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Effect of kuvshinski fluid on double-diffusive convection- radiation interaction on unsteady MHD flow over a vertical moving porous plate with heat generation and soret effects

Arvind Kumar Sharma[1*], G. K. Dubey[2*] and N. K. Varshney[1]

[1]*Department of Mathematics, Agra College, Agra*
[2]*Department and of Mathematics, S. V. College, Aligarh*

ABSTRACT

The objective of present problem is to study effect of Kuvshinski fluid on double-diffusive convection- radiation interaction on unsteady MHD flow over a vertical moving porous plate with heat generation and Soret effects. The fluid is considered to be a gray, absorbing emitting but non scattering medium, and the Rosseland approximation is used to describe the radiative heat flux in the energy equation. The plate moves with constant velocity in the direction of fluid flow while the free stream velocity is assumed to follow the exponentially increasing small perturbation law. A uniform magnetic field acts perpendicular to the porous surface. The effects of various parameters on the velocity, temperature and concentration fields as well as the skin friction coefficient are presented graphically and in tabulated forms.

Keywords: Kuvshinski fluid; MHD; boundary layer; porous medium; thermal radiation; chemical reaction; thermal diffusion; heat generation.

INTRODUCTION

The convection problem in a porous medium has important applications in geothermal reservoirs and geothermal extractions. The process of heat and mass transfer is encountered in aeronautics, fluid fuel nuclear rector, chemical process industries and many engineering applications in which the fluid is the working medium. The wide range of technological and industrial applications has stimulated considerable amount of interest in the study of heat and mass transfer in convection flows. The unsteady fluid flow past a moving plate in the presence of free convection and radiation were studies by Monsure [16], Raptis and Perdikis [22], Ganesan and Loganathan [8], Mbeledogu et. al. [17], Makinde [15], and Abdus –Satter and Hamid Kalim [2]. All these studies have been confined to unsteady flow in a nonporous medium. From the porous literature survey about unsteady fluid flow, we observe that little papers were done in porous medium. The effect of radiation on MHD flow and heat transfer must be considered when high temperatures are reached. El-Hakiem [7] studied the unsteady MHD oscillatory flow on free convection radiation through a porous medium with vertical infinite surface that absorb the fluid with a constant velocity. Ghaly [9] employed a symbolic computation software Mathematical to study the effect of radiation on heat and mass transfer over stretching sheet in the presence of magnetic field. Raptis et. al. [21] studied the effect of radiation on 2D steady MHD optically thin gray gas low along infinite vertical plates taking into account the induced magnetic field.

Cookey et al. [6] researched the influence of viscous dissipation and radiation on steady MHD free convection flow past on infinite heated vertical plate in porous medium with time dependent suction. Abd El-Naby et al. [1] employed an implicit finite difference method to study the effect of radiation on MHD unsteady free convection flow past a semi infinite vertical porous plate but did not take in to account the viscous dissipation. Takhar et. al. [24] describe the radiation effects on MHD free convection flow past a semi infinite vertical plate. Kim [14] studied unsteady MHD convective heat transfer past a semi infinite vertical porous moving plate.

The study of heat generation or absorption effects in moving fluids is important in view of several physical problems, such as fluids undergoing exothermic or endothermic chemical reactions. Hossain et al. [12] studies the problem of natural convection flow along vertical wavy surface with uniform surface temperature in the presence if heat generation / absorption. Alam et al. [3] studied the problem of free convection heat and mass transfer flow past an inclined semi infinite heated surface of an electrically conducting and steady viscous in compressible fluids in the presence of a magnetic field and heat generation. Chamkha [5] investigated unsteady convective heat and mass transfer past a semi infinite porous moving plate with heat absorption. Hady et. al. [10] studied the problem of free convection flow along a vertical wavy surface embedded in electrically conducting fluids saturated porous media in the presence of internal heat generation or absorption effects.

Combined heat and mass transfer problems with chemical reaction are of importance in many processes and have, therefore received a considerable amount of attention in recent years. In processes such as drying, evaporation at the surface of a water body, energy transfer in a wet cooling tower and the flow in a desert cooler heat and the mass transfer occur simultaneously. Possible application if this type of flow can be found in many industries. For example in the power industry among the methods of generation electric power is one in which electrical energy is extracted directly form a moving conducting fluids. Many practical diffusive operations involve the molecular diffusion of a species in the presence of chemical reaction within or at the boundary. There are two types of reactions. A homogeneous reaction is one that occurs uniformly throughout a give phase. The species generation in a homogeneous reaction is analogous to internal source of heat generation. In constraint a heterogeneous reaction takes place in a restricted region or within the boundary of a phase. It can therefore be treated as a boundary condition similar to the constant heat flux condition in heat transfer the study of heat and mass transfer with chemical reaction is of great practical importance to engineers and scientists because of its almost universal occurrence in many branches of science and engineering.

