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Effect of Regularization on Fuzzy Graph

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ABSTRACT

The perception of fuzzy sets was discussed by L. A. Zadeh in 1965, The idea of a membership function defined on a universal set is engrossed in Fuzzy Sets. Later the theory of Graph was introduced by Euler. Using the notion of fuzzy subsets, the perception of FG was introduced by A. Rosenfeld in 1975. In this paper, some definitions of regular FGs, total degree and totally regular FGs are introduced. Also some properties of regular FGs are studied and examined whether they hold for totally regular FGs or not.

INTRODUCTION

Euler in 1736 first introduced the concept of graph theory. L.A. Zadeh (1965) [1] introduced the notion of fuzzy sets which involves the concept of a membership function defined on a universal set and the value of the membership function lies in [0,1]. Kaufmann in 1973 gave the first definition of a fuzzy graph which was based on Zadeh's fuzzy relations.
Rosenfeld [2] (1975) considered fuzzy relations on fuzzy sets and using this concept of fuzzy subsets, the theory of fuzzy graphs was developed. J. N. Mordeson[7] (1993) studied fuzzy line graphs and developed its basic properties.

Here in this paper, the concept of regular fuzzy graphs and totally regular fuzzy graphs are discussed. A study between regular and totally regular fuzzy graphs through various examples is done. Also some properties of regular fuzzy graphs are studied and examined whether they hold for totally regular fuzzy graphs.

PRELIMINARIES

Definition 2.1[6]: A graph $G^* = (V, E)$ is a pair of vertex set (V) and an edge set (E) where $E \subseteq V \times V$ i.e. E is a relation on V

Definition 2.2 [4]: A fuzzy graph $G = (\sigma, \mu)$ is a pair of functions $\sigma : V \to [0, 1]$ and $\mu : V \times V \to [0, 1]$ with $\mu(u, v) \le \sigma(u) \wedge \sigma(v)$, $\forall$ $u, v \in V$, where V is a finite non empty set and $\wedge$ denote minimum.

Remark 2.3[5]: σ is a fuzzy subset of a non empty set V and μ is a symmetric fuzzy relation on σ .

Definition 2.4: The graph $G^* = (V, E)$ is called the underlying crisp graph of the fuzzy graph G $= (\sigma, \mu)$ where $V = \{u : \sigma (u) > 0\}$ = Support of σ and $E = \{(u, v) \in V \times V : \mu(u, v) > 0\}$ = Support of μ.

Definition 2.5[3]: A fuzzy graph $G = (\sigma, \mu)$ is said to be a strong fuzzy graph if
$\mu(u, v) = \sigma(u) \wedge \sigma(v), \forall (u, v) \in E$.

Definition 2.6[3]: A fuzzy graph $G = (\sigma, \mu)$ is said to be a complete fuzzy graph if
$\mu(u, v) = \sigma(u) \wedge \sigma(v), \forall u, v \in V$.

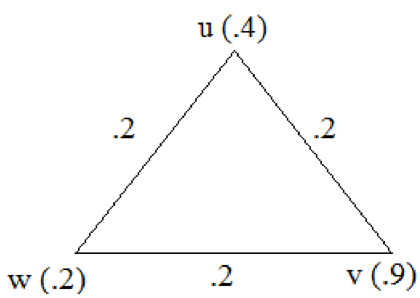
Definition 2.7[3]: The degree of a vertex of G is denoted by

$$d_G(u) = \sum_{u \neq v} \mu(u, v)$$

REGULARIZATION ON FUZZY GRAPH

Definition 3.1: Let G:(σ,μ) be a fuzzy graph. If degree of each vertex is same say m, then G is called m-regular fuzzy graph.

Example 3.2: Let G be a fuzzy graph



Here G is regular fg

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Definition 3.3: Let G:(σ,μ) be a fuzzy graph. The total degree of a vertex of G is denoted by

$$Td_G(u) = \sum_{u \neq v} \mu(u, v) + \sigma(u) = d_G(u) + \sigma(u)$$

If total degree of each vertex is same say m, then G is called m- totally regular fuzzy graph.

