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Effect of body acceleration and slip velocity on the pulsatile flow of casson fluid through stenosed artery

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ABSTRACT

A Mathematical model for the pulsatile blood flow through stenosed artery with the effect of body acceleration and slip velocity is proposed. Blood has been represented by casson fluid equation. Analytic expression for velocity, flow rate, wall shear stress and effective viscosity is derived. Flow variables with the change of parameters are represented graphically. The effect of pulsatility, stenosis, body acceleration, slip velocity, yield stress has been investigated. It is found that yield stress of the fluid and body acceleration highly influenced the velocity of the fluid, shear stress, flow rate in a stenosed artery. High blood viscosity is dangerous in the cardiovascular disorders; the present model may be used as a tool for reducing the blood viscosity by using slip velocity at the constricted wall.

Keywords: Pulsatile, Body acceleration, Stenosis, Casson fluid, Slip velocity.

INTRODUCTION

Blood flow through normal as well as stenosed artery is a very important field of study because of fact that the cause and development of many cardiovascular diseases are depends on the nature of blood flow and mechanical properties of blood vessel walls. The presence of stenosis in one or more locations restricts the blood flow through the lumen of the coronary arteries into the heart leading to cardiac ischemia. The experimental studies and the theoretical treatment of blood flow phenomena are very useful for the diagnosis of a number of cardiovascular diseases and development of pathological patterns in the human and animal physiology and for other clinical purpose and practical application by Sud and Sekhon [15].

To understand the effect of stenosis in the lumen of an artery many researchers investigated the flow of blood through stenosed arteries treating blood as a Newtonian fluid. However, experimental studies show that in the vicinity of the stenosis, the shear rate of the blood is less and therefore the non-Newtonian behavior of blood is quite prominent. Sapna [11] has studied the effect of non-Newtonian behavior of blood flow by considering blood as Power-law fluid model. The non-Newtonian flow behavior of blood for steady flow in stenosed arteries was studied by many researchers [7, 17, 18, 22] by treating blood as Herschel Bulkley fluid. Many researchers have used the Casson fluid model for mathematical modeling of blood flow in narrow arteries at low shear rates. Blair [3] demonstrates that the casson fluid model is adequate for the representation of the simple shear behavior of blood in arrow arteries. Casson [5] studied the validity of casson fluid model in his studies pertaining to the flow characteristics of blood and reported that at low shear rates the yield stress for blood is non-zero. Chaturani and Samy[6] have analyzed the pulsatile blood flow through stenosed arteries.

Externally imposed body acceleration also has major influence on the flow through stenosed artery. In many situations in our life while fast body movements in sports activities, driving vehicles, human body experiences the body accelerations. Due to this body acceleration different health problem such as headache, loss of vision, increase in pulse rate, abnormal pain etc. occurs. Sud and sekhon [15] studied the pulsatile flow of blood through a rigid circular tube subject to the periodic body acceleration, treating blood as a Newtonian fluid. On the basis of experimental results, it is observed that the body acceleration might change the heart beat and might have a negative impact on the circulatory system. So the study of the effect of the magnitude, frequency and duration of the periodic acceleration may play a significant role in the diagnosis and treatment of the health problems**.**

In many situations there may be a partial Slip between the fluid and the boundary for many fluids, the motion of fluid is still governed by the Navier-Stokes equations, but the usual no slip condition at the boundary should be replaced by the slip condition. Several authors in [4, 8, 19] carried out the role of slip velocity in blood flow through stenosed arteries and suggested the presence of red blood cell occurring in slip condition at vessel wall. To understand the existence of slip at the tube wall Nubar [9], Brunn [4] have reviewed the several treatments of slip at the walls of the capillary tubes. In view of theoretical and experimental observations implying the existence of slip at the wall, it is improper to ignore the slip in blood flow. It is also noted that in literature, there is no direct formula to calculate the slip velocity. It is therefore worthwhile to find a formula to calculate the slip velocity at the wall. Pulsatile flow of blood through a catheterized artery in presence of different geometry of stenosis with a velocity slip at a stenotic wall has been investigated by several researchers [1, 2, 17, 20]. It is found that the wall shear stress and effective viscosity decreases while axial velocity increases with velocity slip at wall. Recently several authors [8, 12] have developed Mathematical models for blood flow through stenosed arterial segment by considering velocity slip condition at the constricted wall.

The aim of present paper is to study the effect of blood flow with slip velocity and body acceleration in a stenosed artery. The analytical solution is obtained by using appropriate method. The graphical representations have been presented for the different flow variables with the appropriate discussion. Finally the comparison is made with the other existing results to justify the applicability of the present model.

Formulation of the problem

Let us consider one dimensional pulsatile, axially symmetric, laminar, fully developed flow of blood by considering blood as a casson fluid in the presence of externally imposed periodic body acceleration. It is assumed that the stenosis develops in the arterial wall in an axially non-symmetric but radially symmetric manner and depends upon the axial distance 'z' and the height of it's growth.