Muthucumaraswamy and Ganesan [20] studied first-order chemical reaction on flow past on impulsively started vertical plate with uniform heat and mass flux. Chamkha [4] studied MHD flow of a numerical of uniformly stretched vertical permeable surface in the presence of heat generation/absorption and a chemical reaction. Muthucumaraswamy and Ganesan [19] investigated the effect of the chemical reaction and injection on flow characteristics in an unsteady upward motion of an isothermal plate. Raptis and Perdikis [23] analyzed the effects of a chemical reaction of an electrically conducting viscous fluids on the flow over a non linearly (quadratic) semi-infinite stretching heart in the presence of constant magnetic fields which in normal to the sheet. Ibrahim et al. [13] analyzed the effects of the chemical reaction and radiation absorption on the unsteady MHD free convection flow past a semi infinite vertical permeable moving plate with source and suction. Recently, Mohamed [18] has discussed double-diffusive convection-radiation interaction on unsteady MHD flow over a vertical moving porous plate with heat generation and soret effects.

In spite of all these studies the effect of Kuvshinski fluid on unsteady MHD double diffusive free convection for a heat generating fluids with thermal radiation and chemicals reaction has received more attention. Hence, the main object of the present investigation is to study the effect of Kuvshinski fluid on the unsteady MHD double diffusive free convection fluid flow past a vertical porous plate with a first order homogeneous chemical reaction, thermal radiation, heat source and thermal diffusion in the presence of mass blowing or suction. It is assumed that the plate is embedded in a uniform porous medium and moves with a constant velocity in the flow direction in the presence of a transverse magnetic field. It is also assumed that the free stream to consist of a mean velocity and temperature over which are superimposed an exponentially varying with time.

MATHEMATICAL FORMULATION OF THE PROBLEM

Two dimensional unsteady flow of a laminar, conducting and heat generation/absorption fluid past a semi infinite vertical moving porous plate embedded in a uniform porous medium and subjected to a uniform porous medium and subjected to a uniform magnetic field in the presence of a pressure gradient has been considered with double - diffusive free convection, thermal diffusion, chemical reaction, and thermal radiation effects. According to the coordinate system the x*-axis is taken along the porous plate in the upward direction and y*-axis normal to it. The fluid is assumed to be gray, absorbing-emitting but non-scattering medium. The radiative heat flux in the x*-direction is considered negligible in comparison with that in the y*-direction. It is assumed that there is no applied voltage of which implies the absence of an electric field. The transversely applied magnetic field and magnetic Reynolds number are very small and hence the induced magnetic field is negligible. Viscous and Darcy resistance terms are taken into account in the constant permeability porous medium. The MHD term is derived from an order of magnitude analysis of the full Navier-stokes equation. It is assumed here that the hole size of the porous plate is significantly larger than a characteristics microscopic length scale of the porous medium. We regard the porous medium as an assemblage of small identical spherical particles fixed in space. A homogeneous first-order chemical reaction is between the fluid and the species concentration. The chemical reactions are taking place in the flow and all thermo physical properties are assumed to be constant of the linear momentum equation which is approximation. The fluid properties are assumed to be constants except that the influence of density variation with temperature and concentration has been considered in the body-force term. Due to the semi-infinite place surface assumption furthermore, the flow variable are function of y* and t* only. The governing equation for this investigation is based on the balances of mass, linear momentum energy and concentration species. Taking into consideration the assumptions made above, these equations can be written in Cartesian frame of reference, as follows:

$$\frac{\partial v^*}{\partial y^*}=0 \qquad \text{......................... (1)}$$

$$\rho\{(1+\lambda_1^*\frac{\partial}{\partial t^*})\frac{\partial u^*}{\partial t^*}+v^*\frac{\partial u^*}{\partial y^*}\}=-(1+\lambda_1^*\frac{\partial}{\partial t^*})\frac{\partial p^*}{\partial x^*}+\mu\frac{\partial^2 u^*}{\partial y^{*2}}$$
$$-\rho g-(\sigma B_o^2+\frac{\mu}{K^*})(1+\lambda_1^*\frac{\partial}{\partial t^*})u^* \qquad \text{............ (2)}$$

$$\frac{\partial T^*}{\partial t^*}+v^*\frac{\partial T^*}{\partial y^*}=\frac{k}{\rho C_p}\frac{\partial^2 T^*}{\partial y^{*2}}+\frac{Q_o}{\rho C_p}(T^*-T^*_\infty)-\frac{1}{\rho C_p}\frac{\partial q^*_r}{\partial y^*} \qquad \text{..................... (3)}$$

$$\frac{\partial C^*}{\partial t^*}+v^*\frac{\partial C^*}{\partial y^*}=D_M\frac{\partial^2 C^*}{\partial y^{*2}}+D_T\frac{\partial^2 T^*}{\partial y^{*2}}-R^*(C^*-C^*_\infty) \qquad \text{..................... (4)}$$