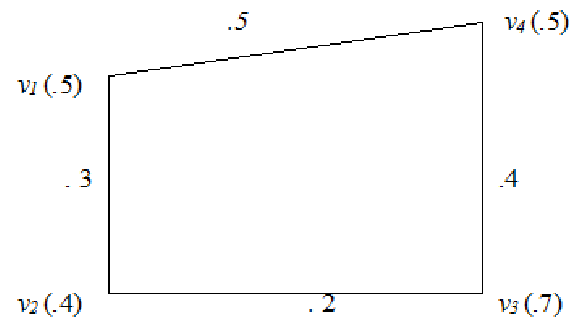
Result 3.4: (I) Every regular fuzzy graph may or may not be a totally regular fuzzy graph

(II) If fuzzy graph is not regular fuzzy graph, it may or may not be a totally regular fuzzy graph

Result 3.5: If fuzzy graph is not regular , it may or may not be a totally regular fuzzy graph

(i) If G is not regular fuzzy graph , G is also not totally regular fuzzy graph.

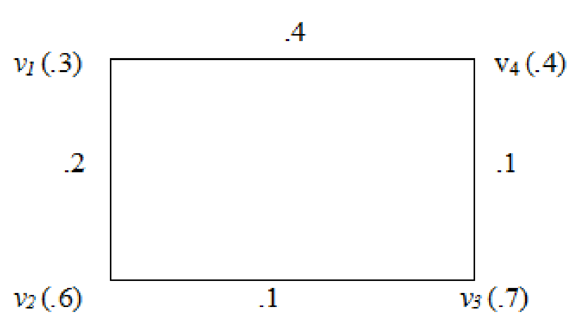
Example 3.6: Consider G*:(V,E) where $V= \{v_1,v_2,v_3,v_4\}$ and $E=\{v_1 v_2,v_2v_3,v_3v_4,v_4v_1\}$. Define fuzzy graphG as below



Then $d(v_i) = 0.8$ and
$d(v_2) = 0.5$
So G is not regular fuzzy graph.
But $Td(v_1) = 1.3$
$Td(v_2).= 0.9$
$Td(v_1). \neq Td(v_2)$.
So G is not totally regular fuzzy graph..

(ii) G is not regular fuzzy graph but G is totally regular fuzzy graph.

Example 3.7: Consider G*:(V,E) where $V= \{v_1,v_2,v_3,v_4\}$ and $E=\{v_1 v_2,v_2v_3,v_3v_4,v_4v_1\}$. Define fuzzy graph G as below

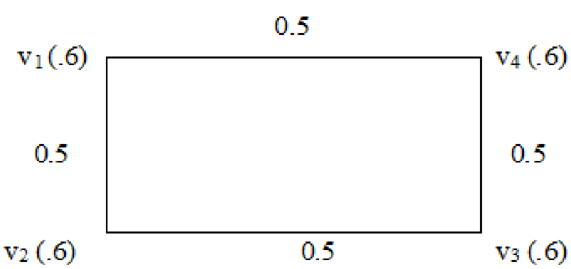


Then $d(v_i) = 0.6$ and
$d(v_2) = 0.3$ So G is not regular fuzzy graph.
But $Td(v_i) = 0.9$ for all $i = 1,2,3,4$
So G is totally regular fuzzy graph.

Result 3.8: Every regular fuzzy graph may or may not be a totally regular fuzzy graph.

(i) A fuzzy graph G is regular as well as totally regular fuzzy graph.

Example3.9: Consider $G^*:(V,E)$ where $V= \{v_1,v_2,v_3,v_4\}$ and $E=\{v_1 v_2,v_2v_3,v_3v_4,v_4v_1\}$. Define fuzzy graph G as below.

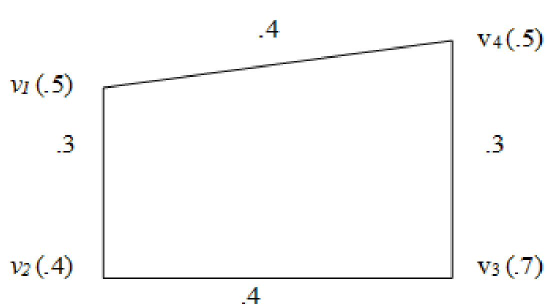


Here $d(v_i) = 1 \ \forall\ i = 1,2,3,4$
So G is regular. Also, $Td(v_i) =1.6 \ \forall\ i = 1,2,3,4$

$\therefore$ G is a totally regular fuzzy graph

(ii) G is regular fuzzy graph , but it is not totally regular fuzzy graph.

Example 3.10: Consider $G^*:(V,E)$ where $V= \{v_1,v_2,v_3,v_4\}$ and $E=\{v_1 v_2,v_2v_3,v_3v_4,v_4v_1\}$. Define fuzzy graph G as below



Then $d(v_i) = 0.7= d(v_2) = d(v_3) = d(v_4)$
So G is regular fuzzy graph.
But $Td(v_1) = 1.2 \neq 1.1 = Td(v_2)$.

