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Effect of rotation on thermosolutal convection in a compressible couple-stress fluid through porous medium

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ABSTRACT

The thermosolutal convection in a compressible couple-stress fluid layer heated and soluted from below through porous medium is considered in the presence of uniform rotation. Following the linearized stability theory and normal mode analysis, the dispersion relation is obtained. For stationary convection, the compressibility, stable solute gradient and rotation have stabilizing effect whereas medium permeability and couple-stress parameter have stabilizing as well as destabilizing effects on the system. The stable solute gradient and rotation introduce oscillatory modes in the system, which were non-existent in their absence. The sufficient conditions for non-existence of overstability are also obtained.

Key words: Thermosolutal convection; compressible couple-stress fluid; rotation; porous medium.

INTRODUCTION

The study of a layer of fluid heated from below in porous media is motivated both theoretically and by its practical applications in engineering. Among the applications in engineering discipline one can find the food process industry, chemical process industry, solidification and centrifugal casting of metals. The development of geothermal power resources has increased general interest in the properties of convection in porous medium. When a fluid permeates an isotropic and homogeneous porous medium, the gross effect is represented by the Darcy's law. A great number of applications in geophysics may be found in the books by Phillips [1], Ingham and Pop [2], and Nield and Bejan [3].

The theoretical and experimental results on thermal convection in a fluid layer, in the absence and presence of rotation, have been given by Chandrasekhar [4]. Thermosolutal convection concerns flow that can arise when a layer of fluid with a dissolved solute (such as salt) is heated from below. The problem of the onset of thermal instability in the presence of a solute gradient is of great importance because of its applications to atmospheric physics and astrophysics, especially in the case of the ionosphere and the outer layer of the atmosphere. The thermosolutal convection problems also arise in oceanography, limnology and engineering. Examples of particular interest are provided by ponds built to trap solar heat [Tabor and Matz [5]] and some Antarctic lakes [Shirtcliffe [6]]. Veronis' [7] has investigated the problem of thermohaline convection in a layer of fluid heated from below and subjected to a stable salinity gradient. The physics is quite similar in the stellar case in that helium acts like salt in raising the density and in diffusing more slowly than heat. The conditions under which convective motions are important in stellar atmospheres are usually far removed from consideration of single component fluid and rigid boundaries, and therefore it is desirable to consider a fluid acted on by a solute gradient and free boundaries. Stommel and Fedorav [8] and Linden [9] have remarked that the length scales characteristics of double-diffusive convective layers in the ocean may be sufficiently large that the Earth's rotation might be important in the formation. Moreover, the rotation of the Earth distorts the boundaries of a hexagonal convection cell in a fluid through a porous medium and the distortion plays an important role in the extraction of energy in the geothermal regions. Brakke [10] explained a double-diffusive instability that occurs when a solution of a slowly diffusing protein is layered over a denser solution of more rapidly diffusing sucrose. Nason et al. [11] found that this instability, which is deleterious to certain biochemical separation, can be suppressed by rotation in the ultracentrifuge. The use of the Boussinesq approximation has been made throughout, which states that the variations of density in the equations of motion can safely be ignored everywhere except in its association with the external force. The approximation is well justified in the case of incompressible fluids.

When the fluids are compressible, the equations governing the system become quite complicated. To simplify them, Boussinesq tried to justify the approximation for compressible fluids when the density variations arise principally from thermal effects. Spiegel and Veronis' [12] have simplified the set of equations governing the flow of compressible fluids under the following assumptions:

(a) the depth of the fluid layer is much less than the scale height, as defined by them; and (b) the fluctuations in temperature, density and pressure, introduced due to motion, do not exceed their total static variations.