The boundary conditions at the wall and in the free stream are:

$$u^*=u_p^*, T^*=T_w^*+\varepsilon(T_w^*-T_\infty^*)e^{n^*t^*}, C^*=C_w^*+\varepsilon(C_w^*-C_\infty^*)e^{n^*t^*} \text{ at } y^*=0 \qquad \text{........ (5)}$$

$$u^*\to U_\infty^*=U_o(1+\varepsilon e^{n^*t^*}), T^*\to T_\infty^*,\ C^*\to C_\infty^* \text{ at } y^*\to\infty \qquad \text{................ (6)}$$

where u^*, v^* - Velocity components in X, Y directions respectively, g - Gravitational acceleration, t^* -Time, ν - Kinematic coefficient of viscosity, σ - Electrical conductivity, □ - The viscosity, ρ - Density of the fluid, λ_1^* - the coefficient of Kuvshinski fluid, T^* - Temperature of the fluid, T_w^* - The temperature at the plate, T_∞^* - The temperature of the fluid in free stream, k - Thermal conductivity, C_p - Specific heat at constant pressure, q^*_r - Radiative heat flux, K*- Permeability parameter of the porous medium, D_M - the coefficient of chemical molecular diffusivity, D_T - The coefficient of thermal diffusivity, C^* - The dimensional concentration, C_w^* -The concentration at the wall, C_∞^* - The concentration in free stream, R* - the reaction rate constant, Q_o - Source/sink constant, U_o and n* - constants. The magnetic and viscous dissipations are neglected in this study. It is assumed that the porous plate moves with a constant velocity, u_p^* in the direction of fluid flow, and the free stream velocity U_∞^* follows the exponentially increasing small perturbation law. In addition, it is assumed that the temperature and concentration at the wall as well as the suction velocity are exponentially varying with time.

It is clear from equation (1) that the suction velocity at the plate surface is a function of time only. Assuming that it takes the following exponential form:

$$v^* = -V_o(1+\varepsilon A e^{n^* t^*}) \qquad \text{................ (7)}$$

where A is a real positive constant, ε and εA are small less than unity, and V_o is a scale of suction velocity which has non-zero positive constant.

In the free stream, from equation (2) we get

$$\rho\frac{dU_\infty^*}{dt^*} = -\frac{\partial p^*}{\partial x^*} - \rho_\infty g - \sigma B_o^2 U_\infty^* - \frac{\mu}{K^*}U_\infty^* \qquad \text{........................ (8)}$$

Eliminating $\frac{\partial p^*}{\partial x^*}$ between equation (2) and equation (8), we obtain

$$\rho\{(1+\lambda_1^*\frac{\partial}{\partial t^*})\frac{\partial u^*}{\partial t^*} + v^*\frac{\partial u^*}{\partial y^*}\} = \rho(1+\lambda_1^*\frac{\partial}{\partial t^*})\frac{dU_\infty^*}{dt^*} + \mu\frac{\partial^2 u^*}{\partial y^{*2}} + (\rho_\infty - \rho)g + (\sigma B_o^2 + \frac{\mu}{K^*})(1+\lambda_1^*\frac{\partial}{\partial t^*})(U_\infty^* - u^*) \qquad \text{...... (9)}$$

by making use the equation of state

$$(\rho_\infty - \rho) = \rho\beta(T^* - T_\infty^*) + \rho\beta^*(C^* - C_\infty^*) \qquad \text{..................... (10)}$$

where β is the volumetric coefficient of thermal expansion, β^* is the volumetric coefficient of expansion with concentration and ρ_∞ is the density of the fluid far away the surface. Then substituting from equation (10) into equation (9) we obtain

$$(1+\lambda_1^*\frac{\partial}{\partial t^*})\frac{\partial u^*}{\partial t^*} + v^*\frac{\partial u^*}{\partial y^*} = (1+\lambda_1^*\frac{\partial}{\partial t^*})\frac{dU_\infty^*}{dt^*} + \nu\frac{\partial^2 u^*}{\partial y^{*2}} + g\beta(T^* - T_\infty^*) + g\beta^*(C^* - C_\infty^*) + (\frac{\sigma B_o^2}{\rho} + \frac{\nu}{K^*})(1+\lambda_1^*\frac{\partial}{\partial t^*})(U_\infty^* - u^*) \qquad \text{....... (11)}$$