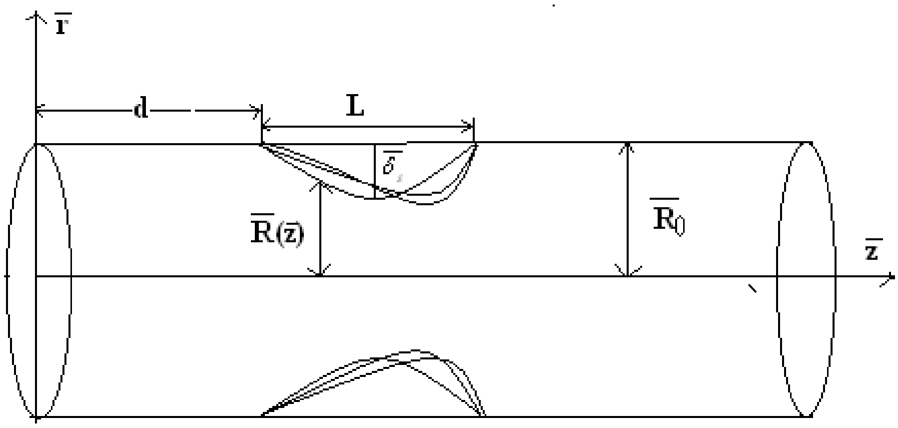

Figure 1. Geometry of an axially nonsymmetrical stenosis

The geometry of the flow is shown in Figure (1) and is given by

$$\begin{aligned}\frac{\bar{R}(\bar{Z})}{\bar{R}_0} &= 1-\bar{A}\left[\bar{L}_0^{(n-1)}\left(\bar{z}-\bar{d}\right)-\left(\bar{z}-\bar{d}\right)^n\right], \quad \bar{d}\le\bar{z}\le\bar{d}+\bar{L}_0 \\ &= 1, \qquad\qquad\qquad\qquad\qquad\qquad otherwise\end{aligned} \qquad (1)$$

Where $\bar{R}(\bar{z})$ and R_0 is the radius of the artery with and without stenosis respectively. L_0 is the length of the stenosis and d indicates it's location, $n \ge 2$ is the stenosis shape parameter and the parameter 'A' is given by

$$A=\frac{\delta}{R_0 L_0^n}\frac{n^{n/(n-1)}}{(n-1)},$$

Where δ denotes the maximum height of the stenosis at $z=(d+L_0)/n^{1/(n-1)}$ such that $\delta / R_0 < 1$The periodic body acceleration $\bar{F}(\bar{t})$ in the axial direction is given by

$$\bar{F}\left(\bar{t}\right)=a_0 \cos\cos\left(\overline{\omega_b}\,\bar{t}+\varphi\right), \qquad (2)$$

where a_0 is the amplitude of body acceleration, $\overline{\omega_b}=2\pi \overline{f_b}$; $\overline{f_b}$ is it's frequency in Hz. The frequency of the body acceleration $\overline{f_b}$ is assumed to be small so that wave effect can be neglected.

Since the pressure gradient is the function of $\bar{z}$ and $\bar{t}$, we take

$$\frac{-\partial\bar{p}}{\partial\bar{z}}\left(\bar{z},\bar{t}\right)=A_0+A_1\cos\cos\left(\overline{\omega_p}\,\bar{t}\right), t\ge 0, \qquad (3)$$

where A_0 is the steady state pressure gradient, A_1 is the amplitude of the fluctuating component, $\overline{\omega_p}=2\pi\overline{f_p}$, where $\overline{f_p}$ is the pulse rate frequency.

The Navier-Stokes equations governing the fluid flow is given by Schlichting and Gersten [12].

$$\bar{\rho}\left(\partial\bar{u}/\partial\bar{t}\right)=-\left(\partial\bar{p}/\partial\bar{z}\right)-(1/r)\frac{\partial}{\partial\bar{r}}\left(\bar{r}\bar{\tau}\right)+\bar{F}\left(\bar{t}\right), \qquad (4)$$

$$\partial\bar{p}/\partial\bar{r}=0, \qquad (5)$$

where $\bar{u}$ represent the axial velocity along z-direction, $\bar{p}$ is the pressure, $\bar{\rho}$ is the density, $\bar{t}$ the time, $\bar{\tau}$ the shear stress and $\bar{F}\left(\bar{t}\right)$ the body acceleration. Mathematically $\bar{F}\left(\bar{t}\right)$ is described in equation (2).

The casson fluid equation is given by

$$\left.\begin{aligned}\sqrt{\bar{\tau}} &= \sqrt{\bar{\tau}_y} + \sqrt{\bar{\mu}\left(-\frac{\partial \bar{u}}{\partial \bar{r}}\right)}; \quad if\ \bar{\tau} \geq \bar{\tau}_y \\ \frac{\partial \bar{u}}{\partial \bar{r}} &= 0, \qquad\qquad if\ \bar{\tau} < \bar{\tau}_y\end{aligned}\right\} \tag{6}$$

where $\bar{\tau}_y$ denotes yield stress and $\bar{\mu}$ denotes the viscosity of the blood.

