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Effect of magnetic field on peristaltic transport of a couple stress fluid in a channel

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ABSTRACT

The effect of magnetic field on peristaltic transport of a couple stress fluids in non-uniform two dimensional channels has been studied. Long wavelength and low Reynolds number approximation is used to linearize the governing equations. The expressions for pressure gradient and frictional force are obtained by using boundary conditions. The effects of various parameters on channel are discussed with the help of graphs.

Keywords: Peristaltic transport, Magnetic field, Couple stress fluid, Pressure rise.

INTRODUCTION

Peristaltic pumping is now well known to physiologists to be one of the major mechanisms for fluid transport in many biological systems. In particular, peristalsis is an important mechanism generated by the propagation of waves along the walls of a channel or tube. It occurs in the gastrointestinal, urinary, reproductive tracts and many other glandular ducts in a living body. The initial mathematical model of peristalsis obtained by train of sinusoidal waves in an infinitely long symmetric channel or tube has been investigated by Shapiro et al., [1] and Fung and Yahi [5]. After these investigations, many authors have studied the peristaltic pumping for newtonian and non- newtonian fluids in different situations. Effect of thickness of the porous material on the peristaltic pumping of a Jeffry fluid with non-erodible porous lining wall is studied by Rathod and Mahadev [13]. Rathod and Mahadev [23] studied slip effects and heat transfer on MHD peristaltic flow of Jeffrey fluid in an inclined channel. Rathod and Laxmi Devindrappa [24] studied slip effect on peristaltic transport of a conducting fluid through a porous medium in an asymmetric vertical channel by Adomian decomposition method. Rathod and Laxmi Devindrappa [25] studied peristaltic transport of a conducting fluid in an asymmetric vertical channel with heat and mass transfer. Rathod and Laxmi Devindrappa [26] studied effects of heat transfer on the peristaltic MHD flow of a Bingham fluid through a porous medium in a channel. Vijayaraj et al., [12] studied the peristaltic pumping of a fluid of variable viscosity in a non-uniform tube with permeable wall. Ravi Kumar et al., [8] studied the unsteady peristaltic pumping in a finite length tube with permeable wall. Effect of thickness of the porous material on the peristaltic pumping when the tube wall is provided with non-erodible porous lining has been studied by Hemadri et al., [6].

A number of studies containing couple stress have been investigated [2,4,9,10,11]. Couple stress in peristaltic transport of fluids is studied by Elshehawey and Mekheimer [3]. Peristaltic transport of a couple stress fluid in a uniform and non- uniform channels is studied by Mekheimer [7]. Peristaltic pumping of couple stress fluid through non-erodible porous lining tube wall with thickness of porous material is studied by Rathod et al., [20]. Srivastava et al., [21] studied peristaltic transport of a physiological fluid: part I flow in non-uniform geometry. Gupta and

Sheshadri [22] studied peristaltic pumping in non-uniform tubes. It is now well known that blood behaves like a magneto hydrodynamic (MHD) fluid Stud et al., [17]. The consideration of blood as a MHD fluid helps in controlling blood pressure and has potential for therapeutic use in the diseases of heart and blood vessels by Mekheimer [16]. Peristaltic transport to a MHD third order fluid in a circular cylindrical tube was investigated by Hayat and Ali [14]. Hayat et al., [15] have investigated peristaltic transport of a third order fluid under the effect of a magnetic field. Subba Reddy et al., [18] have studied the peristaltic transport of Williamson fluid in a channel under the effect of a magnetic field. Jayarami Reddy et al., [19] have studied the peristaltic flow of a Williamson fluid in an inclined planar channel under the effect of a magnetic field. Rathod and Sridhar [27] studied peristaltic flow of a couple stress fluid in an inclined channel. Rathod et.al., [28] studied peristaltic flow of a couple stress fluid in an inclined channel under the effect of magnetic field. Rathod et.al., [29] studied peristaltic transport of a conducting couple stress fluid through a porous medium in a channel.