The radiation flux on the basis of the Rosseland diffusion model for radiation heat transfer is expressed as:

$$q_r^* = -\frac{4\sigma^*}{3k_1^*}\frac{\partial T^{*4}}{\partial y^*} \qquad \text{..................... (12)}$$

where σ^* and k_1^* are respectively the Stefan – Boltzmann constant and the mean absorption coefficient. We assume that the temperature difference within the flow are sufficiently small such that T^{*4} may be expressed as a linear function of the temperature. This is accomplished by expanding in Taylor series about T_∞^* and neglecting higher order terms, thus

$$T^{*4} \cong 4T_\infty^{*3}T^* - 3T_\infty^{*4} \qquad \text{..................... (13)}$$

By using equation (12) and (13), into equation (3) is reduced to

$$\frac{\partial T^*}{\partial t^*}+v^*\frac{\partial T^*}{\partial y^*}=\frac{k}{\rho C_p}\frac{\partial^2 T^*}{\partial y^{*2}}+\frac{Q_o}{\rho C_p}(T^*-T^*_\infty)-\frac{16\sigma^* T_\infty^{*3}}{3\rho C_p k_1^*}\frac{\partial^2 T^{*4}}{\partial y^{*2}} \quad \text{................ (14)}$$

We now introduce the dimension less variables, as follows:

$$\left.\begin{array}{l} u=\dfrac{u^*}{U_o},\ v=\dfrac{v^*}{V_o},\quad y=\dfrac{y^* V_o}{\nu},\ U_\infty^*=U_\infty U_o,\quad u_p^*=U_p U_o \\ t=\dfrac{t^* V_o^2}{\nu},\quad n=\dfrac{n^*\nu}{V_o^2},\quad \theta=\dfrac{T^*-T_\infty^*}{T_w^*-T_\infty^*},\quad \phi=\dfrac{C^*-C_\infty^*}{C_w^*-C_\infty^*} \end{array}\right\} \quad \text{............(15)}$$

Then substituting from equation (15) into equations (11), (14) and (4) and taking into account equation (7), we obtain

$$(1+\lambda_1\frac{\partial}{\partial t})\frac{\partial u}{\partial t}-(1+\varepsilon Ae^{nt})\frac{\partial u}{\partial y}=(1+\lambda_1\frac{\partial}{\partial t})\frac{dU_\infty}{dt}+\frac{\partial^2 u}{\partial y^2}$$
$$+G_{rT}\theta+G_{rC}\phi+N(1+\lambda_1\frac{\partial}{\partial t})(U_\infty-u) \quad \text{....... (16)}$$

$$\frac{\partial\theta}{\partial t}-(1+\varepsilon Ae^{nt})\frac{\partial\theta}{\partial y}=\frac{1}{P_r}(1+\frac{4R}{3})\frac{\partial^2\theta}{\partial y^2}+\eta\theta \quad \text{................(17)}$$

$$\frac{\partial\phi}{\partial t}-(1+\varepsilon Ae^{nt})\frac{\partial\phi}{\partial y}=\frac{1}{S_c}\frac{\partial^2\phi}{\partial y^2}+S_o\frac{\partial^2\theta}{\partial y^2}-\delta\phi \quad \text{................(18)}$$

where

$G_{rT}=\dfrac{\nu g\beta(T_w^*-T_\infty^*)}{V_o^2 U_o}$ (Thermal Grashof number)

$G_{rC}=\dfrac{\nu g\beta^*(C_w^*-C_\infty^*)}{V_o^2 U_o}$ (Solutal Grashof number)

$M=\dfrac{\sigma B_0^2\nu}{\rho V_o^2}$ (Magnetic field parameter)

$K=\dfrac{K^* V_o^2}{\nu^2}$ (Permeability parameter)

$\eta=\dfrac{\nu Q_o K^*}{\rho V_o^2 C_p}$ (Heat generation/absorption parameter)

$R=\dfrac{4\sigma^* T_\infty^{*3}}{kk_1^*}$ (Thermal radiation parameter)

$\delta=\dfrac{R^*\nu}{V_o^2}$ (Chemical reaction parameter)

$P_r=\dfrac{\rho\nu C_p}{k}$ (Prandtl number)

$$S_c = \frac{\nu}{D_M} \text{ (Schmidt number)}$$

$$S_o = \frac{D_T(T_w^* - T_\infty^*)}{\nu(C_w^* - C_\infty^*)} \text{ (Soret number)}$$

$$\lambda_1 = \frac{\lambda_1^* V_o^2}{\nu} \text{ (Visco-elastic parameter)}$$

$$N = \left(M + \frac{1}{K}\right) \quad \text{..........(19)}$$

The corresponding boundary conditions are

$$u = U_p, \theta = 1 + \varepsilon e^{nt}, \phi = 1 + \varepsilon e^{nt} \text{ at } y = 0 \quad \text{.......... (20)}$$

$$u \to U_\infty = (1 + \varepsilon e^{nt}), \theta \to 0, \ \phi \to 0 \text{ at } y \to \infty \quad \text{................ (21)}$$