Boundary conditions
The boundary conditions are

$$\bar{u} = \bar{u}_s \ at \ \bar{r} = \bar{R}(\bar{z}), \tag{7}$$

$$\tau \text{ is finite at } \bar{r} = 0 \tag{8}$$

where $\bar{u}_s$ is the slip velocity at the stenotic wall.

By introducing the following non-dimensional variables

$$\left.\begin{aligned}&u = \frac{\bar{u}}{A_0 \bar{R}_0^2 / 4\bar{\mu}},\ z = \bar{z}/\bar{R}_0,\ R(z) = \bar{R}(\bar{z})/\bar{R}_0,\ r = \bar{r}/\bar{R}_0,\ t = \bar{t}\bar{\omega}_p, \\ &\omega = \bar{\omega}_b/\bar{\omega}_p,\ \delta_s = \bar{\delta}_s/\bar{R}_0,\ u_s = \frac{\bar{u}_s}{A_0 \bar{R}_0^2/4\bar{\mu}},\ \tau = \frac{\bar{\tau}}{A_0 R_0/2},\ \alpha^2 = \frac{\bar{R}_0^2 \bar{\omega}_p \bar{\rho}}{\bar{\mu}}, \\ &e = A_1/A_0,\ B = a_0/A_0,\ \theta = \frac{\bar{\tau}_y}{A_0 R_0/2}\end{aligned}\right\} \tag{9}$$

The non-dimensional equation (4) becomes

$$\alpha^2\left(\partial u/\partial t\right) = 4\left(1 + eCos\ t\right) + 4Bcos\left(\omega t + \phi\right) - \left(2/r\right)\frac{\partial}{\partial r}\left(r\tau\right), \tag{10}$$

where $\alpha^2 = \omega_p R_o^2 / (\mu/\rho)$, is called Womersley frequency parameter.

Equation (6) can be written as

$$\left.\begin{aligned}\sqrt{\tau} &= \sqrt{\theta} + \frac{1}{\sqrt{2}}\sqrt{-\frac{\partial u}{\partial r}}; \quad if\ \tau \geq \theta \\ \frac{\partial u}{\partial r} &= 0, \qquad\qquad if\ \tau < \theta\end{aligned}\right\} \tag{11}$$

The boundary conditions reduces to

$$u = u_s \; at \; r = R(z) \tag{12}$$

and τ is finite at $r = 0$ (13)

The geometry of stenosis in the non-dimensional form is given by

$$\left.\begin{aligned} R(Z) &= 1 - A\left[L_0^{(n-1)}(z-d) - (z-d)^n\right], \; d \le z \le d + L_0 \\ &= 1, \qquad\qquad otherwise \end{aligned}\right\} \tag{14}$$

The non-dimensional volumetric flow rate is defined by

$$Q(z,t) = 4\int_0^{R(z)} r\, u(z,r,t)\, dr, \tag{15}$$

where $Q(z,t) = \dfrac{\bar{Q}(\bar{z},\bar{t})}{\pi A_0 (\bar{R}_0)^4 / 8\bar{\mu}}$; $\bar{Q}(\bar{z},\bar{t})$ is the volumetric flow rate.

Effective viscosity $\bar{\mu_e}$ defined as

$$\bar{\mu_e} = \pi\left(-\frac{\partial \bar{p}}{\partial \bar{z}}\right)(\bar{R}(\bar{z}))^4 \Big/ \bar{Q}(\bar{z},\bar{t}), \tag{16}$$

can be expressed in the dimension less form as

$$\mu_e = R^4 (1 + e\cos t)/Q(z,t). \tag{17}$$

Analysis

Let the velocity *u* and shear stress τ can be expressed in the following form

$$u(z,r,t) = u_0(z,r,t) + \alpha^2 u_1(z,r,t) + ... \tag{18}$$

$$\tau(z,r,t) = \tau_0(z,r,t) + \alpha^2 \tau_1(z,r,t) + ... \tag{19}$$

Substituting the value of u and τ from equation (18) and (19) in equation (10) and equating the constant term and α^2 term, we get

$$\frac{\partial}{\partial r}(r\tau_0) = -2r[(1 + e\cos t) + B\cos(\omega t + \phi)], \tag{20}$$

$$\partial u_0/\partial t = \frac{2}{r}\frac{\partial}{\partial r}(r\tau_1), \tag{21}$$

Integrating equating (20) and using boundary condition (13), we get

$$\tau_0 = -f(\mathrm{t})r \tag{22}$$

where, $f(\mathrm{t}) = \left[(1 + e\cos t) + B\cos(\omega t + \phi)\right]$ (23)

