Available online at www.pelagiaresearchlibrary.com

Pelagia Research Library

Advances in Applied Science Research, 2011, 2 (2): 167 -178

ISSN: 0976-8610
CODEN (USA): AASRFC

Effect of thickness of the porous material on the peristaltic pumping when the tube wall is provided with non-erodible porous lining

R. Hemadri Reddy[1]*, A. Kavitha[2], S. Sreenadh[2], P.Hariprabakaran[2]

[1]*School of Advanced Sciences, VIT University, Vellore, India*
[2]*Department of Mathematics, Sri Venkateswara University, Tirupati*

ABSTRACT

We investigate the effect of thickness of the porous material on the peristaltic pumping when the tube wall is provided with non-erodible porous lining .The motion is caused by the movement of peristaltic waves on the flexible walls of the tube. The effects of yield stress and permeability on the pumping characteristics are studied in detail.

Keywords: Peristaltic transport; Newtonian fluid; volume flow rate; pressure rise; pumping characteristics.

INTRODUCTION

Peristaltic pumping is a form of fluid transport, generally from a region of lower to higher pressure, by means of a progressive wave of area contraction or expansion which propagates along the length of a tube-like structure. Peristalsis occurs naturally as a means of pumping biofluids from one place of the body to another. This mechanism occurs in the gastrointestinal, urinary and reproductive tracts and many other glandular ducts in the living body. The early reviews of Ram Chandra Rao and Usha [12], Jaffrin and Shapiro [4,5], Brasseur et al [2], Srivastava and Srivastava [15], Provost and Schwarz [11], Shukla and Gupta [13], Subba Reddy et al [16,17], Srinivas et al [8,14], Kavitha et al [6,7] and Vajravelu et al [19,20,21] deal with the peristaltic transport of viscous fluids through tubes and channels having impermeable flexible walls. Recently Usha, Sreenadh and Arunachalam [18], Mishra [9], Misra [10] made detailed analysis on the peristaltic transport through uniform and non-uniform tubes with permeable walls. Channabasappa et al [3] discussed the effect of porous lining on the walls of tubes and channels in ordered to have a better understanding of the increased mass flow rate due to permeable boundaries.

Motivated by these facts, it is interesting to study the fluid mechanical aspects of peristaltic pumping when the walls are provided with non-erodible porous lining. This study can be applied to blood flow in small blood vessels and biofluid flow in the stomach.

In this chapter peristaltic transport of a viscous fluid in a tube of radius ‘a’ is investigated. The wall of the tube is lined with non-erodible porous material of thickness h^1. The free flow past the porous material is governed by Navier-Stokes equations and the flow in the permeable wall is described by Darcy’s law. Applying Beavers and Joseph [1] slip condition at the permeable wall, the velocity field, the stream function, the volume flow rate, the pressure rise and the frictional force are determined. The effect of thickness of porous lining on the pumping characteristics is discussed.

Mathematical formulation and solution

Consider the peristaltic transport of a viscous fluid in a tube of radius 'a’. The wall of the tube is lined with porous material of permeability ‘k’. The thickness of the porous lining is h^1 (see Figure 1). The flow surrounded by the porous lining is governed by Nervier-Stokes equations and the flow in the porous layer is according to Darcy’s law. The flow is axisymmetric, cylindrical polar co-ordinate system (R, Θ, Z) is used. The wall deformation due to the infinite train of peristaltic waves is represented by

$$R = H(Z,\ t) = a + b\sin\frac{2\pi}{\lambda}(Z - ct) \qquad (1)$$

where b is the amplitude, λ is the wavelength and c is the wave speed.

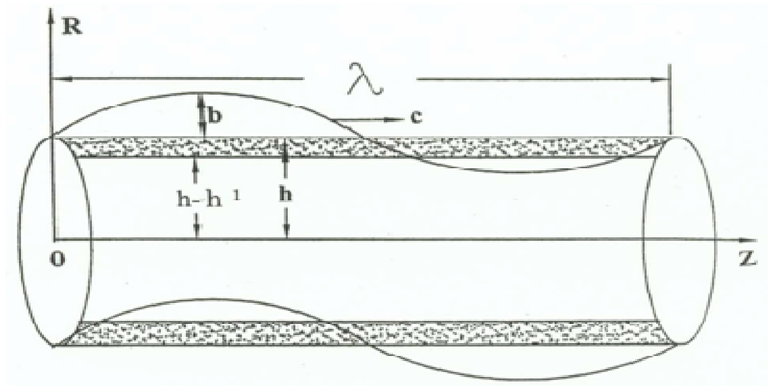

Figure1 .Physical Model

Equations of motion

Under the assumption that the tube length is an integral multiple of the wavelength λ and the pressure difference across the ends of the tube is a constant, the flow is inherently unsteady in the laboratory frame (R, Θ, Z) and becomes steady in the wave frame (r, θ, z) which is moving with velocity c along the wave. The transformation between these two frames is given by

$$r = R; \quad \theta = \Theta; \quad z = Z - ct; \quad p(z) = P(Z, t); \quad \psi = \Psi - \frac{R^2}{2} \qquad (2)$$

where ψ and Ψ are stream functions in the wave and laboratory frames respectively. We assume that the flow is inertia-free and the wavelength is infinite.