ANALYTICAL APPROXIMATE SOLUTION

In order to reduce the above system of partial differential equations to a system of ordinary equations in dimension less form, we may represent the velocity, temperature and concentration as

$$u = u_o(y) + \varepsilon e^{nt} u_1(y) + O(\varepsilon^2) \quad \text{...............(22)}$$

$$\theta = \theta_o(y) + \varepsilon e^{nt} \theta_1(y) + O(\varepsilon^2) \quad \text{...............(23)}$$

$$\phi = \phi_o(y) + \varepsilon e^{nt} \phi_1(y) + O(\varepsilon^2) \quad \text{...............(24)}$$

By substituting the above equations (22) to (24) into equations (16) to (18), equating the harmonic and non-harmonic term and neglecting the higher-order terms of $O(\varepsilon^2)$, we obtain the following pairs of equations for (u_o, θ_o, ϕ_o) and (u_1, θ_1, ϕ_1).

$$u_o'' + u_o' - Nu_o = -N - G_{rT}\theta_o - G_{rC}\phi_o \quad \text{...............(25)}$$

$$u_1'' + u_1' - (N+n)(1+n\lambda_1)u_1 = -(1+n\lambda_1)(N+n) - Au_o' - G_{rT}\theta_1 - G_{rC}\phi_1 \quad \text{...............(26)}$$

$$(3+4R)\theta_o'' + 3P_r\theta_o' + 3\eta P_r\theta_o = 0 \quad \text{...............(27)}$$

$$(3+4R)\theta_1'' + 3P_r\theta_1' - 3(n-\eta)\theta_1 = -3AP_r\theta_o' \quad \text{...............(28)}$$

$$\phi_o'' + S_c\phi_o' - \delta S_c\phi_o = -S_c S_o \theta_o'' \quad \text{...............(29)}$$

$$\phi_1'' + S_c\phi_1' - (n+\delta)S_c\phi_1 = -AS_c\phi_o' - S_o S_c \theta_1'' \quad \text{...............(30)}$$

where, the primes denote differentiation with respect to y.

The corresponding boundary conditions are

$$u_o = U_p, u_1 = 0, \theta_o = 1, \theta_1 = 1, \phi_o = 1, \phi_1 = 1 \text{ at } y = 0 \quad \text{.......... (31)}$$

$$u_o = 1, u_1 = 1, \theta_o \to 0, \theta_1 \to 0, \phi_o \to 0, \phi_1 \to 0 \quad \text{as} \quad y \to \infty \quad \text{............... (32)}$$

The analytical solutions of equation (25) to (30) with satisfying boundary conditions (31) and (32) are given by

$$u_o = (U_p - 1 + L_2 + L_3)e^{-L_1 y} + 1 - L_2 e^{-R_3 y} - L_3 e^{-R_1 y} \quad \text{............ (33)}$$

$$u_1 = 1 + (-1 - C_1 + D + E + F + G)e^{-m_1 y} + C_1 e^{-L_1 y} - De^{-R_1 y}$$
$$-Ee^{-R_2 y} - Fe^{-R_3 y} - Ge^{-H_1 y} \quad \text{............ (34)}$$

$$\theta_o = e^{-R_3 y} \quad \text{............ (35)}$$

$$\theta_1 = e^{-H_1 y} + Z_1(e^{-R_3 y} - e^{-H_1 y}) \quad \text{............ (36)}$$

$$\phi_o = e^{-R_1 y} + Z_2(e^{-R_1 y} - e^{-R_3 y}) \quad \text{............ (37)}$$

$$\phi_1 = (1 - Z_3 + Z_4 + Z_5)e^{-R_2 y} + Z_3 e^{-R_1 y} - Z_4 e^{-R_3 y} - Z_5 e^{-H_1 y} \quad \text{............ (38)}$$

where

$$L_1 = \frac{1}{2}(1 + \sqrt{1 + 4N}) \qquad m_1 = \frac{(1 + \sqrt{1 + 4(1 + n\lambda_1)(n + N)})}{2}$$

$$H_1 = \frac{3P_r}{2(3 + 4R)}\left(1 + \sqrt{1 + \frac{4(n - \eta)(3 + 4R)}{3P_r}}\right) \qquad R_1 = \frac{1}{2}(S_c + \sqrt{S_c^2 + 4\delta S_c})$$

$$R_2 = \frac{1}{2}(S_c + \sqrt{S_c^2 + 4(n + \delta)S_c}) \qquad R_3 = \frac{3P_r}{2(3 + 4R)}\left(1 + \sqrt{1 - \frac{4\eta(3 + 4R)}{3P_r}}\right)$$

$$L_2 = \frac{G_{rT} - G_{rC}Z_2}{R_3^2 - R_3 - N} \qquad L_3 = \frac{G_{rC}(1 + Z_2)}{R_1^2 - R_1 - N} \qquad C_1 = \frac{L_1 A(U_p + L_2 + L_3 - 1)}{L_1^2 - L_1 - (n + N)(1 + n\lambda_1)}$$