We propose to study peristaltic transport of a viscous incompressible fluid (creeping flow) in non-uniform two dimensional channels at long wavelength and low Reynolds number. A motivation of the present analysis has been the hope that such a theory of a couple stress fluid is very useful in understanding the role of peristaltic muscular contraction in transporting bio-fluids behaving like couple stress fluid, besides its numerous applications to other physiological and engineering problems. In this paper, peristaltic transport of a couple stress fluid with effect of magnetic field in a channel is investigated.

2. FORMULATION OF THE PROBLEM

Consider the peristaltic flow of a couple stress fluid through a two-dimensional channel of non-uniform thickness with a sinusoidal wave travelling down its wall. We choose a rectangular coordinate system for the channel with x^* along the centerline in the direction of wave propagation and y^* transverse to it, see in Fig. 1. The geometry of the wall surface is defined as

$$h^*(x^*,t^*) = a(x^*) + bSin(\frac{2\pi}{\lambda}(x^* - ct^*)) \tag{2.1}$$

With $a(x^*) = a_{20} + kx^*$ (2.2)

Where $a(x^*)$ is the half-width of the channel at any axial distance x^* from inlet, a_{20} is the half-width at the inlet, k (<<1) is the constant whose magnitude depends on the length of the channel exit and inlet dimensions, b is the wave amplitude, λ is the wavelength, c is the propagation velocity and t is time.

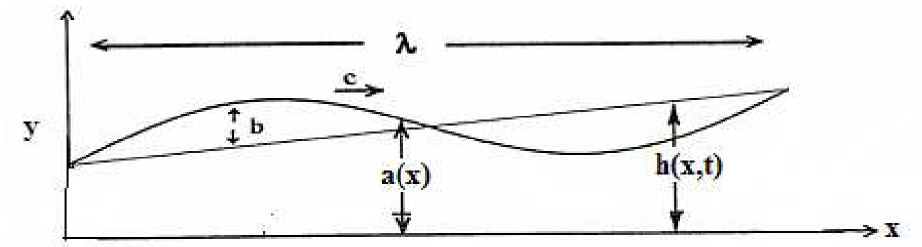

Fig 1: Peristaltic transport in a non-uniform channel

In the absence of the body force and body couples, the equations of motion in the laboratory frame are:

$$\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} = 0 \tag{2.3}$$

$$\rho\{\frac{\partial u^*}{\partial t^*} + u^*\frac{\partial u^*}{\partial x^*} + v^*\frac{\partial u^*}{\partial y^*}\} = -\frac{\partial p^*}{\partial x^*} + \mu\nabla^2(u^*) - \eta\nabla^4(u^*) - \sigma B_\circ^2(u^*) \tag{2.4}$$

$$\rho\{\frac{\partial v^*}{\partial t^*}+u^*\frac{\partial v^*}{\partial x^*}+v^*\frac{\partial v^*}{\partial y^*}\}=-\frac{\partial p^*}{\partial y^*}+\mu\nabla^2(v^*)-\eta\nabla^4(v^*)-\sigma B_\circ^2(v^*) \qquad (2.5)$$

Where $\nabla^2=\frac{\partial^2}{\partial x^{*2}}+\frac{\partial^2}{\partial y^{*2}}$

Where u^* and v^* are velocity components in the corresponding coordinates. ρ is density, μ is viscosity, η is couple stress parameter, σ is electric conductivity and $B_\circ$ is applied magnetic field.