Under the above approximations, the flow equations are the same as those for incompressible fluids, except that the static temperature gradient is replaced by its excess over the adiabatic one and C_v is replaced by C_p. Sharma [13] has considered the effect of rotation and magnetic field on the thermal instability in compressible fluids. The fluid has been considered to be Newtonian in all the above studies. The effect of magnetic field and rotation on thermosolutal convection in Walters B' elastico-viscous fluid has been considered by Kango and Rana [14]. Saravana et al. [15] have considered the heat and mass transfer on the unsteady viscoelastic second order Rivlin-Ericksen fluid past an impulsive started infinite vertical plate in the presence of a foreign mass and constant mass flux on taking into account of viscous dissipative heat at the plate under the influence of a uniform transverse magnetic field.

With the growing importance of non-Newtonian fluids in modern technology and industries, the investigations on such fluids are desirable. Stokes [16] formulated the theory of couple-stress fluid. One of the applications of couple-stress fluid is its use to the study of mechanisms of lubrications of synovial joints, which has become the object of scientific research. A human joint is a dynamically loaded bearing which has articular cartilage as the bearing and synovial fluid as the lubricant. When a fluid film is generated, squeeze-film action is capable of providing considerable protection to the cartilage surface. The shoulder, ankle, knee and hip joints are the loaded-bearing synovial joints of the human body and these joints have a low friction coefficient and negligible wear. Normally synovial fluid is a viscous, non-Newtonian fluid and is clear or yellowish. According to the theory of Stokes [16], couple-stress appears in noticeable magnitudes in fluids with very large molecules. Since the long chain hyaluronic acid molecules are found as additives in synovial fluids, Walicki and Walicka [17] modeled the synovial fluid as a couple-stress fluid. The synovial fluid is the natural lubricant of joints of the vertebrates. The detailed description of the joint lubrication has very important practical implications. Practically, all diseases of joints are caused by or connected with malfunction of the lubrication. The efficiency of the physiological joint lubrication is caused by several mechanisms. The synovial fluid is due to its content of the hyaluronic acid, a fluid of high viscosity, near to gel. Goel et al. [18] have studied the hydromagnetic stability of an unbounded couple-stress binary fluid mixture under rotation with vertical temperature and concentration gradients. Sharma et al. [19] have considered a couple-stress fluid with suspended particles heated from below. In another study, Sunil et al. [20] have considered a couple-stress fluid heated from below in a porous medium in the presence of a magnetic field and rotation. Kumar et al. [21] have considered the thermal instability of layer of couple-stress fluid acted on by a uniform rotation, and have found that for stationary convection the rotation has a stabilizing effect whereas couple-stress has both stabilizing and destabilizing effects. Thermosolutal convection in a couple-stress fluid in presence of magnetic field and rotation, separately, has been investigated by Kumar and Singh ([22], [23]). Singh and Kumar [24] have considered the problem of thermal instability of compressible, electrically conducting couple-stress fluid in the presence of a uniform magnetic field. The electrically conducting flow of couple-stress fluid in a vertical porous layer has been investigated by Sreenadh et al. [25].

Keeping in mind the importance in geophysics, soil sciences, ground water hydrology, astrophysics and various applications mentioned above, the thermosolutal convection in compressible couple-stress fluid in the presence of uniform rotation through porous medium has been considered in the present paper.

2. Formulation of the problem and perturbation equations

Here we consider an infinite, horizontal compressible couple-stress fluid layer of thickness d in a porous medium, heated and soluted from below so that the temperatures, densities and solute concentrations at the bottom surface z = 0 are T_o, ρ_0 and C_0, and at the upper surface z = d are T_d, ρ_d and C_d, respectively, z-axis being taken as vertical, and that a uniform temperature gradient $\beta\left(=\left|dT/dz\right|\right)$ and a uniform solute gradient $\beta'\left(=\left|dC/dz\right|\right)$ are maintained. This layer is acted on by a uniform vertical rotation $\vec{\Omega}\,(0,0,\Omega)$ and gravity field $\vec{g}(0,0,-g)$.