Using the non-dimensional quantities

$$\bar{r}=\frac{r}{a};\quad \bar{Q}=\frac{Q}{c};\quad \in=\frac{h^1}{a};\quad \bar{z}=\frac{z}{\lambda};\quad \bar{h}=\frac{h}{a};\quad \bar{w}=\frac{w}{c}$$

$$\bar{p}=\frac{a^2 p}{\lambda\mu_1 c};\ \bar{w}_B=\frac{w_B}{c};\quad Da=\frac{k}{a^2};\ \bar{u}=-\frac{1}{\bar{r}}\frac{\partial\bar{\psi}}{\partial\bar{z}};\quad \bar{w}=\frac{1}{\bar{r}}\frac{\partial\psi}{\partial\bar{r}}$$

where $\bar{u}$ and $\bar{w}$ are the radial and axial velocities in the wave frame. Now the equations governing the motion becomes (dropping the bars).

$$\frac{\partial p}{\partial z}=\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right) \tag{3}$$

$$\frac{\partial p}{\partial r}=0 \tag{4}$$

The dimensionless boundary conditions are

$$\frac{\partial w}{\partial r}=0 \quad \text{at} \quad r=0 \tag{5}$$

$$w=-1+w_B \quad \text{at} \quad r=h-\in \tag{6}$$

$$\frac{\partial w}{\partial r}=\frac{\alpha}{\sqrt{Da}}\left(w_B-Q\right) \quad \text{at} \quad r=h-\in \tag{7}$$

where $Q=-\frac{Da}{\mu}\frac{\partial p}{\partial z}$

$$\mu=\frac{\mu_1}{\mu_2}$$

$\mu_1=$ Viscosity in the free flow region

$\mu_2=$ Viscosity in the porous flow region

$\alpha=$ Slip parameter

Solution

Solving (3) and (4) subject to the boundary conditions (5) to (7), we obtain the velocity as

$$w=\frac{p}{4}\left[r^2-\left(h-\in\right)^2+2\left(h-\in\right)\frac{\sqrt{Da}}{\alpha}-4\frac{Da}{\mu}\right]-1 \tag{8}$$

where $p=\frac{\partial p}{\partial z}$

Integrating the equation (8) and using the conditions $\psi=0$ at $r=0$, we get the stream function as

$$\psi=\frac{ph^2}{4}\left[\frac{r^2}{4h^2}-\frac{r^2}{2}\left\{\left(1-\frac{\in}{h}\right)^2-\frac{2}{h}\left(1-\frac{\in}{h}\right)\frac{\sqrt{Da}}{\alpha}-\frac{4}{h^2}\frac{Da}{\mu}\right\}\right]-\frac{r^2}{2} \tag{9}$$

The volume flux q through each cross-section in the wave frame is given by

$$q = 2\int_0^{h-\epsilon} wr\,dr$$

$$q = -\frac{p(h-\epsilon)^4}{8}\left[1-4\frac{\sqrt{Da}}{(h-\epsilon)\alpha}+\frac{8Da}{\mu(h-\epsilon)^2}\right]-\frac{(h-\epsilon)^2}{2} \qquad (10)$$

The instantaneous volume flow rate $Q(z,t)$ in the laboratory frame between the centre line and the wall is

$$Q(Z,t) = 2\int_0^{(h-\epsilon)}(w+1)r\,dr$$

$$= -\frac{p(h-\epsilon)^4}{8}\left[1-\frac{4\sqrt{Da}}{(h-\epsilon)\alpha}+\frac{8Da}{\mu(h-\epsilon)^2}\right] \qquad (11)$$

From equation (10) we have

$$\frac{dp}{dz} = \frac{-8\left[q+(h-\epsilon)^2\right]}{(h-\epsilon)^4\left(1-\frac{4\sqrt{Da}}{\alpha(h-\epsilon)}+\frac{8Da}{\mu(h-\epsilon)^2}\right)} \qquad (12)$$