Substituting u from equation (18) into condition (12), we get

$$u_0 = u_s, u_1 = 0 \; at \, r = R(\mathrm{z}) \tag{24}$$

Substituting equation (18) and (19) in equation (11), we get

$$-\frac{\partial u_0}{\partial r} = 2\left[\theta + |\tau| - 2\sqrt{\theta/\tau_0}\right] \tag{25}$$

$$-\frac{\partial u_1}{\partial r} = 2|\tau_1|\left[1 - \sqrt{\theta/\tau_0}\right] \tag{26}$$

Integrating equation (25), and using relation (22) and relation (24), we obtain

$$u_0 = u_s + f(\mathrm{t})R^2\left[1 - \left(\frac{r}{R}\right)^2 - \frac{8}{3}\frac{k}{\sqrt{R}}\left\{1 - \left(\frac{r}{R}\right)^{3/2}\right\} + \frac{2k^2}{R}\left\{1 - \left(\frac{r}{R}\right)\right\}\right] \tag{27}$$

Where $k^2 = \dfrac{\theta}{f(\mathrm{t})}$

Similarly the solution for u_1 and τ_1 can be obtained by using equations (21), (26) and (27).

$$\tau_1 = \frac{f'(t)R^3}{8}\left[22\left(\frac{r}{R}\right) - \left(\frac{r}{R}\right)^3 - \frac{8}{21}\frac{k}{\sqrt{R}}\left\{7\left(\frac{r}{R}\right) - 4\left(\frac{r}{R}\right)^{5/2}\right\}\right] \tag{28}$$

$$u_1 = \frac{f'(t)R^4}{16}\left[\begin{array}{l}\left(\frac{r}{R}\right)^4 + 4\left(\frac{r}{R}\right)^2 + 3 + \frac{k}{\sqrt{R}}\left\{\frac{16}{3}\left(\frac{r}{R}\right)^2 - \frac{424}{147}\left(\frac{r}{R}\right)^{7/2} + \frac{16}{3}\left(\frac{r}{R}\right)^{3/2} - \frac{1144}{147}\right\} + \\ \frac{k^2}{R}\left\{\frac{128}{63}\left(\frac{r}{R}\right)^3 - \frac{64}{9}\left(\frac{r}{R}\right)^{3/2} + \frac{320}{63}\right\}\end{array}\right] \tag{29}$$

On substituting the value of u_0 and u_1 in equation (18) we get the velocity as

$$u = f(t)R^2\left[1-\left(\frac{r}{R}\right)^2 - \frac{8}{3}\frac{k}{\sqrt{R}}\left\{1-\left(\frac{r}{R}\right)^{3/2}\right\} + \frac{2k^2}{R}\left\{1-\left(\frac{r}{R}\right)\right\}\right.$$
$$+\frac{\alpha^2R^2C}{16}\left\{\left(\frac{r}{R}\right)^4 + 4\left(\frac{r}{R}\right)^2 + 3 + \frac{k}{\sqrt{R}}\left\{\frac{16}{3}\left(\frac{r}{R}\right)^2 - \frac{424}{147}\left(\frac{r}{R}\right)^{7/2} + \frac{16}{3}\left(\frac{r}{R}\right)^{3/2} - \frac{1144}{147}\right\}\right.$$
$$\left.\left.+\frac{k^2}{R}\left\{\frac{128}{63}\left(\frac{r}{R}\right)^3 - \frac{64}{9}\left(\frac{r}{R}\right)^{3/2} + \frac{320}{63}\right\}\right\}\right] + u_s \tag{30}$$

The wall shear stress τ_ω can be written as

$$\tau_\omega = f(t)R\left\{1+\frac{\alpha^2R^2C}{8}\left(1-\frac{8k}{7\sqrt{R}}\right)\right\} \tag{31}$$

where $C = \frac{f'(t)}{f(t)}$

The volumetric flow rate Q is given by

$$Q = 4\int_0^{R(z)} ru(r,z,t)dr$$
$$= f(t)R^4\left[\frac{1}{4}+\frac{4}{7}\frac{k}{\sqrt{R}}+\frac{1}{3}\left(\frac{k}{\sqrt{R}}\right)^2 + \frac{\alpha^2R^2C}{16}\left\{\frac{2}{3}+\frac{120}{177}\frac{k}{\sqrt{R}}+\frac{32}{35}\left(\frac{k}{\sqrt{R}}\right)^2\right\}\right] + 2u_sR^2 \tag{32}$$

The effective viscosity in the non-dimensional form is given by

$$\mu_e = \frac{(R(z))^4}{Q(z,t)}(1+e\cos t)$$
$$= R^2(1+e\cos t)\left[f(t)R^4\left[\frac{1}{4}+\frac{4}{7}\frac{k}{\sqrt{R}}+\frac{1}{3}\left(\frac{k}{\sqrt{R}}\right)^2 + \frac{\alpha^2R^2C}{16}\left\{\frac{2}{3}+\frac{120}{177}\frac{k}{\sqrt{R}}+\frac{32}{35}\left(\frac{k}{\sqrt{R}}\right)^2\right\}\right] + 2u_sR^2\right]^{-1} \tag{33}$$

RESULTS AND DISCUSSION

The aim of resent model is to analyze the combined effects of body acceleration and slip velocity

on the flow variables viz., axial velocity, flow rate, shear stress and effective viscosity of blood, flowing in an artery with the axially non-symmetric stenosis at the arterial wall. On using perturbation method, the velocity u is expanded in terms of womersely frequency parameter α^2

The assumption of the small value of α is valid for the physiological situations in small blood vessels. In the present analysis blood is modeled as casson fluid model. In our analysis the value of shape parameter of stenosis is considered to be 2. The body acceleration parameter B is considered in the range 0-2. The pressure gradient parameter e is taken in the range0-7, magnitude of lead angle ϕ is taken as 0.2.