Assume that X_m is the constant space distribution of X, X_0 is the variation in X in the absence of motion and $X'(x,y,z,t)$ is the fluctuations in X due to the motion of the fluid. Spiegel and Veronis' [12] defined X as any of the state variables (pressure (p), density (ρ) or temperature (T)) and expressed these in the form

$$X(x,y,z,t)=X_m+X_0(z)+X'(x,y,z,t). \tag{1}$$

The initial state is, therefore, a state in which the fluid velocity, temperature, solute concentration, pressure and density at any point in the fluid are given by

$$\vec{q}=0, T=T(z), C=C(z),\ p=p(z), \rho=\rho(z), \tag{2}$$

respectively, where

$$\begin{aligned}
&T(z)=T_0-\beta z,\\
&C(z)=C_0-\beta' z,\\
&p(z)=p_m-g\int_0^z(\rho_m+\rho_0)dz,\\
&\rho(z)=\rho_m[1-\alpha_m(T-T_m)+\alpha'_m(C-C_m)+K_m(p-p_m)]
\end{aligned} \tag{3}$$

and $\alpha_m=-\left(\frac{1}{\rho}\frac{\partial\rho}{\partial T}\right)_m (=\alpha, say),\quad \alpha'_m=-\left(\frac{1}{\rho}\frac{\partial\rho}{\partial C}\right)_m (=\alpha', say),$

$$K_m=\left(\frac{1}{\rho}\frac{\partial\rho}{\partial p}\right)_m.$$

Let $\delta p, \delta\rho, \theta, \gamma$ and $\vec{q}(u,v,w)$ denote, respectively, the perturbations in pressure p, density ρ, temperature T, solute concentration C and fluid velocity (0, 0, 0). The linearized perturbation equations, relevant to the problem, are

$$\frac{1}{\varepsilon}\frac{\partial\vec{q}}{\partial t}=-\frac{1}{\rho_m}\nabla\delta p-\vec{g}(\alpha\theta-\alpha'\gamma)-\frac{1}{k_1}\left(\nu-\frac{\mu'}{\rho_m}\nabla^2\right)q+\frac{2}{\varepsilon}\left(\vec{q}\times\vec{\Omega}\right), \tag{4}$$

$$\nabla.\vec{q}=0, \tag{5}$$

$$E\frac{\partial\theta}{\partial t}=\left(\beta-\frac{g}{C_p}\right)w+\kappa\nabla^2\theta, \quad (6)$$

$$E'\frac{\partial\gamma}{\partial t}=\beta' w+\kappa'\nabla^2\gamma. \quad (7)$$

Here $\frac{g}{C_p}$ is the adiabatic gradient; $\nu\left(=\mu/\rho_m\right), \mu', \kappa, \kappa', \varepsilon \text{ and } k_1$ stand for kinematic viscosity, couple-stress viscosity, thermal diffusivity, solute diffusivity, medium porosity and medium permeability, respectively. $E=\varepsilon+(1-\varepsilon)\left(\rho_s C_s/\rho_0 C\right)$ is a constant and E' is a constant analogous to E but corresponding to solute rather heat; $\rho_s, C_s \text{ and } \rho_0, C$ stand for density and heat capacity of solid (porous matrix) material and fluid, respectively.

The equation of state is

$$\rho=\rho_m[1-\alpha(T-T_0)+\alpha'(C-C_0)], \quad (8)$$

where α is the coefficient of thermal expansion and α' analogous solute coefficient. The suffix zero refers to the values at the reference level z = 0. The change in density δρ caused mainly by the perturbations θ and γ in temperature and concentration, is given by

$$\delta\rho=-\rho_m(\alpha\theta-\alpha'\gamma). \quad (9)$$

In writing Eq. (4), use has been made of Eq. (9).
Equations (4)-(7) give

$$\left[\frac{1}{\varepsilon}\frac{\partial}{\partial t}+\frac{1}{k_1}\left(\nu-\frac{\mu'}{\rho_m}\nabla^2\right)\right]\nabla^2 w=g\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)(\alpha\theta-\alpha'\gamma)-\frac{2\Omega}{\varepsilon}\frac{\partial\zeta}{\partial z}, \quad (10)$$