Averaging equation (11) over one period yields the time mean flow (time-averaged volume flow rate) $\bar{Q}$ as

$$\bar{Q} = \frac{2}{T}\int_0^T\int_0^{(h-\epsilon)}(w+1)r\,dr\,dt$$

$$= q+\frac{1}{T}\int_0^T(h-\epsilon)^2 dt$$

$$= q+(1-\epsilon)^2+\frac{\phi^2}{2} \qquad (13)$$

The pumping characteristics

Integrating the equation (12) with respect to z over one wavelength, we get the pressure rise (drop) over one cycle of the wave as

$$\Delta p = \int_0^1 \frac{-8\left[\left(\bar{Q}-(1-\epsilon)^2-\frac{\phi^2}{2}\right)+(h-\epsilon)^2\right]}{(h-\epsilon)^4\left(1-\frac{4\sqrt{Da}}{\alpha(h-\epsilon)}+\frac{8Da}{\mu(h-\epsilon)^2}\right)}dz \qquad (14)$$

The pressure rise required to produce zero average flow rate is denoted by Δp_0. Hence Δp_0 is given by

$$\Delta p_0 = \int_0^1 \frac{-8\left[\left(-(1-\epsilon)^2 - \frac{\phi^2}{2}\right) + (h-\epsilon)^2\right]}{(h-\epsilon)^4\left(1 - \frac{4\sqrt{Da}}{\alpha(h-\epsilon)} + \frac{8Da}{\mu(h-\epsilon)^2}\right)} dz \qquad (15)$$

It is observed that $Da \to 0$ and $\epsilon \to 0$, equation (9), (10) and (14) reduce to the corresponding results of Jaffrin and Shapiro [14] for the peristaltic transport of a Newtonian fluid in a circular tube.

The dimensionless frictional force F at the wall across one wavelength in the tube is given by

$$F = \int_0^1 (h-\epsilon)^2\left(-\frac{dp}{dz}\right)dz$$

$$= \int_0^1 (h-\epsilon)^2 \left[\frac{-8\left[\left(\bar{Q} - (1-\epsilon)^2 - \frac{\phi^2}{2}\right) + (h-\epsilon)^2\right]}{(h-\epsilon)^4\left(1 - \frac{4\sqrt{Da}}{\alpha(h-\epsilon)} + \frac{8Da}{\mu(h-\epsilon)^2}\right)}\right] dz \qquad (16)$$

DISCUSSIONS AND RESULTS

From equation (14), we have calculated the pressure difference as a function of $\bar{Q}$ for different values of ϵ (thickness of the porous lining), when $\phi = 0.6$, $Da = 0.01$, $\mu = 0.1$, $\alpha = 0.5$, and is shown in Figure.2. It is observed that the peristaltic wave passing over the tube wall pumps against more pressure rise (Δp) due to increase in the thickness of the porous lining. Further there is no difference in the flux due to variation in ϵ for free pumping case $(\Delta p = 0)$.

The variation of pressure rise with time averaged flow rate is calculated from equation (14) for different Darcy numbers and is shown in Figure.3, for fixed $\phi = 0.6$, $\epsilon = 0.01$, $\mu = 0.1$, $\alpha = 0.5$, we observe that the smaller the Darcy number, the greater the pressure rise against which the pump works. For a given Δp, the flux $\bar{Q}$ depends on Da and it decreases with the increasing Darcy number.

From equation (14) we have calculated the pressure rise as a function of $\bar{Q}$ for different values of α and is shown in Figure.4, for fixed $Da = 0.01$, $\epsilon = 0.01$, $\mu = 0.1$, $\phi = 0.6$. We observe that with a given $\bar{Q}$, the value of Δp decreases with an increase in the slip parameter 'α'. For free pumping there is no difference in the flux due to an increment in α.

The variation of pressure rise with time averaged flow rate is calculated from (11) for different μ (ratios of viscosities in the free flow region and the porous region) and is shown in

Figure.5, for fixed $Da = 0.01$, $\in = 0.01$, $\alpha = 0.5$, $\phi = 0.6$, we observe that the larger the value of μ, the greater the pressure rise against which the pump works. For a given Δp, the flux $\bar{Q}$ depends on μ and it increases with increasing μ.

From equation (14) we have calculated the pressure difference as a function of $\bar{Q}$ for different values of amplitude ratios and is shown in Figure.6, for fixed $Da = 0.01$, $\in = 0.01$, $\alpha = 0.5$, $\mu = 0.1$, for given amplitude ratios the pressure difference Δp decreases with increasing $\bar{Q}$. For a given Δp the flux $\bar{Q}$ increases with increasing amplitude ratio ϕ. For free pumping the flux increases with increasing amplitude ratio. For a given $\bar{Q}$ the pressure difference increases with increasing ϕ.