Introducing a wave frame (x, y) moving with velocity c away from the fixed frame (x^*, y^*) by the transformation

$$x=x^*-ct^*, y=y^*, u=u-c, v=v^*, p=p^*(x^*,t^*)$$

It is convenient to non-dimensionalize variable appearing in equation (2.1-2.5) introducing Reynolds number Re and wave number ratio δ as follows:

$$x=\frac{x^*}{\lambda},\ y=\frac{y^*}{a_{20}},\ u=\frac{u^*}{c},\ v=\frac{\lambda v^*}{a_{20}c}\ p=\frac{a_{20}^2}{\lambda\mu c}p^*(x^*),\ t=\frac{t^*c}{\lambda},\ \mathrm{Re}=\frac{\rho c a_{20}}{\mu},\ \delta=\frac{a_{20}}{\lambda},$$
$$M=B_\circ\sqrt{\frac{\sigma}{\mu a_{20}^2}},\ h=\frac{h^*}{a_{20}}=1+\frac{\lambda kx}{a_{20}}+\phi Sin[2\pi(x-t)], where\quad \phi(amplitude)=\frac{b}{a_{20}}\le 1 \qquad (2.6)$$

The equation of motion and boundary conditions in dimensionless form becomes

$$\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=0 \qquad (2.7)$$

$$\mathrm{Re}\,\delta\{\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\}=-\frac{\partial p}{\partial x}+\nabla^2(u)-\frac{1}{\gamma^2}\nabla^4(u)-M^2(u) \qquad (2.8)$$

$$\mathrm{Re}\,\delta\{\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\}=-\frac{\partial p}{\partial y}+\delta^2\nabla^2(v)-\frac{\delta^2}{\gamma^2}\nabla^4(v)-\delta^2M^2(v) \qquad (2.9)$$

Where, $\gamma=\sqrt{\frac{\eta}{\mu a_{20}^2}}$ couple-stress parameter & $M=B_\circ\sqrt{\frac{\sigma}{\mu a_{20}^2}}$ is the Hartmann number

Using long wavelength approximation (δ <<1) & low Reynolds number (Re $\rightarrow$ 0), it follows from Eqs (2.7)–(2.9) that appropriate eqn. describing the flow in laboratory frame of reference are:

$$\frac{\partial p}{\partial y}=0 \qquad (2.10)$$

$$\frac{\partial p}{\partial x}=\frac{\partial^2 u}{\partial y^2}-\frac{1}{\gamma^2}\frac{\partial^4 u}{\partial y^4}-M^2(u) \qquad (2.11)$$

with dimensionless boundary conditions

$$\frac{\partial u}{\partial y}=0,\quad \frac{\partial^3 u}{\partial y^3}=0 \quad \text{at} \quad y=0$$

$$u = 0, \quad \frac{\partial^2 u}{\partial y^2} = 0 \quad \text{at} \quad y = h \tag{2.12}$$

Integrating equation (2.11) and using boundary condition (2.12), one finds the expression for viscosity profile as

$$u(x,y,t) = \frac{\partial p}{\partial z}\left[\frac{(y^2-h^2)}{(2-M^2y^2)} + \frac{(h^4-y^4)}{2\gamma^2}\right] \tag{2.13}$$

The instantaneous volume flow rate Q (x, t) is given by

$$Q(x,t) = \int_0^h u\,dy = \frac{dp}{dx}\left[\frac{2h^5}{5\gamma^2} - \left(\frac{2hM + \sqrt{2}(-2+h^2M^2)\text{ArcTanh}[\frac{\text{hM}}{\sqrt{2}}]}{2hM^3}\right)\right] \tag{2.14}$$

Or

$$\frac{dp}{dx} = \frac{Q(x,t)}{\left[\frac{2h^5}{5\gamma^2} - \left(\frac{2hM + \sqrt{2}(-2+h^2M^2)\text{ArcTanh}[\frac{\text{hM}}{\sqrt{2}}]}{2hM^3}\right]\right.} \tag{2.15}$$