$$Z_1 = \frac{3AP_r R_3}{(3 + 4R)R_3^2 - 3P_r R_3 - 3P_r(n - \eta)} \qquad Z_2 = \frac{S_o S_c R_3^2}{R_3^2 - R_3 - \delta S_c}$$

$$Z_3 = \frac{AS_c R_1(1 + Z_2)}{R_1^2 - S_c R_1 - S_c(n + \delta)} \qquad Z_4 = \frac{S_c R_3(AZ_2 + S_o Z_1 R_3)}{R_3^2 - S_c R_3 - S_c(n + \delta)}$$

$$Z_5 = \frac{S_o S_c H_1^2(1 - Z_1)}{H_1^2 - S_c H_1 - S_c(n + \delta)} \qquad D = \frac{AR_1 L_3 + G_{rC} Z_3}{R_1^2 - R_1 - (n + N)(1 + n\lambda_1)}$$

$$E = \frac{G_{rC}(1 - Z_3 + Z_4 + Z_5)}{R_2^2 - R_2 - (n + N)(1 + n\lambda_1)} \qquad F = \frac{AR_3 L_2 + G_{rT} Z_1 - G_{rC} Z_4}{R_3^2 - R_3 - (n + N)(1 + n\lambda_1)}$$

$$G = \frac{G_{rT}(1 - Z_1) + G_{rC} Z_5}{H_1^2 - H_1 - (n + N)(1 + n\lambda_1)}$$

In view of the above solution, the velocity, temperature and concentration distribution in the boundary layer become

$$u(y,t) = (U_p - 1 + L_2 + L_3)e^{-L_1 y} + 1 - L_2 e^{-R_3 y} - L_3 e^{-R_1 y} + \varepsilon e^{nt}\{1 + (-1 - C_1 + D$$
$$+ E + F + G)e^{-m_1 y} + C_1 e^{-L_1 y} - De^{-R_1 y} - Ee^{-R_2 y} - Fe^{-R_3 y} - Ge^{-H_1 y}\} \quad \text{....(39)}$$

$$\theta(y,t) = e^{-R_3 y} + \varepsilon e^{nt}\{e^{-H_1 y} + Z_1(e^{-R_3 y} - e^{-H_1 y})\} \quad \text{............ (40)}$$

$$\phi(y,t) = e^{-R_1 y} + Z_2(e^{-R_1 y} - e^{-R_3 y}) + \varepsilon e^{nt}\{(1 - Z_3 + Z_4 + Z_5)e^{-R_2 y}$$
$$+ Z_3 e^{-R_1 y} - Z_4 e^{-R_3 y} - Z_5 e^{-H_1 y}\} \quad \text{............ (41)}$$

It is now important to calculate the physical quantities of primary interest, which are the local wall shear stress, the local surface heat and mass flux. Given the velocity field in the boundary layer, we can now calculate the local wall shear stress (i.e., skin-friction) is given by

$$\tau_w^* = \mu\left(\frac{\partial u^*}{\partial y^*}\right)_{y^*=0} \qquad \text{…………(42)}$$

and in dimensionless form, we obtain

$$C_f = \frac{\tau_w^*}{\rho U_o V_o} = \left(\frac{\partial u}{\partial y}\right)_{y=0} = -L_1(U_p - 1 + L_2 + L_3) + L_2R_3 + L_3R_1 + \varepsilon e^{nt}\{-m_1(-1 - C_1 + D + E + F + G) - L_1C_1 + R_1D + ER_2 + R_3F + GH_1\} \qquad \text{....(43)}$$

RESULTS AND DISCUSSION

Fluid velocity distribution of fluid flow is tabulated in Table -1 and plotted in Fig. -1 having five graphs at P_r = 0.71, S_c = 0.47, G_{rT} = 5, G_{rC} = 3, n = 0.2, t = 1, R = 1, A = 1, U_p = 1, S_o = 4, δ = 3, η = 0.01, ε = 0.002 for following different value of M, K and λ_1.

	M	K	λ_1
For Graph-1	2	5	2
For Graph-2	4	5	2
For Graph-3	2	10	2
For Graph-4	2	5	5

It is observed from Fig.-1 that all velocity graphs are increasing sharply up to y = 1.2 after that velocity in each graph begins to decrease and tends to zero with the increasing in y. It is also observed from Fig. -1 that velocity increases with the increase in K, but it decreases with the increase in M and λ_1.

The temperature and concentration do not change with the change in above parameters taken for velocity.

The skin friction distribution is tabulated in Table -2 and plotted in Fig. -2 having five graphs. It is observed from Fig. -2 that skin friction increases with the increase in K, but it decreases with the increase in M and λ_1.