$$\left[\frac{1}{\varepsilon}\frac{\partial}{\partial t}+\frac{1}{k_1}\left(\nu-\frac{\mu'}{\rho_m}\nabla^2\right)\right]\zeta=\frac{2\Omega}{\varepsilon}\frac{\partial w}{\partial z}, \quad (11)$$

$$\left(E\frac{\partial}{\partial t}-\kappa\nabla^2\right)\theta=\left(\beta-\frac{g}{C_p}\right)w, \quad (12)$$

$$\left(E'\frac{\partial}{\partial t}-\kappa'\nabla^2\right)\gamma=\beta' w, \quad (13)$$

Here $\zeta=\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}$ stands for the z-component of vorticity.

Consider the case in which both the boundaries are free and the temperatures, concentrations at the boundaries are kept constant. Then the boundary conditions appropriate to the problem are

$$w=\frac{\partial^2 w}{\partial z^2}=0,\ \theta=0,\ \gamma=0 \text{ at } z=0 \text{ and } z=d. \quad (14)$$

The constitutive equations for the couple-stress fluid are

$$\tau_{ij} = (2\mu - 2\mu'\,\nabla^2)e_{ij}; \qquad e_{ij} = \frac{1}{2}\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right). \tag{15}$$

The conditions on a free surface are the vanishing of tangential stresses τ_{xz} and τ_{yz}, which yield

$$\tau_{xz} = (\mu - \mu'\,\nabla^2)\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) = 0, \tag{16}$$

$$\tau_{yz} = (\mu - \mu'\,\nabla^2)\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right) = 0. \tag{17}$$

Since w vanishes for all x and y on the bounding surface, it follows from (16) and (17) that

$$(\mu - \mu'\,\nabla^2)\frac{\partial u}{\partial z} = 0, \qquad (\mu - \mu'\,\nabla^2)\frac{\partial v}{\partial z} = 0. \tag{18}$$

From the equation of continuity (5) differentiated with respect to z, we conclude that

$$\left[\mu - \mu'\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)\right]\frac{\partial^2 w}{\partial z^2} = 0, \tag{19}$$

which on using (12) and (16) implies that

$$\frac{\partial^4 w}{\partial z^4} = 0 \ at\ z = 0\ and\ z = d. \tag{20}$$

3. Dispersion relation

We now analyze the disturbances into normal modes, assuming that the perturbation quantities are of the form

$$[w, \theta, \gamma, \zeta] = [W(z), \Theta\,(z), \Gamma\,(z), Z\,(z)]\exp\,(ik_x x + ik_y y + nt), \tag{21}$$

where k_x , k_y are the wave numbers along x- and y- directions respectively, $k(=\sqrt{k_x^2 + k_y^2}\,)$ is the resultant wave number and n is the growth rate which is, in general, a complex constant.

Using expression (21), Eqs. (12) – (15), in non-dimensional form, become

$$\left[\frac{\sigma}{\varepsilon} + \frac{1}{P_l}\{1 - F(D^2 - a^2)\}\right](D^2 - a^2)W + \frac{ga^2 d^2}{\nu}(\alpha\,\Theta - \alpha'\,\Gamma) + \left(\frac{2\Omega d^3}{\varepsilon\nu}\right)DZ = 0, \tag{22}$$

$$\left[\frac{\sigma}{\varepsilon} + \frac{1}{P_l}\{1 - F(D^2 - a^2)\}\right]Z = \left(\frac{2\Omega d}{\varepsilon\nu}\right)DW, \tag{23}$$

$$(D^2 - a^2 - Ep_1\sigma)\Theta = -\frac{d^2}{\kappa}\frac{g}{C_p}(G - 1)W, \tag{24}$$

$$\left(D^2 - a^2 - Eq\sigma\right)\Gamma = -\left(\frac{\beta' d^2}{\kappa'}\right)W, \tag{25}$$