The pressure rise $\Delta p_L(t)$ and friction force $F_L(t)$ (at the wall) in the channel of length L, in their non-dimensional forms, are given by

$$\Delta p_L(t) = \int_0^A \frac{dp}{dx}dx \tag{2.16}$$

$$\Delta F_L(t) = \int_0^A h\left(-\frac{dp}{dx}\right)dx \tag{2.17}$$

Where A = L / λ,

Use of Eq. (2.6) & Eq. (2.15) in Eqns. (2.16) and (2.17) gives

$$\Delta p_L(t) = \int_0^A \left[\frac{Q(x,t)}{\left\{\frac{2h^5}{5\gamma^2} - \left(\frac{2hM + \sqrt{2}(-2+h^2M^2)\text{ArcTanh}[\frac{\text{hM}}{\sqrt{2}}]}{2hM^3}\right\}\right.}\right]dx \tag{2.18}$$

$$\Delta F_L(t) = \int_0^A \left[\frac{-hQ(x,t)}{\left\{\frac{2h^5}{5\gamma^2} - \left(\frac{2hM + \sqrt{2}(-2+h^2M^2)\text{ArcTanh}[\frac{\text{hM}}{\sqrt{2}}]}{2hM^3}\right\}\right.}\right]dz, \tag{2.19}$$

Putting k=0 in equations (2.18) and (2.19), reduces to the expressions for pressure rise and friction force in a uniform channel. The analytical interpretation of our analysis with other theories are difficult to make at this stage, as the integral equations (2.18) and (2.19) are not solvable in closed form , neither for non-uniform nor uniform geometry (k=0).Thus, further studies of our analysis are only possible after numerical evaluation of these integrals.

3. NUMERICAL RESULT AND DISCUSSION

To discuss the results obtained above quantitatively we shall assume form of instantaneous volume flow rate Q (x, t), periodic in (x-t) as [21,22]

$$Q(x,t) = \bar{Q} + \phi \sin(2\pi(x-t)) \tag{3.20}$$

Where $\bar{Q}$ is time-average of flow over one period of wave .This form of Q(x, t) has been assumed in view of fact that the constant value of Q(x,t) gives $\Delta P_L(t)$ always negative, and hence there will be no pumping action. Using this form of Q (x, t), we shall now compute dimensionless pressure rise $\Delta P_L(t)$ and friction force F_L over channel length for various value of dimensionless time t, dimensionless flow average(rate) $\bar{Q}$, amplitude ratio ϕ , couple stress parameter γ and M is magnetic field. Average rise in pressure $\Delta \bar{P}_L$ and friction force $F_{\bar{(L)}}$ are then evaluated by averaging $\Delta P_L(t)$ and $F_L(t)$ over one period of wave. Using the following value of the parameters in equations (18) and (19) as:

a_{20}=0.01cm, L= λ =10cm $k = \frac{0.5a_{20}}{\lambda} = 0.0005$

The integral in Eqs. (2.18) and (2.19) are numerically evaluated. Figs.[2-5] represents the variation of dimensionless pressure rise with dimensionless time t for different values of magnetic field, amplitude ratio, flow rate and couple stress for non-uniform channels respectively. Fig.2.shows that as the magnetic field M increases the pressure rise decreases. Fig.3.which shows the effect of increasing the amplitude ratio is to increase the pressure rise. Fig.4.it is clear that an increase in the flow rate decreases the pressure rise. Fig.5.shows that as the couple stress fluid parameter increases the pressure rise decreases.

The friction force F with dimensionless time t under the influence of all emerging parameters such as magnetic field, amplitude ratio, flow rate and couple stress. It is observed that the effect of all the parameters on friction force are opposite to the effects on pressure with time is observed in Figs. [6-9].

Average pressure rise verses time average mean flow rate is plotted for different values of magnetic field, amplitude ratio and couple stress for a non-uniform channel. Figs.[10-12] which shows a linear relation between them. It is clear that an increase in the flow rate decreases the pressure rise. Fig.10. the effect of increases the magnetic field M is to increases the pressure rise. Fig.11.also, the effect of increasing the amplitude ratio is to increase the pressure rise and fig.12 also, shows the effect of couple stress fluid, where the pressure rise increases as couple stress increases.

Figs.[13-15] represents the friction force F verses time average mean flow rate is plotted for different values of magnetic field, amplitude ratio and couple stress for a non-uniform channel. Here, it is observed that the effect of all the parameters on friction force are opposite to the effects on pressure with time average mean flow rate is observed.