Finally from equation (16) we have calculated frictional force F as a function of $\bar{Q}$ fixed $\in$, Da, α, μ and ϕ depicted in Figures.7 to 11. It is observed that the frictional force F has the opposite behavior compared to pressure rise (Δp).

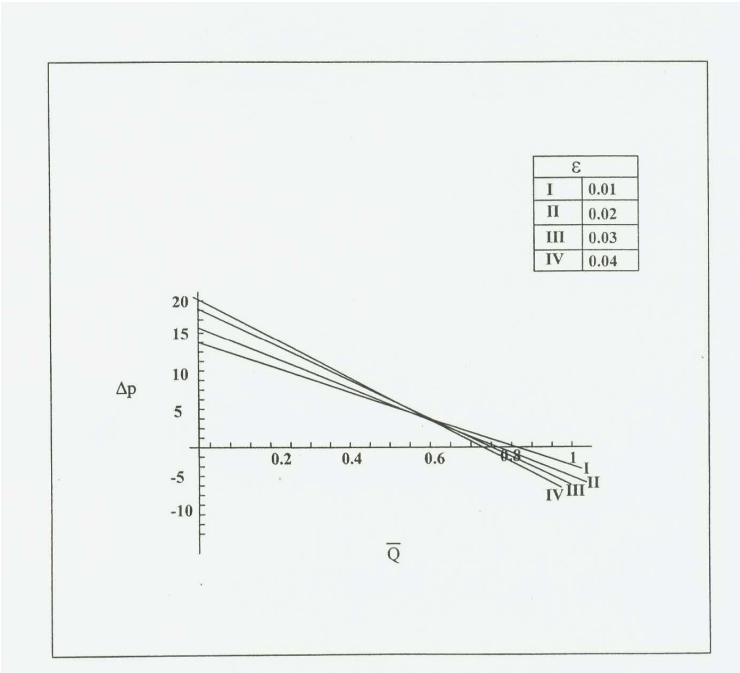

Figure 2. The variation of Δp with $\bar{Q}$ for different values of $\in$ with $\phi = 0.6$, $Da = 0.01$, $\mu = 0.1$, $\alpha = 0.5$

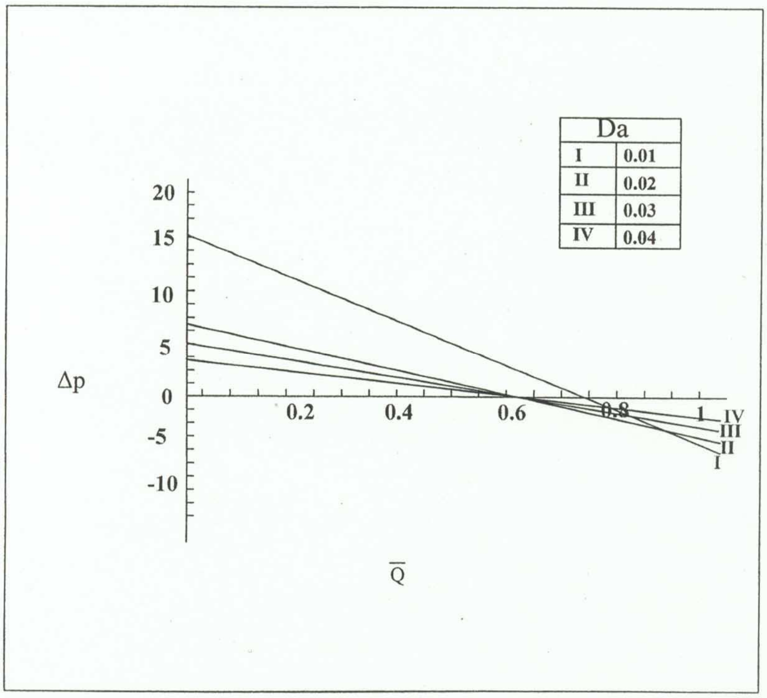

Figure 3. The variation of Δp **with** $\overline{Q}$ **for different values of Da with** $\phi = 0.6, \ \in = 0.01, \ \ \mu = 0.1, \ \ \alpha = 0.5$