PARTICULAR CASE

When λ_1 is equal to zero, this problem reduces to the problem of Mohamed [18].

CONCLUSION

Velocity and skin friction increase slightly with the increase in λ_2 (Visco- elastic parameter).

Table-1: Value of velocity u for Fig-1 at P_r = 0.71, S_c = 0.47, G_{rT} = 5, G_{rC} = 3, n = 0.2, t = 1, R = 1, A = 1, U_p = 1, S_o = 4, δ = 3, η = 0.01, ε = 0.002 for following different value of M, K and λ_1.

y	Graph 1	Graph 2	Graph 3	Graph 4
0	1.00000	1.00000	1.00000	1.00000
1	2.56237	1.93704	2.62082	2.56114
2	2.26808	1.70635	2.32287	2.26802
3	1.93866	1.51430	1.98083	1.93858
4	1.69599	1.38070	1.72749	1.69579
5	1.51912	1.28422	1.54261	1.51899

Table-2: Value of skin friction τ for Fig-2 at $P_r = 0.71$, $S_c = 0.47$, $G_{rT} = 5$, $G_{rC} = 3$, $n = 0.2$, $t = 1$, $R = 1$, $A = 1$, $U_p = 1$, $S_o = 4$, $\delta = 3$, $\eta = 0.01$, $\varepsilon = 0.002$ for following different value of M, K and λ_1.

y	Graph 1	Graph 2	Graph 3	Graph 4
0	6.09041	4.73140	6.24627	6.07238
0.2	6.09466	4.73377	6.25199	6.07589
0.4	6.09908	4.73624	6.25795	6.07955
0.6	6.10369	4.73882	6.26416	6.08336
0.8	6.10848	4.74150	6.27061	6.08732
1	6.11347	4.74428	6.27733	6.09145

