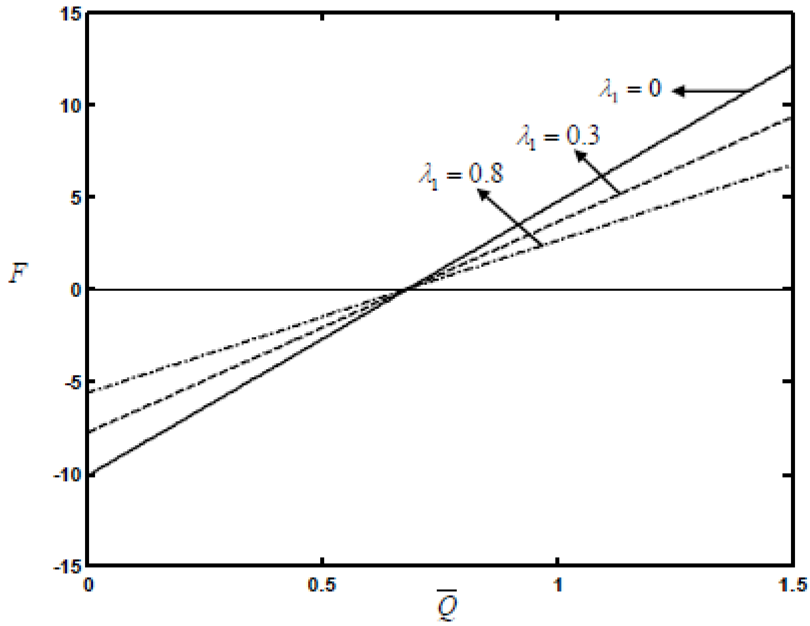

Fig. 5. The variation of friction force F with time averaged volume *flow rate* $\overline{Q}$ *for different values of* λ_1 *with* $\phi = 0.6$ *and* $\alpha = 0.1$.

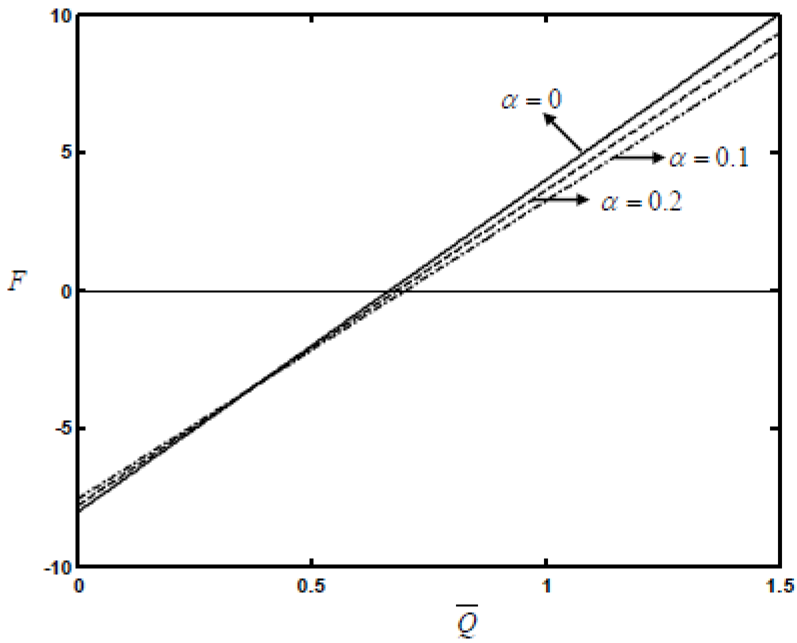

Fig. 6. The variation of friction force F with time averaged volume *flow rate $\overline{Q}$ for different values of α with $\phi = 0.6$ and $\lambda_1 = 0.3$.*

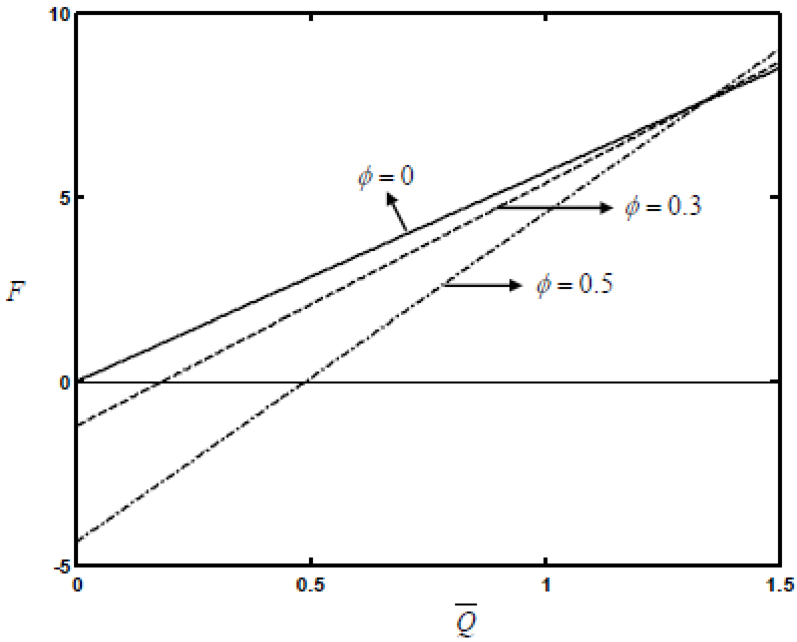

Fig. 7. The variation of friction force F with time averaged volume *flow rate $\overline{Q}$ for different values of ϕ with $\alpha = 0.1$ and $\lambda_1 = 0.3$.*

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