In figure 2 and figure 3 the variation of axial velocity *u* at the throat of the stenosis i.e. at *z=0* with radial distance *r*, for fixed values of stenosis height δ_s, pressure gradient *e*, time *t* and for different values of body acceleration parameter B, are presented. It is observed from the figure that axial velocity is maximum at $r = 0$ and decreases with the increase in the radius of artery *r*. it is also found that axial velocity attains it's minimum value at the stenotic wall at $r = R(z)$. Use of slip at the wall increases the velocity. It is found that body acceleration parameter B plays a very important role in flow; it brings not only quantitative changes but also qualitative changes in velocity profiles. In the presence of body acceleration more flow takes place because with the increase in body acceleration the plug flow region shrinks and hence velocity is more. It is observed that the magnitude of velocity is almost doubled when the body acceleration is 2 to the case when body acceleration is not present, for the same values of pressure gradient and yield stress.

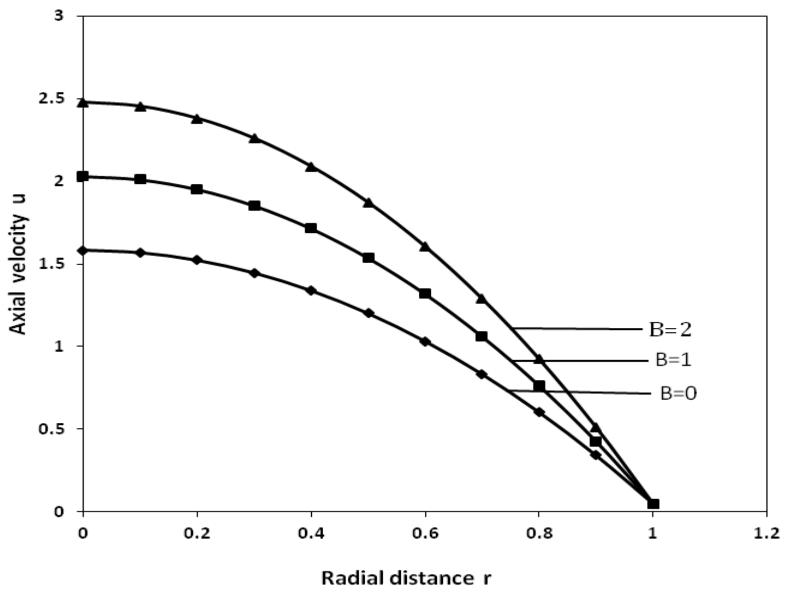

Figure 2. Varition of axial velocity u with radial distance r for
$\delta_s = 0.2, \alpha = 0.2, \phi = 0.1, e = 1, \theta = 0$

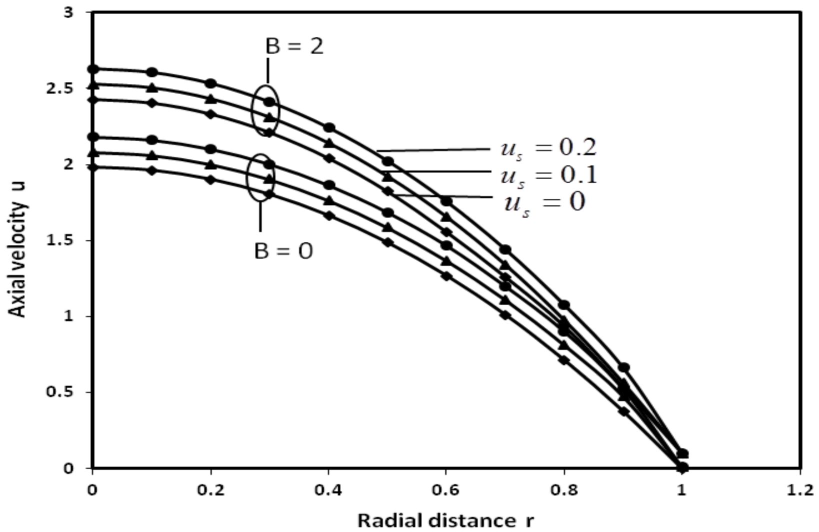

Figure 3. Vartiation of axial velocity with radial distance r for
$\delta_s = 0.2, t = 1, \alpha = 0.1, \phi = 0$