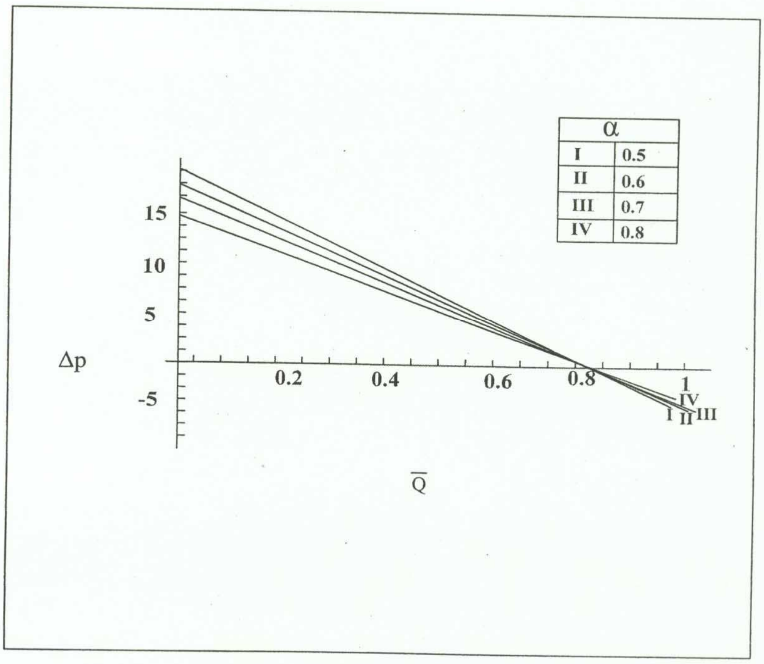

Figure 4. The variation of Δp **with** $\overline{Q}$ **for different values of** α **With** $Da = 0.01, \ \in = 0.01, \ \ \mu = 0.1, \ \ \phi = 0.6$

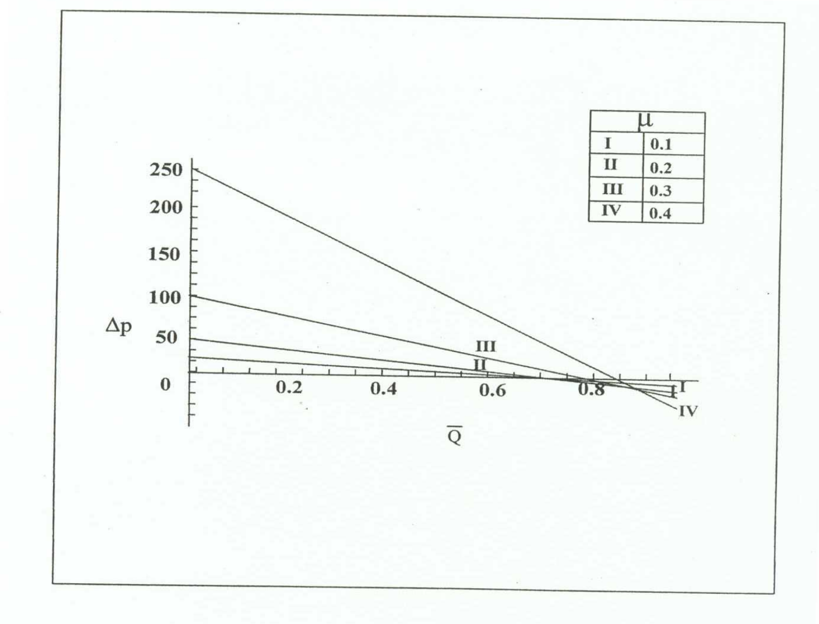

Figure 5. The variation of Δp **with** $\bar{Q}$ **for different values of** μ
With $Da = 0.01, \quad \in = 0.01, \quad \alpha = 0.5, \quad \phi = 0.6$