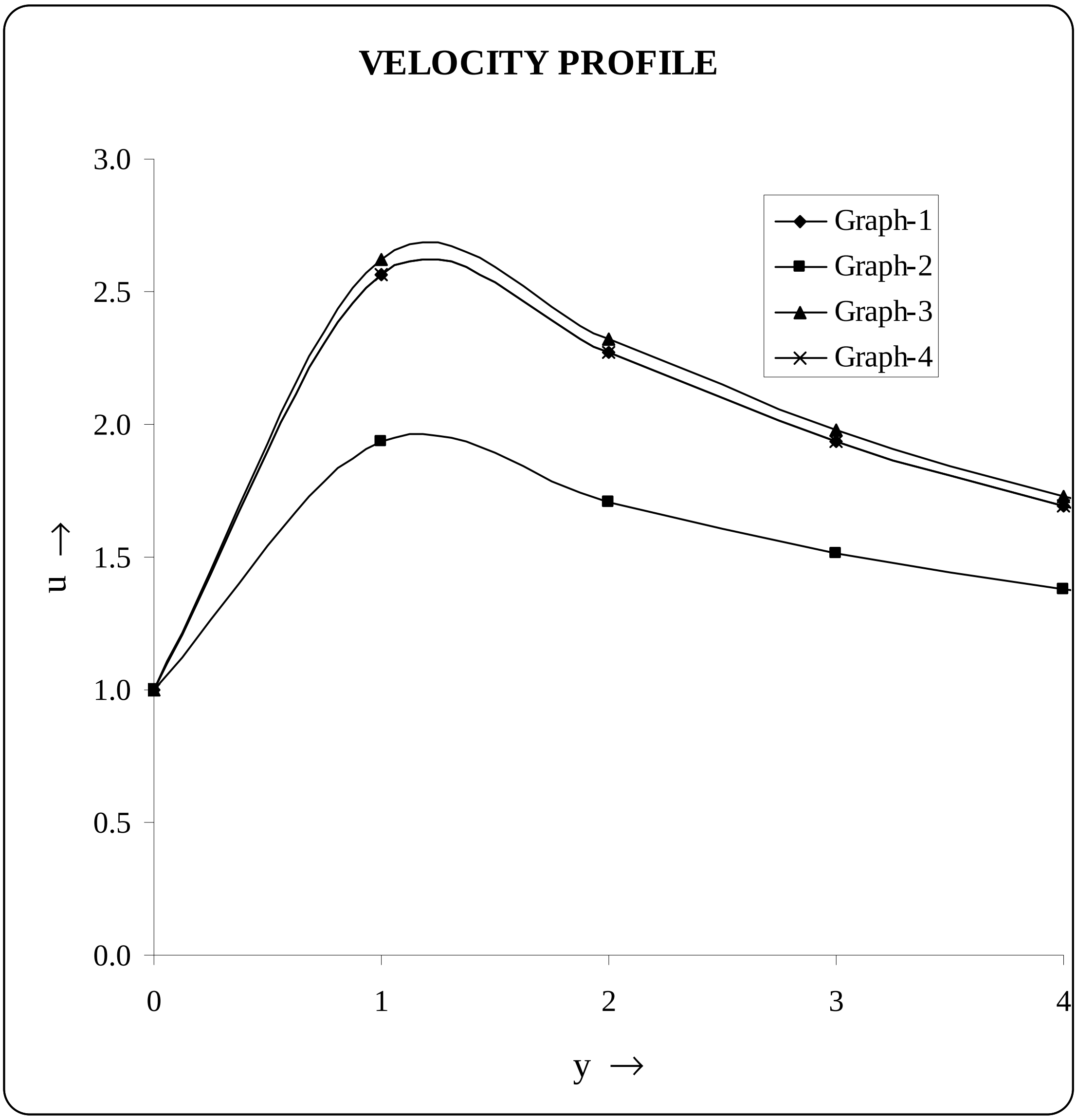

Fig.-1

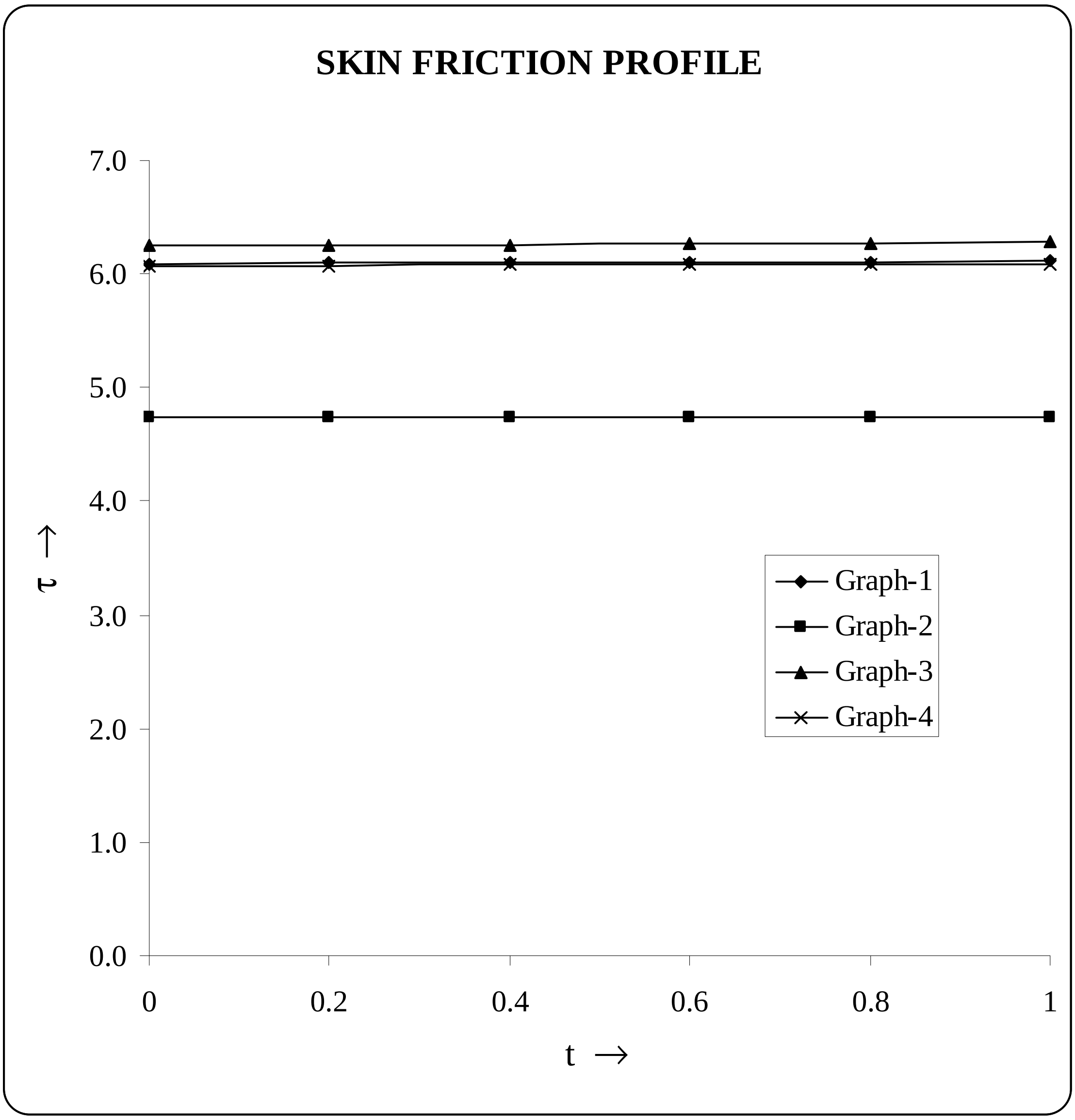

Fig.-2

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