So G is not totally regular.

Theorem 3.11: Let σ be a constant function of fuzzy graph G. Prove that G is a regular fuzzy graph if G is a totally regular fuzzy graph.

Proof: Let σ be a constant function say c,
Let G be a m_1 – totally regular fuzzy graph.

$\therefore\ Td(u) = m_1,$ for all $u \in V$.

$\Rightarrow d(u) + \sigma(u) = m_1,$ for all $u \in V$.

$\Rightarrow d(u) + c = m_1,$ for all $u \in V$.

$\Rightarrow d(u) = m_1 - c,$ for all $u \in V$.

$\therefore$ G is a regular fuzzy graph.
Thus G is a regular fuzzy graph if G is a totally regular fuzzy graph.

Theorem 3.12: Let σ be a constant function of fuzzy graph G. Prove that if G is a regular fuzzy graph , G is a totally regular fuzzy graph.

Proof: Let σ be a constant function say c,
Now , Let G be a m_2 – regular fuzzy graph.
Then $d(u) = m_2,\ \forall\ u \in V$.
$\Rightarrow Td(u) = d(u) + \sigma(u),\ \forall\ u \in V$.
$\Rightarrow Td(u) = m_2 + c,\ \forall\ u \in V$.

Hence G is a totally regular fuzzy graph. Thus if G is a regular fuzzy graph then G is a totally regular fuzzy graph.

Theorem 3.13: When a fuzzy graph G is regular as well as totally regular, then vertex set is a constant function.

Proof: Let σ be a vertex set and G be a m_1– regular and m_2– totally regular fuzzy graph.
So $d(u) = m_1$ and $Td(u) = m_2,\ \forall\ u \in V$.

Now $Td(u) = m_2,\ \forall\ u \in V$.

$\Rightarrow d(u) + \sigma(u) = m_2,\ \forall\ u \in V$.

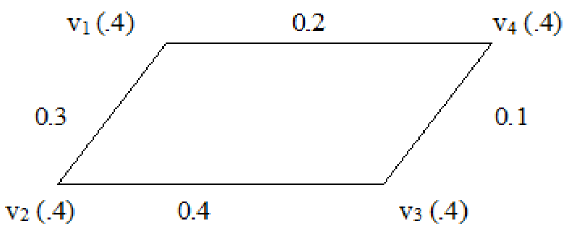
$\Rightarrow m_1 + \sigma(u) = m_2,\ \forall\ u \in V$.

$\Rightarrow \sigma(u) = m_2 - m_1,\ \forall\ u \in V$.

Hence σ is a constant function.

Remark 3.14: Converse of the above theorem 3.13 need not be true.

Example 3.15: : Consider G*:(V,E) where V= $\{v_1,v_2,v_3,v_4\}$ and E=$\{v_1 v_2,v_2v_3,v_3v_4,v_4v_1\}$. Define fuzzy graphG as below



Then $d(v_i) = 0.5$, $d(v_2) = 0.7$
So G is not regular fg .
But $Td(v_1) = 0.9 \neq 1.1 = Td(v_2)$.
So G is not totally regular whereas σ is a constant function .

CONCLUSION

There is no relation between regular fuzzy graphs and totally regular fuzzy graphs. A regular fuzzy graph may or may not be a totally regular fuzzy graph and in addition if fuzzy graph is not regular, it does not provide assurance for it to be a totally regular fuzzy graph

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REFERENCES

1. L.A. Zadeh,1965, Fuzzy Sets, Information and Control, 8: 338-353.
2. A. Rosenfeld,1975, Fuzzy Graphs, In Fuzzy Sets and their Applications to Cognitive and Decision Process, Zadeh. L. A., Fu, K. S., Shimura, M., Eds; Academic Press, New York, 77-95.
3. Sunitha,M.S., and Vijayakumar, A.A., Characterization of Fuzzy Trees, Information Sciences, 113(1999), 293-300.
4. T. Dinesh and T. V. Ramakrishnan, On Generalised Fuzzy Graph Structures , Applied Mathematical Sciences, Vol. 5, 2011, no. 4, 173 – 180.
5. Bhutani, K.R., On Automorphism of fuzzy Graphs, Pattern Recognition Letters 12:413-420, 1991.
6. Frank Harary, Graph Thoery, Narosa /Addison Wesley, Indian Student Edition, 1988.
7. John N. Mordeson and Premchand S.Nair, Fuzzy Graphs and Fuzzy Hypergrphs, Physica-verlag, Heidelberg 2000.