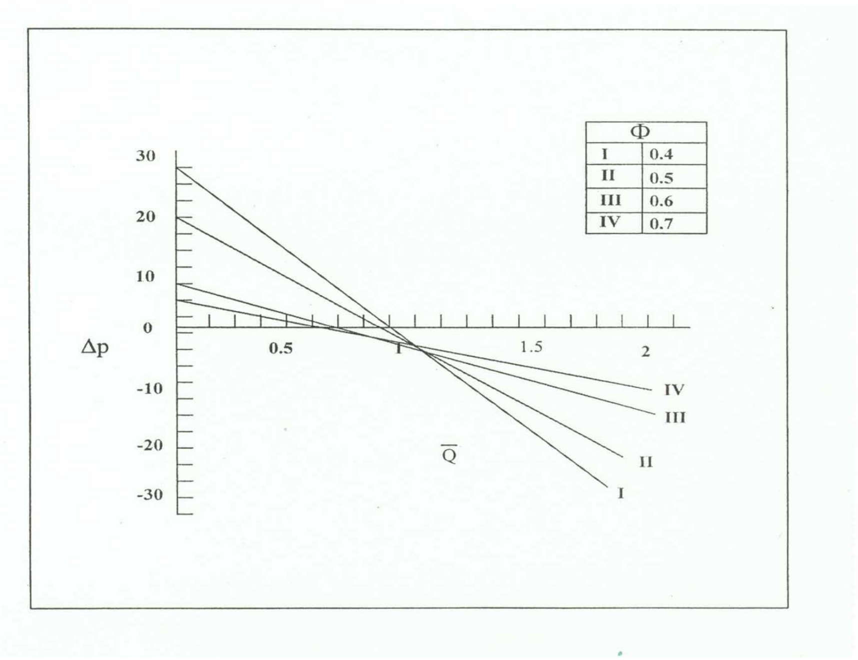

Figure 6. The variation of Δp **with** $\bar{Q}$ **for different values of** ϕ
With $Da = 0.01, \quad \in = 0.01, \quad \alpha = 0.5, \quad \mu = 0.1$

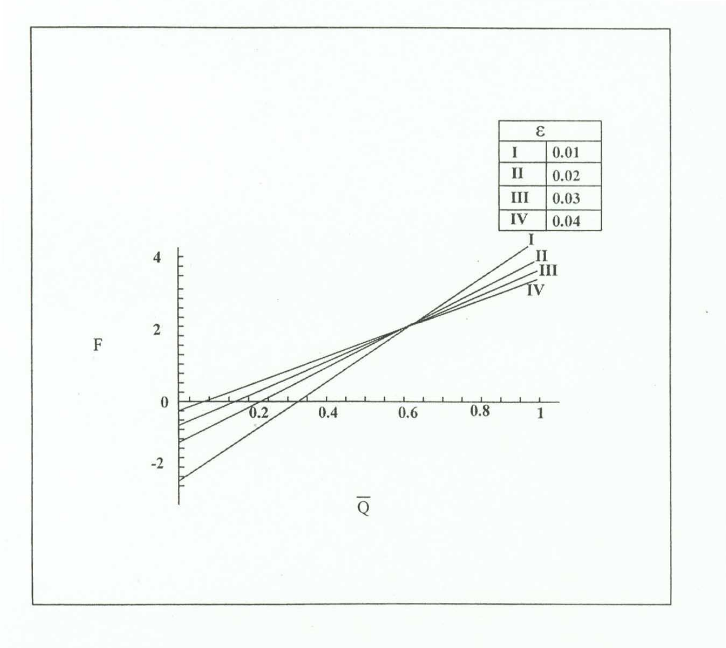

Figure 7. The Variation of F with $\bar{Q}$ for different values of $\in$ with $\phi = 0.6, \quad Da = 0.01, \quad \mu = 0.1, \quad \alpha = 0.5$

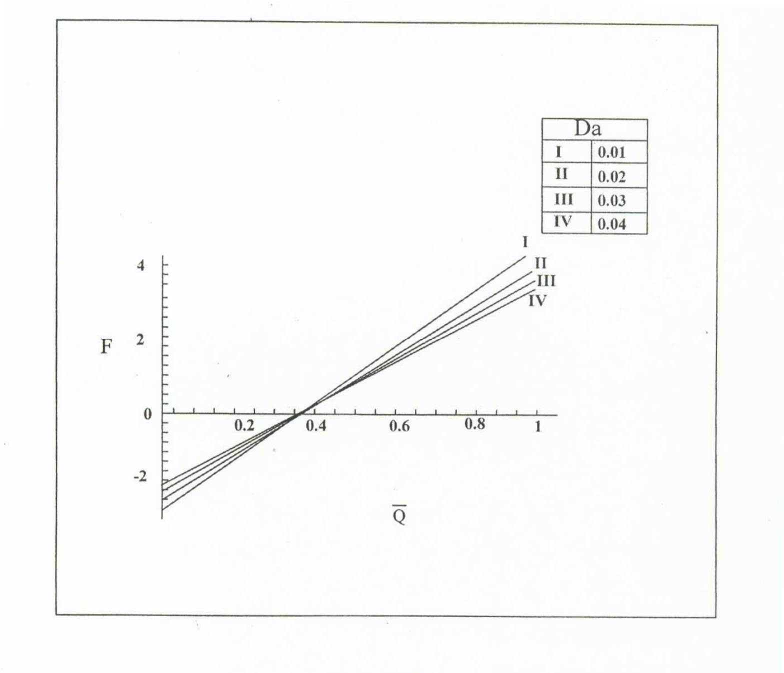

Figure 8. The Variation of F with $\bar{Q}$ for different values of Da with $\phi = 0.6, \quad \in = 0.01, \quad \mu = 0.1, \quad \alpha = 0.5$