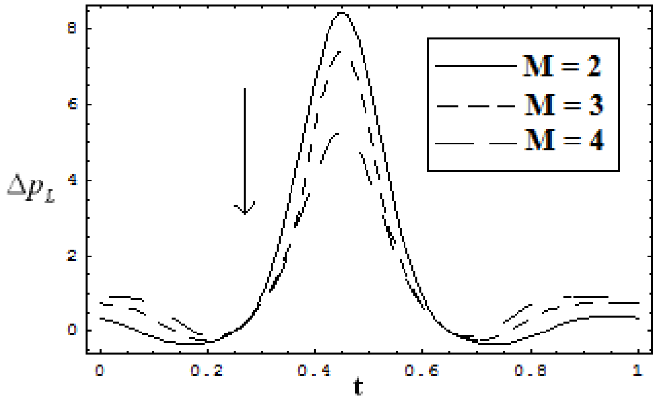

Fig 2: Variation of dimensionless Pressure rise over dimensionless time for $\phi = 0.7$, $\gamma = 3$, $\bar{Q} = 0.22$ & different values of M

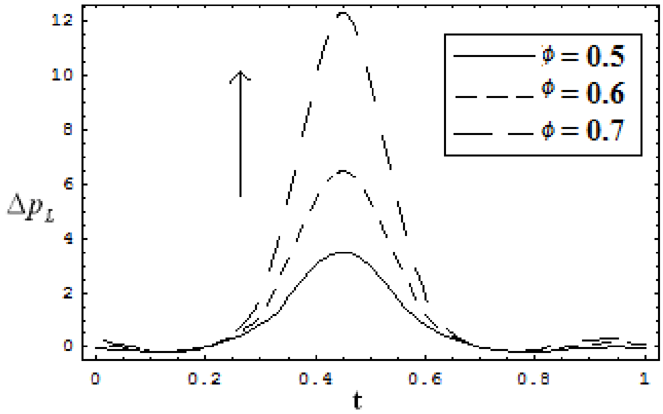

Fig 3: Variation of dimensionless Pressure rise over dimensionless time for M = 2, $\gamma = 3$, $\bar{Q} = 0$ & different values of ϕ

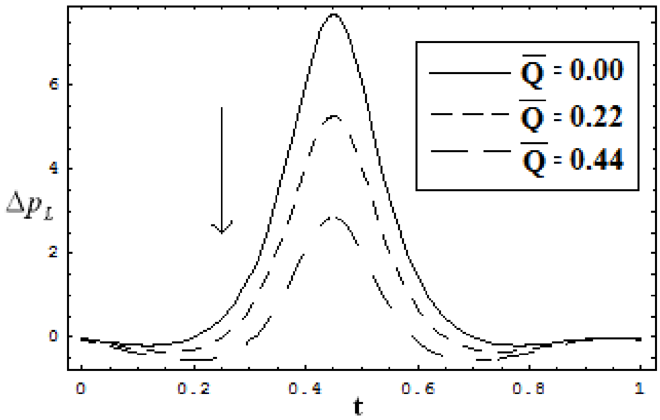

Fig 4: Variation of dimensionless Pressure rise over dimensionless time for M = 4, $\gamma = 8$, $\phi = 0.7$ & different values of $\bar{Q}$

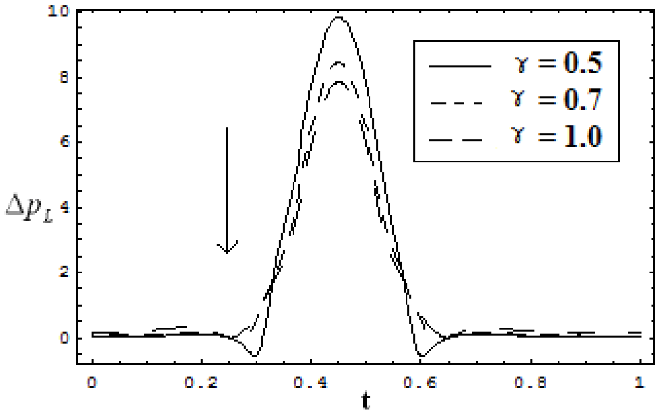

Fig 5: Variation of dimensionless Pressure rise over dimensionless time for M = 3, $\bar{Q}$ = 0.22, ϕ = 0.7 & different values of γ