where we have put $a = kd, \sigma = \frac{nd^2}{\nu}, x = x^* d, y = y^* d, z = z^* d$ and $D = \frac{d}{dz^*}$. Here $p_1 = \frac{\nu}{\kappa}$ is the Prandtl number, $q = \frac{\nu}{\kappa'}$ is the Schmidt number, $P_l = \frac{k_1}{d^2}$ is the dimensionless permeability, F = $\frac{\mu'}{\rho_0 d^2 \nu}$ is dimensionless couple-stress parameter and G = $\frac{C_p \beta}{g}$ is the dimensionless compressibility parameter. We shall suppress the star (*) for convenience hereafter.

Eliminating Θ, Γ *and* Z between Eqs. (22)-(25), we obtain

$$\left[\frac{\sigma}{\varepsilon} + \frac{1}{P_l}\left\{1 - F\left(D^2 - a^2\right)\right\}\right]^2 \left(D^2 - a^2\right)\left(D^2 - a^2 - Ep_1\sigma\right)\left[D^2 - a^2 - E'q\sigma\right]W + Sa^2$$
$$\left[\frac{\sigma}{\varepsilon} + \frac{1}{P_l}\left\{1 - F\left(D^2 - a^2\right)\right\}\right]\left[D^2 - a^2 - Ep_1\sigma\right]W + T_A\left(D^2 - a^2 - Ep_1\sigma\right)\left(D^2 - a^2 - E'q\sigma\right)D^2W$$
$$= Ra^2\left(\frac{G-1}{G}\right)\left[\frac{\sigma}{\varepsilon} + \frac{1}{P_l}\left\{1 - F\left(D^2 - a^2\right)\right\}\right]\left(D^2 - a^2 - E'q\sigma\right)W, \tag{26}$$

where R = $\frac{g\alpha\beta d^4}{\nu\kappa}$ is the Rayleigh number, $S = \frac{g\alpha'\beta' d^4}{\nu\kappa'}$ is the solute Rayleigh number and $T_A = \frac{4\Omega^2 d^4}{\varepsilon^2 \nu^2}$ is the modified Taylor number.

The boundary conditions (14), (20), in non-dimensional form, using expression (21) transform to

$$W = D^2W = 0,\ \Theta = 0, \Gamma = 0\ DZ = 0\ \ at\ z = 0\ and\ z = 1. \tag{27}$$

Using the boundary conditions (27), it can be shown with the help of Eqs. (22)-(25) that all the even order derivatives of W must vanish at z = 0 and z =1. Hence the proper solution of W characterizing the lowest mode is

$$W = W_0 \sin \pi z, \tag{28}$$

where W_0 is a constant. Substituting the proper solution (28) in Eq. (26), we obtain the dispersion relation

$$R_1 = \left(\frac{G}{G-1}\right)\left[\frac{(1+x)\left(1 + x + Ep_1\frac{\sigma}{\pi^2}\left\{\frac{\sigma}{\varepsilon\pi^2} + \frac{1}{P}\left(1 + \pi^2 F\overbrace{1+x}\right)\right\}\right)}{x} + S_1\frac{\left(1 + x + Ep_1\frac{\sigma}{\pi^2}\right)}{\left(1 + x + E'q\frac{\sigma}{\pi^2}\right)}\right.$$

$$+T_{A_1}\frac{\left(1+x+Ep_1\frac{\sigma}{\pi^2}\right)}{x\left\{\frac{\sigma}{\varepsilon\pi^2}+\frac{1}{P}\left(1+\pi^2F\overbrace{1+x}\right)\right\}}\Bigg], \tag{29}$$

$$where\ R_1=\frac{R}{\pi^4},\ S_1=\frac{S}{\pi^4},\ T_{A_1}=\frac{T_A}{\pi^4},\ P=\pi^2P_l\ and\ x=\frac{a^2}{\pi^2}.$$