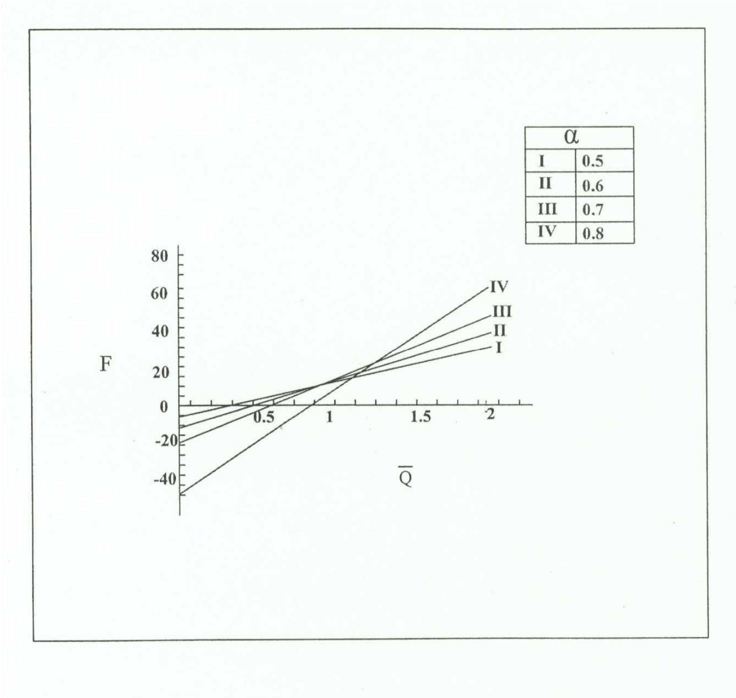

Figure 9. The Variation of F with $\overline{Q}$ for different values of α
With $Da = 0.01, \ \in = 0.01, \ \ \mu = 0.1, \ \ \phi = 0.6$

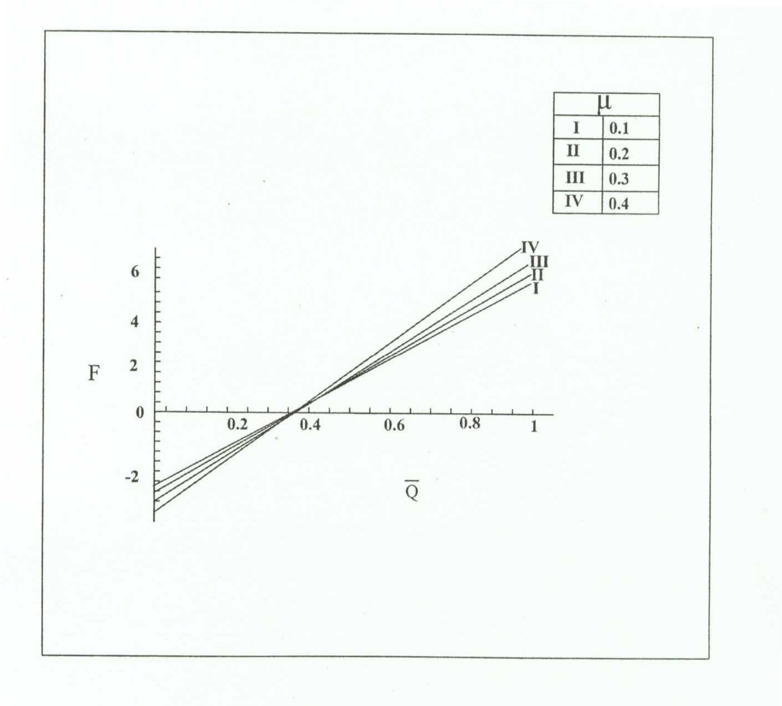

Figure10. The variation of F with $\overline{Q}$ for different values of μ
With $Da = 0.01, \ \ \in = 0.01, \ \ \alpha = 0.5, \ \ \phi = 0.6$