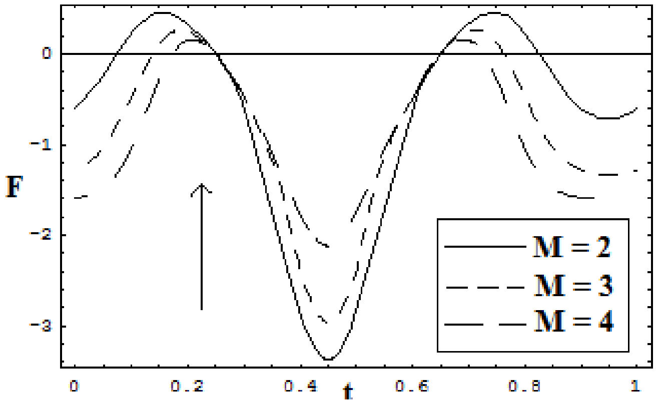

Fig.6. Effect of M on friction force of non-uniform when ϕ = 0.7, γ = 3, $\bar{Q}$ = 0.22

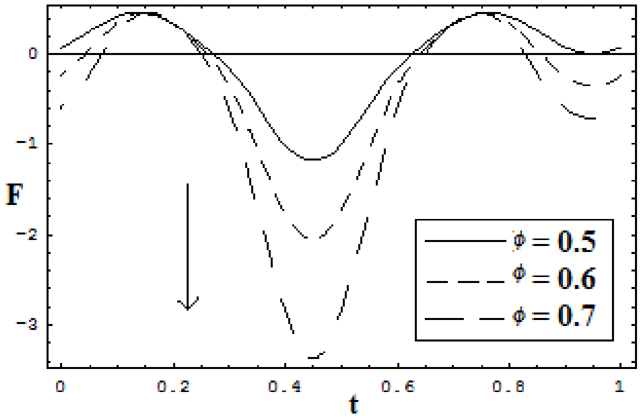

Fig.7. Effect of ϕ on friction force of non-uniform when M = 2, γ = 3, $\bar{Q}$ = 0.22

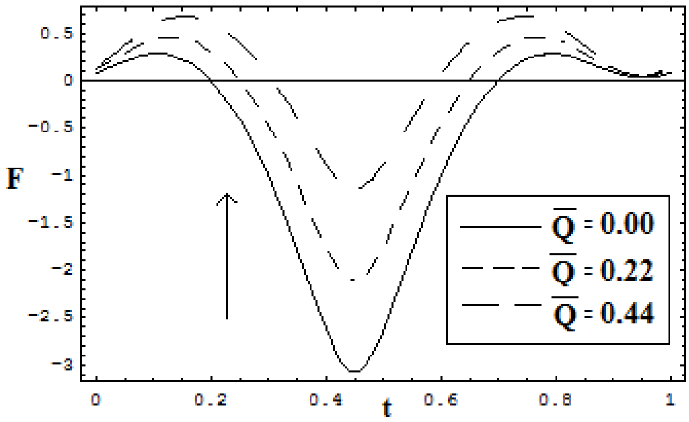

Fig.8. Effect of $\overline{Q}$ on friction force of non-uniform when M = 4, γ = 8, ϕ = 0.7