Figures (4, 5) also shows the variation of axial velocity u with radial distance r. it is depicted that when the yield stress is not present i. e. for Newtonian fluid velocity increases sharply on the axis of the tube. On the application of yield stress velocity profile is reduced and becomes blunt in the mid region of the tube which indicates the plug flow. Figure 5 show that axial velocity decreases with time in a stenosed artery for a fixed value of body acceleration and yield stress. Figure 6 represent the variation of flow rate with pressure gradient *e* for different flow parameters $(t, \theta, B, \delta_s, u_s)$. It is observed that flow rate increases gradually with the increases in pressure gradient e for any value of B and θ . However the magnitude of flow rate in the presence of yield stress is less than it's magnitude in the absence of yield stress (θ=0). Increase in θ results substantial decreases in flow rate. This occurs due to the increases in width of plug flow region It is further observed that employment of body acceleration as well as slip velocity enhances the flow rate.

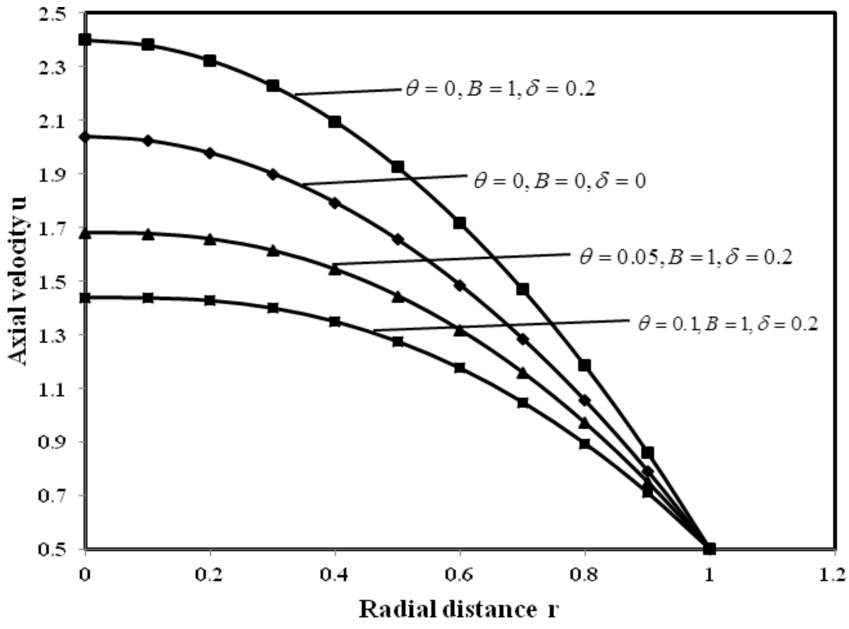

Figure 4. Axial velocity profile for $e = 1, w = 1, z = 0, \phi = 0.2, \alpha = 0.1, t = 1$

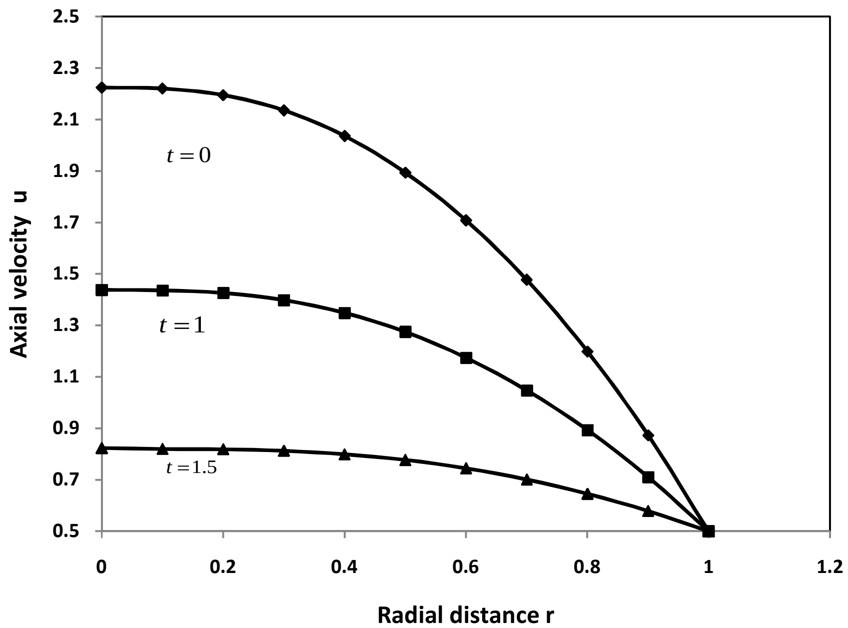

Figure 5. Axial velocity profiles for $e = 1, w = 1, z = 0, \phi = 0.2, \alpha = 0.2, u_s = 0.5$