4. The stationary convection

When the instability sets in as stationary convection, the marginal state will be characterized by $\sigma=0$. Putting $\sigma=0$, the dispersion relation (29) reduces to

$$R_1=\left(\frac{G}{G-1}\right)\left[\frac{(1+x)^2}{xP}\left(1+\pi^2F\overbrace{1+x}\right)+S_1+T_{A_1}\frac{(1+x)P}{x\left(1+\pi^2F\overbrace{1+x}\right)}\right]. \tag{30}$$

Equation (30) expresses the modified Rayleigh number R_1 as a function of the dimensionless wave number x and the parameters $G, P, F, T_{A_1}\ and\ S_1$. For fixed $P, F, T_{A_1}\ and\ S_1$, let G (accounting for the compressibility effects) also be kept fixed.

Then we find that

$$\overline{R_C}=\left(\frac{G}{G-1}\right)R_C, \tag{31}$$

where $\overline{R_C}\ and\ R_C$ denote respectively the critical Rayleigh numbers in the presence and absence of compressibility. $G>1$ is relevant here. The cases $G<1\ and\ G=1$ correspond to negative and infinite values of the critical Rayleigh numbers in the presence of compressibility, which are not relevant in the present study. The effect of compressibility is thus to postpone the onset of thermosolutal convection.
Equation (30) yield

$$\frac{dR_1}{dS_1}=\left(\frac{G}{G-1}\right), \tag{32}$$

$$\frac{dR_1}{dT_{A_1}}=\left(\frac{G}{G-1}\right)\frac{(1+x)P}{x\left(1+\pi^2F\overbrace{1+x}\right)}, \tag{33}$$

$$\frac{dR_1}{dP}=\left(\frac{G}{G-1}\right)\frac{(1+x)\left(1+\pi^2 F\overbrace{1+x}\right)}{x}\left[\frac{T_{A_1}}{\left(1+\pi^2 F\overbrace{1+x}\right)^2}-\frac{1+x}{P^2}\right]. \qquad (34)$$

$$\frac{dR_1}{dF}=\left(\frac{G}{G-1}\right)\frac{\pi^2 P(1+x)^2}{x}\left[\frac{(1+x)}{P^2}-\frac{T_{A_1}}{\left(1+\pi^2 F\overbrace{1+x}\right)^2}\right]. \qquad (35)$$

Equations (32) and (33) imply the stabilizing effect of stable solute gradient and rotation on the system.

In the absence of rotation $\left(T_{A_1}=0\right)$, equation (34) gives that $\frac{dR_1}{dP}$ is negative which means that permeability of the medium has a destabilizing effect for a non-rotating system. However, for a rotating system, the medium permeability has still a destabilizing effect if $T_{A_1}\langle\frac{(1+x)\left(1+\pi^2 F\overbrace{1+x}\right)^2}{P^2}$ but has a stabilizing effect also if

$$T_{A_1}\rangle\frac{(1+x)\left(1+\pi^2 F\overbrace{1+x}\right)^2}{P^2}.$$

It is evident from Eq. (35) that couple-stress parameter has stabilizing or destabilizing effect according as

$$T_{A_1}\langle \; or \; \rangle\frac{(1+x)\left(1+\pi^2 F\overbrace{1+x}\right)^2}{P^2}.$$

5. Some important theorems

Theorem 1: The modes may be oscillatory or non-oscillatory in contrast to the case of no rotation and in absence of stable solute gradient where modes are non-oscillatory, for G > 1.