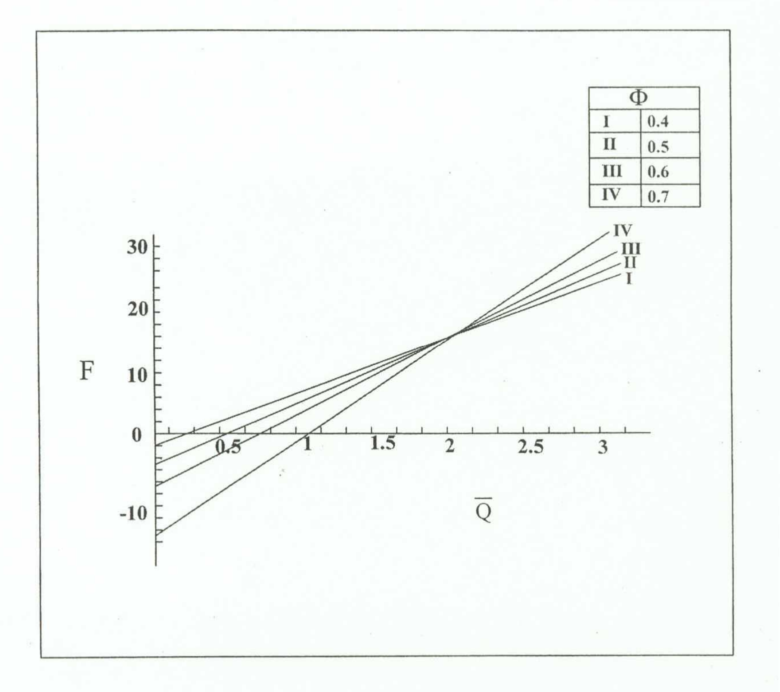

Figure 11. The variation of F with $\overline{Q}$ for different values of ϕ
With $Da = 0.01, \ \in = 0.01, \ \alpha = 0.5, \ \mu = 0.1$

Acknowledgement

The author **Dr.S. Sreenadh** expresses his grateful thanks to DST authorities for providing financial support to undertake this work.

REFERENCES

[1] Beavers, G.S. And Joseph, D.D., *J.Fluid Mech.*, 13,**1967**, 197-207
[2] Brasseur, J.G., Corrsin, S. and Lu, Nan Q., *J.Fluid Mech.*, 174,**1987**, 495-519
[3] Channabasappa, M.N, Umapathy, K.G., And Nayak. I.V., *Proc. Indian Acad. Sci., Vol. 88a, Part Iii*, **1979**, Pp. 163-167
[4] Jaffrin, M.Y. And Shapiro, A.H. , *Ann. Rev. Fluid Mech.*, 3, **1971**, 13-36
[5] Jaffrin, M.Y. And Shapiro, A.H. , *Trans ASME J. Appl, Mech.*, 36,**1969**, 379-381
[6] Kavitha.A, Hemadri Reddy .R, Sreenadh.S, Saravana.R And Srinivas.A.N.S, *Advances In Applied Science Research*, 2,**2011**,269-279
[7] Kavitha.A, Hemadri Reddy.R, Sreenadh.S, Saravana.R, *International Journal Of Innovative Technology And Creating Engineering*, 1,**2011**,Pp.52-57
[8] Kothandapani. M, Srinivas. S , *Physics Letters A, Volume* 372, Issue 8, **2008**, Pp. 1265-1276
[9] Mishra, M. And Ramachandra Rao, *Proceedings Of International Conference On Mathematical Biology, Iit, Kanpur, Feb*. 19-21,**2004**
[10] Misra, J.C. And Pandey, S.K., *Int. J.Engg Sci.*, 37, **1999**, Pp. 1841-1858
[11] Provost, A.M. And Schwarz, W.H. *J. Fluid Mech.*, 279,**1994**, 177-195
[12] Ramachandra Rao, A., And Usha, S. , *J.Fluid Mech.*, 298,**1995**, 271-285
[13] Shukla, J.B. And Gupta, S.P., *Trans. Asme J.Biomech.Engng.*, 104,**1982**, 182-186
[14] Srinivas.S, Kothandapani. M,*International Communications In Heat And Mass Transfer*, Volume 35, Issue4, **2008**, Pages514-522
[15] Srivastava, L.M. And Srivastava, V.P, *J.Biomech.*, 17,**1984**, 821-829

[16] Subba Reddy, M.V. Manoranjan Mishra, Sreenadh, S. And Ramachandra Rao, A. , *Journal Of Fluids Engineering,* 127, **2005**, Pp. 824-827
[17] Subba Reddy.M.V, A.Ramachandra Rao And S.Sreenadh,*International Journal Of Nonlinear Mechanics,* 42, **2006**, ,Pp. 1153-1161
[18] Usha, S., Sreenadh, S., Arunachalam, P.V., *Narosa Publishing House* ,**2001**, 391-398
[19] Vajravelu, K. Sreenadh, S. And Ramesh Babu, V, *Appl. Math. And Computation,* 169,**2005a**, 726-735
[20] Vajravelu, K. Sreenadh, S. And Ramesh Babu, V., *Int. J. Non-Linear Mech.* 40,**2005b**, 83-90
[21] Vajravelu. K, Sreenadh. S, Hemadri Reddy. R, And Murugeshan.K, *International Journal Of Fluid Mechanics Research,* 36 (3), **2009**, Pp. 244-254