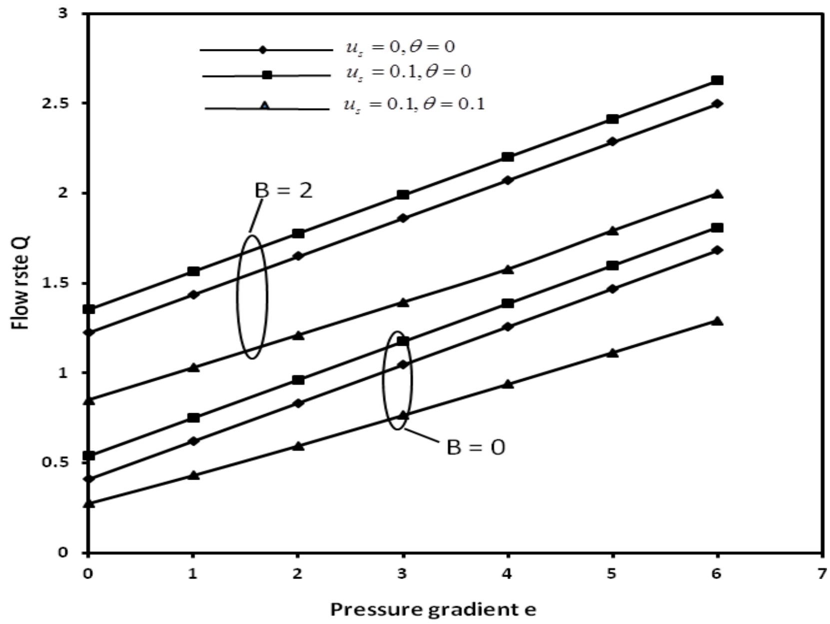

Figure 6. Variation of flow rate with pressure gradient for $\delta_s = 0.2, \alpha = 0.5, t = 1, z = 1$

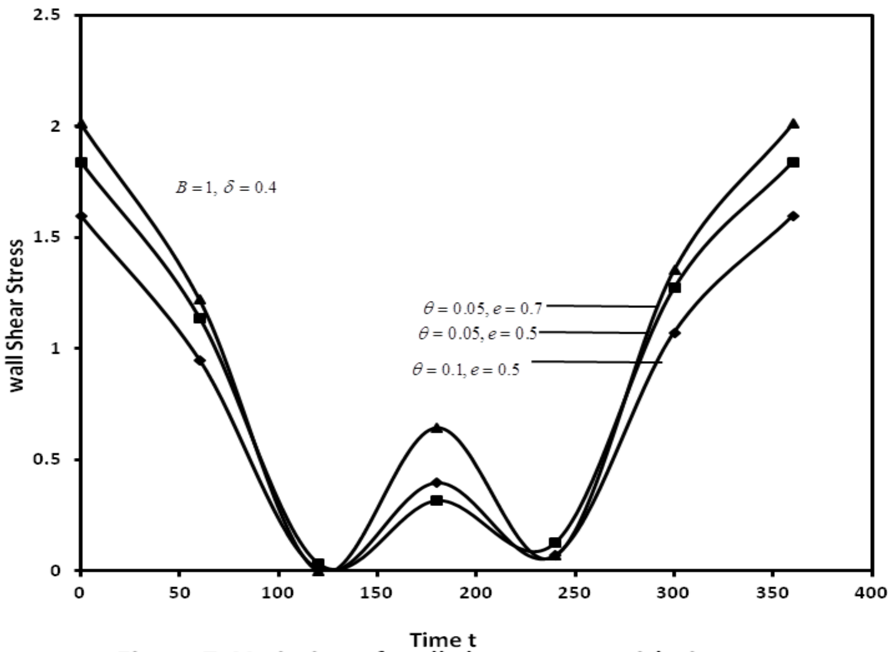

Figure 7. Variation of wall shear stress with time t for different values of θ and e

Wall shear stress is a very important factor in hemodynamics. Figure 7 and figure 8 show the variation of wall shear stress with time t for different values of B, e, δ_s, θ.It is observed that the behavior of wall shear stress is symmetrical about $t = 180^0$ .wall shear stress decreases with the increase in t till a certain limit, then increases with t. in the absence of body acceleration, wall shear stress is less compared to the case when body acceleration is present and it steadily decreases with time with the point of minimum at $t = 180^0$ which is showing that there could be a chance of more friction on the wall in a stenosed artery under the influence of body acceleration. It is noticed that the effect of yield stress and pressure gradient is small but enhances shear stress.