Proof: Multiplying Eq. (22) by W*, the complex conjugate of W, integrating over the range of z and using Eqs. (23)-(25) together with boundary conditions (27), we obtain

$$\left(\frac{\sigma}{\varepsilon}+\frac{1}{P_l}\right)I_1+\frac{F}{P_l}I_2+\frac{g\alpha'\kappa' a^2}{\nu\beta'}\left(I_5+E'q\sigma^* I_6\right)-d^2\left(\frac{\sigma}{\varepsilon}+\frac{1}{P_l}\right)I_7-\frac{F}{P_l}d^2 I_8$$
$$=\left(\frac{1}{G-1}\right)\frac{C_p\alpha\kappa a^2}{\nu}\left(I_3+Ep_1\sigma^* I_4\right), \qquad (36)$$

where

$$\left.\begin{aligned}
I_1 &= \int_0^1 (|DW|^2 + a^2|W|^2)\,dz, \\
I_2 &= \int_0^1 \left(|D^2W|^2 + 2a^2|DW|^2 + a^4|W|^2\right)dz, \\
I_3 &= \int_0^1 (|D\Theta|^2 + a^2|\Theta|^2)\,dz, \\
I_4 &= \int_0^1 |\Theta|^2\,dz, \\
I_5 &= \int_0^1 (|D\Gamma|^2 + a^2|\Gamma|^2)\,dz, \\
I_6 &= \int_0^1 |\Gamma|^2\,dz, \\
I_7 &= \int_0^1 |Z|^2\,dz, \\
I_8 &= \int_0^1 \left(|DZ|^2 + a^2|Z|^2\right)dz,
\end{aligned}\right\}, \qquad (37)$$

and σ* is the complex conjugate of σ. The integrals $I_1 - I_8$ are all positive definite.
Putting $\sigma = \sigma_r + i\sigma_i$ in (36) and equating real and imaginary parts, we obtain

$$\sigma_r\left(\frac{I_1}{\varepsilon} + \frac{g\alpha'\kappa' a^2}{\nu\beta'}E'qI_6 - \frac{d^2}{\varepsilon}I_7 - \frac{1}{G-1}\frac{C_p\alpha\kappa a^2}{\nu}Ep_1I_4\right)$$
$$= -\left(\frac{I_1}{P_l} + \frac{F}{P_l}I_2 + \frac{g\alpha'\kappa' a^2}{\nu\beta'}I_5 - \frac{d^2}{P_l}I_7 - \frac{F}{P_l}d^2I_8 - \frac{1}{G-1}\frac{C_p\alpha\kappa a^2}{\nu}I_3\right), \qquad (38)$$

and

$$\sigma_i\left(\frac{I_1}{\varepsilon} - \frac{g\alpha'\kappa' a^2}{\nu\beta'}E'qI_6 - \frac{d^2}{\varepsilon}I_7 + \frac{1}{G-1}\frac{C_p\alpha\kappa a^2}{\nu}Ep_1I_4\right) = 0. \qquad (39)$$

Equation (39) yields that $\sigma_i = 0$ *or* $\sigma_i \neq 0$, which means that **modes may be non-oscillatory or oscillatory**. In the absence of rotation and stable solute gradient, Eq. (39) reduces to

$$\sigma_i\left(\frac{I_1}{\varepsilon} + \frac{1}{G-1}\frac{C_p\alpha\kappa a^2}{\nu}Ep_1I_4\right) = 0, \qquad (40)$$

and the terms in brackets are positive definite when G > 1. Thus $\sigma_i = 0$, which means that oscillatory modes are not allowed and the **principle of exchange of stabilities is satisfied for a porous medium in compressible, couple-stress fluid in the absence of stable solute gradient and rotation.** This result is true for porous medium and compressible, couple-stress fluid as well as for non-porous medium and incompressible Newtonian fluid [Chandrasekhar [4]]. The presence of each, the stable solute gradient and rotation, brings oscillatory modes (as σ_i may not be zero) which were non-existent in their absence. Equation (38) simply tells us that there may be stability or instability in the presence of rotation and stable solute gradient for a porous medium in compressible couple-stress fluid which is also true in their absence.

Theorem 2: $\left[\varepsilon + (1-\varepsilon)\frac{\rho_S C_S'}{\rho_m C'}\right]\kappa < \left[\varepsilon + (1-\varepsilon)\frac{\rho_S C_S}{\rho_m C}\right]\kappa' \ and\ \kappa < \frac{\mu'\varepsilon}{\rho_m k_1}\left[\varepsilon + (1-\varepsilon)\frac{\rho_S C_S}{\rho_m C}\right]$,

are the sufficient conditions for the non-existence of overstability.