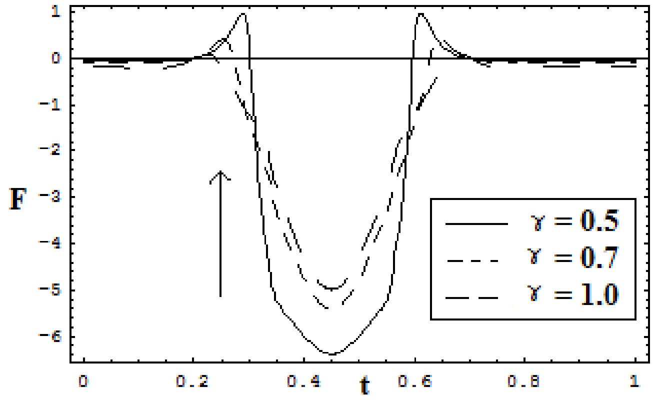

Fig.9. Effect of γ on friction force of non-uniform when M = 2.5, $\overline{Q}$ = 0, ϕ = 0.7

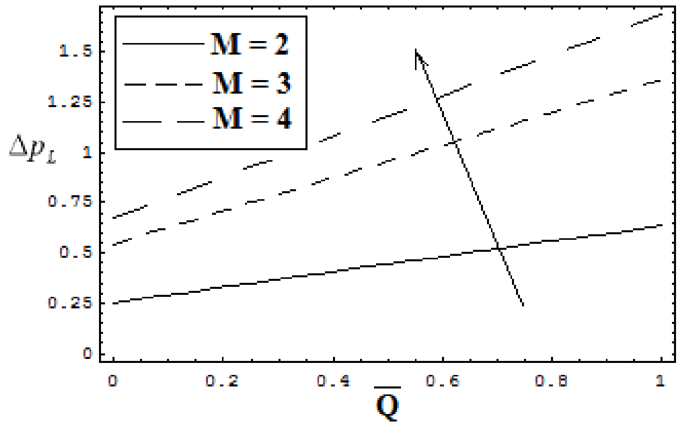

Fig.10. Effect of M on Pressure rise when γ = 3, t = 0, ϕ = 0.7

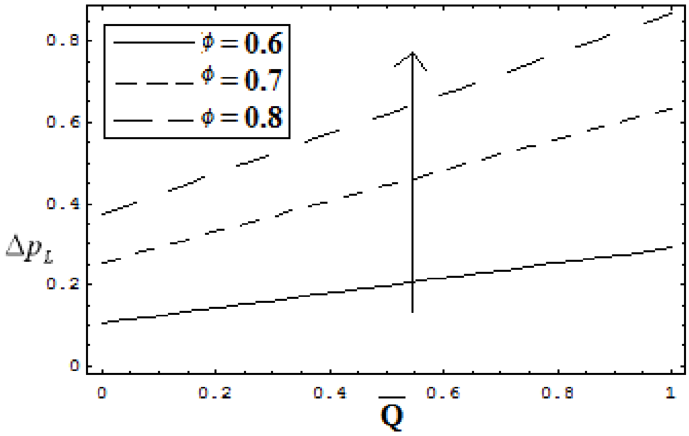

Fig.11. Effect of ϕ on Pressure rise when γ = 3, t = 0, M = 2

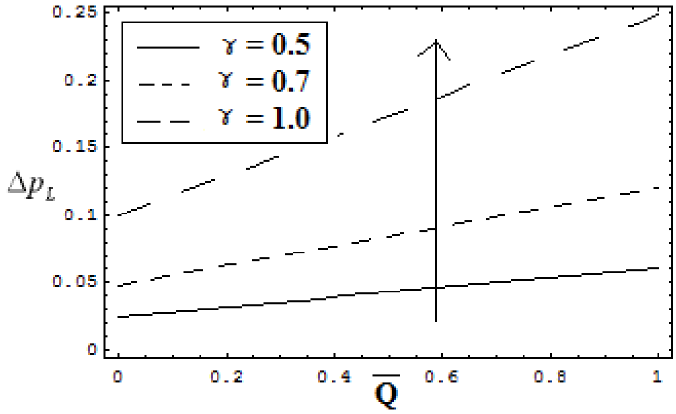

Fig.12. Effect of γ on Pressure rise when ϕ = 0.7, t = 0, M = 3

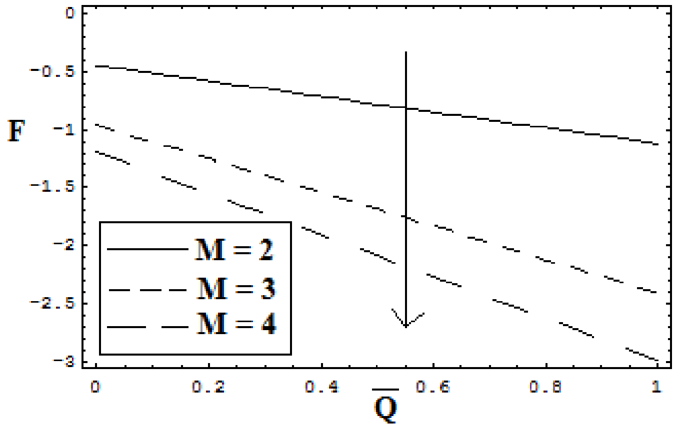

Fig.13. Effect of M on friction force of non-uniform when ϕ =0.7, γ = 3, t = 0

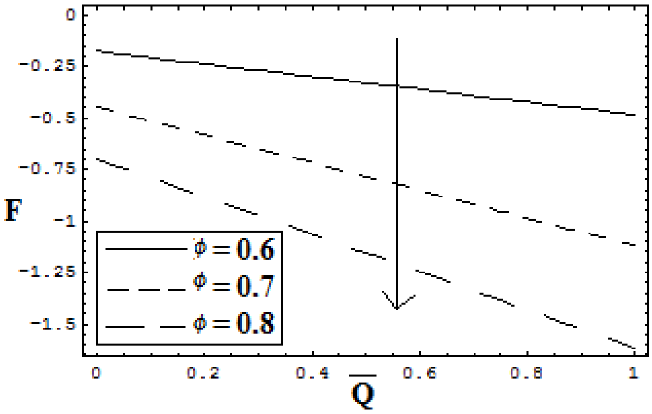

Fig.14. Effect of ϕ on friction force of non-uniform when M = 2, γ = 3, t = 0

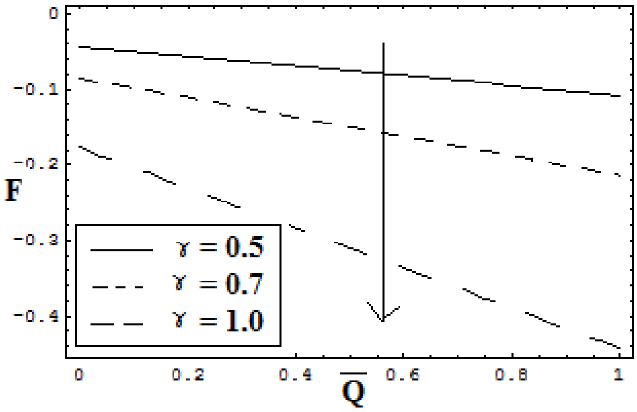

Fig.15. Effect of γ on friction force of non-uniform when M = 2.5, t = 0, ϕ = 0.7

CONCLUSION

In this paper we presented a theoretical approach to study the peristaltic flow of a couple stress fluid with effect of magnetic field in non-uniform two dimensional channel. The governing Equations of motion are solved analytically. Furthermore, the effect of various values of parameters on Pressure rise and Friction force have been computed numerically and explained graphically. We conclude the following observations:

- Pressure with time:

Pressure decreases with increase in magnetic field M, low rate $\bar{Q}$ & couple stress parameter γ and increases with increasing in amplitude ϕ.

- Pressure with averaged flow rate:

Pressure increases with increase in magnetic field M, amplitude ϕ & couple stress parameter γ.

- The variations of friction force with time and averaged flow rate shows opposite behavior to that of pressure.

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