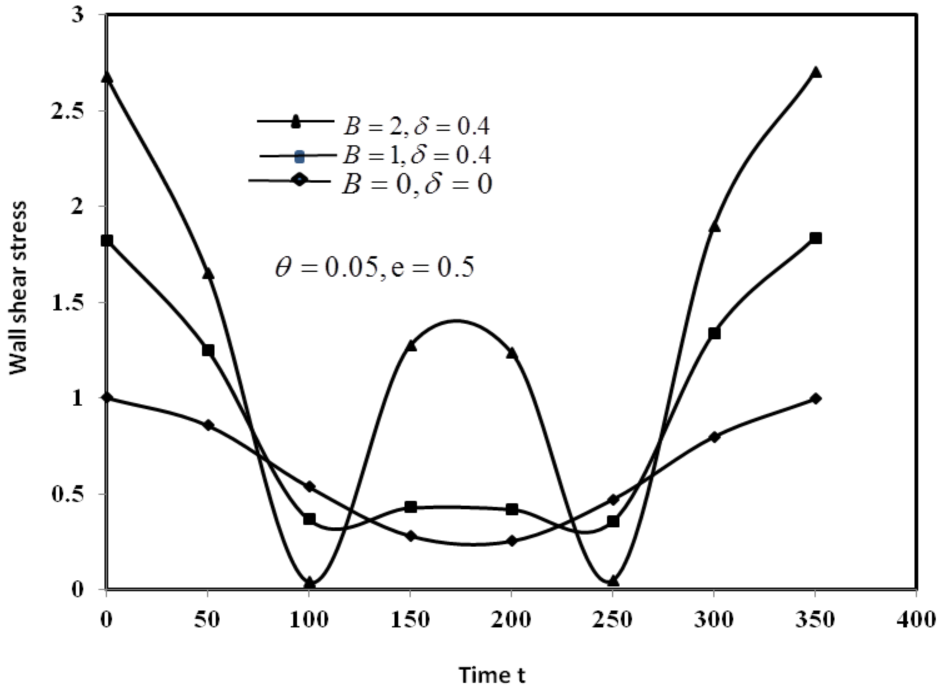

Figure 8.Wall shear stress distribution for different values of B

CONCLUSION

In the present mathematical model, pulsatile blood flow through stenosed artery with periodic body acceleration and axial slip velocity at the constricted wall has been considered. Analytic expressions for flow variables are obtained and their variations with different flow parameters are presented graphically. It is observed that effective viscosity and wall shear stress decreases with body acceleration but velocity and flow rate increases. It is also found that axial velocity and flow rate increases but effective viscosity decreases due to the wall slip. This model concludes that slip velocity play a very important role in blood flow modeling in a stenosed artery. It may also be concluded that with slip, damages to the vessel wall could be reduced. Reduction in wall shear stress and effective viscosity gould be exploited for good function of stenosed arterial system. Therefore we use the devices and medicines so that slip can be produced and use them for treatment of arterial disease. So this study may help the physicians in estimating the severity of stenosis and it's consequences. This study may further extend by the introduction of more rheological and physical parameters in the case of more sever stenosis.

REFERENCES

[1] Biswas D and Chakraborty US, *Application and Applied Mathematics*, **2009**, 4, 329.
[2] Biswas D and Chakraborty US, *Applied Mathematical Sciences*, **2010**, 4, 2865.
[3] Blair GW S, *Nature*, **1959**, 183,613.
[4] Brunn P, *Rheologica Acta*, **1975** 14,1039.
[5] Casson N, C. C. Mill, Ed., Pergamon press, London, UK, **1959**, 84-102.
[6] Chaturani P, Samy RP, *Biorheology*, **1985,** 22, 521.
[7] McDonald DA, *J. Biomech.*, **1979**, 12,13.
[8] Misra JC, Shit GC, *Journal of Mechanics in medicine and Biology*, **2007**,7, 337.

[9] Nubar Y, *Rheology*, **1967,** 4,133.
[10] Rathod VP, Asha SK, *Adv. in Appl. Sci. & Res.*, **2011**, 2, 240.
[11] Sapna, *International journal of research in pure and applied physics*, **2011**,1, 1.
[12] Schlichting H, Gersten K,*Springer-Verlag*, **2004**.
[13] Sharma MK, Bansal K, Bansal S, *Korea-Australia Rheology Journal*, **2012**, 24, 181.
[14] Sinha A, Shit GC, Kundu PK, *Biomedical engineering*, **2013**, 2013, 10 pages.
[15] Sud VK, Sekhon GS, *Bull. Math. Biol.*, **1985**, 47, 35.
[16] Tanwar VK, Agarwal R, Varshney NK, *Applied Mathematical Sciences*, **2012**, 6, 5959.
[17] Verma NK, Siddiqui SU, Gupta RS, *e-Journal of Science and Technology*, **2011**, 6, 61.
[18] Verma NK, Siddiqui SU, Gupta RS, Mishra S, *Applied Mathematics and Computation*, **2010**, 210, 49.
[19] Verma NK, Siddiqui SU, Gupta RS, Mishra S, *Applied Mathematics*, **2011**, 2, 764.
[20] Verma NK, Siddiqui SU, Gupta RS, Mishra S, *Journal of science and technology*, **2010**, 5, 49.
[21] Verma NK, Siddiqui SU, Gupta RS, Mishra S, *Adv. in Appl. Sci. & Res.*, **2011**, 2, 114.
[22] Young DF, *J. Biomech, Engg.*, **1979**, 101, 157.