Proof: For overstability, we put $\frac{\sigma}{\pi^2} = i\sigma_1$ where σ_1 is real, in Eq. (29); equating real and imaginary parts and eliminating R_1 between them, we obtain

$$A_2 c_1^2 + A_1 c_1 + A_0 = 0, \qquad (41)$$

where

$$A_2 = \left[\frac{bq^2E'^2}{\varepsilon^2}\left(\frac{b}{\varepsilon} + \frac{Ep_1}{P}\right) + \frac{q^2 p_1 E'^2 b^2 \pi^2 F}{\varepsilon^2 P}\right],$$

$$A_1 = \left[\frac{b^4}{\varepsilon^2} + \frac{b^3}{\varepsilon^2 P}(1+\pi^2 Fb)(Ep_1 - E'q) + \frac{qE'b^3}{\varepsilon P}(1+\pi^2 Fb) + \frac{q^2 p_1 EE'^2 b}{P^3}(1+\pi^2 Fb)^3\right.$$
$$\left. + \frac{q^2E'^2b^2}{\varepsilon P^2}(1+\pi^2 Fb)^2 + \frac{S_1 b}{\varepsilon^2}(b-1)(Ep_1 - E'q) + T_{A_1}\left\{bq^2E'^2\left(\frac{Ep_1\pi^2F}{P} - \frac{1}{\varepsilon}\right) + \frac{p_1 q^2 EE'^2}{P}\right\}\right],$$

$$A_0 = \frac{b^3 Ep_1}{P^3}(1+\pi^2 Fb)^3 + \frac{b^4}{\varepsilon P^2}(1+\pi^2 Fb)^2 + \frac{S_1 b}{P^2}(b-1)(1+\pi^2 Fb)^2(Ep_1 - E'q)$$
$$+ T_{A_1}\left\{\frac{b^2 Ep_1}{P} + b^3\left(\frac{Ep_1\pi^2 F}{P} - \frac{1}{\varepsilon}\right)\right\} = 0, \qquad (42)$$

where we have written $c_1 = \sigma_1^2\ and\ b = 1 + x$.

Since σ_1 is real for overstability, both the values of $c_1 (= \sigma_1^2)$ are positive. Equation (41) is quadratic in c_1 and does not involve any of its roots to be positive, if

$$Ep_1 > E'q\ and\ Ep_1 > \frac{P}{\varepsilon\pi^2 F} \qquad (43)$$

which imply that

$$\left[\varepsilon + (1-\varepsilon)\frac{\rho_S C_S'}{\rho_m C'}\right]\kappa < \left[\varepsilon + (1-\varepsilon)\frac{\rho_S C_S}{\rho_m C}\right]\kappa' \ and\ \kappa < \frac{\mu'\varepsilon}{\rho_m k_1}\left[\varepsilon + (1-\varepsilon)\frac{\rho_S C_S}{\rho_m C}\right], \qquad (44)$$

where $\rho_m C, \rho_S C_S; \rho_m C', \rho_S C_S'$ denote respectively the heat capacities of fluid, solid matrix and analogous solute capacities of fluid, solid matrix.

Thus $\left[\varepsilon+(1-\varepsilon)\frac{\rho_S C'_S}{\rho_m C'}\right]\kappa<\left[\varepsilon+(1-\varepsilon)\frac{\rho_S C_S}{\rho_m C}\right]\kappa' \text{ and } \kappa<\frac{\mu'\varepsilon}{\rho_m k_1}\left[\varepsilon+(1-\varepsilon)\frac{\rho_S C_S}{\rho_m C}\right]$, are the sufficient conditions for the non-existence of overstability, the violation of which does not necessarily imply the occurrence of overstability.

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