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Dynamic response under moving concentrated loads of non uniform rayleigh beam resting on pasternak foundation

P. B. Ojih*, M. A. Ibiejugba and B. O. Adejo

Department of Mathematical Sciences, Kogi State University, Anyigba, Nigeria

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ABSTRACT

In this study, the dynamic response of non uniform Rayleigh beam resting on Pasternak foundation and subjected to concentrated loads travelling at varying velocity with simply supported boundary condition has been investigated. Analytical solution which represents the transverse displacement response of the beam under both concentrated forces and masses travelling at non uniform velocities was obtained. To obtain the solution of the fourth order partial differential equation with singular and variable coefficients, a technique based on the Generalized Galerkin's Method and the struble's asymptotic technique was employed. Numerical results in plotted curves are presented. The results show that as the Rotatory inertia R_0 *increases, the response amplitudes of the non uniform Rayleigh beam decreases for both moving force and moving mass problems. Furthermore, the results show that the response amplitudes of the non uniform Rayleigh beam decreases with an increase in the values of the shear modulus* G_0 *for fixed values of foundation modulus* K_0 *and Rotatory inertia* R_0*. Similarly, as* K_0 *increases, the response amplitudes decreases but the effect of* G_0 *is more noticeable than that of* K_0*. Finally, the critical speed for the moving mass problem is reached prior to that of the moving force for the non uniform Rayleigh beam problem in the illustrative example considered. Hence, the moving force solution is not a safe approximation to the moving mass problem, therefore, we cannot guarantee safety for a design based on the moving force solution since resonance is reached earlier in the moving mass problem than in the moving force problem.*

Keywords: moving mass moving force, Rayleigh beam, Pasternak foundation, resonance.

INTRODUCTION

The study of the behavior of elastic solid bodies (beams, plates or shell) subjected to moving loads has been the concern of several researchers in applied mathematics and engineering. More specifically, several dynamical problems involving the response of beams on a foundation and without foundation have variously been tackled by Fryba [4] and Sadiku and Leipholz [11]. Among the earliest work in this area of study was the work of Stokes [13] who obtained an approximate solution for the response of a beam by neglecting the mass of the beam. This is because the introduction inertia effect of the moving mass would make the governing equation cumbersome to solve as reported in Stanistic et al [12] , recognizing this difficulty, pestel [10] applied Rayleigh–Ritz techniques to reduce the moving mass problem defined by a continuous differential equation to an approximate system of discrete differential equations with analytic coefficients. The system was reduced by a finite difference scheme for solution, but no numerical results were presented. After this, several researchers have approached this problem by assuming that the inertia of the moving load was negligible. In fact, Arye et al [2] pointed out, in their summary of work done prior to 1952 that the fundamental mathematical difficulties encountered in the problem lie in the fact that one of the coefficients of the linear operator describing the motion is a function of both space and time. They added that it is caused by the presence of a Dirac-Delta function as a coefficient necessary for a proper description of the motion. It is remarked at this juncture that, physically, this term represents the interplay of the inertial forces due to the discrete masses distributed over the structure during the motion Fryba L [4]. Arye et al [2] also considered the problem of elastic beam under the action of moving loads. They assumed the mass of the beam to be smaller than the mass of the moving load and obtained an approximate solution to the problem. This is followed by the other extreme case when the mass of the load was smaller than the mass of the beam. In particular, the dynamic response of a simply supported beam transverse by a constant force moving at a uniform speed was first studied by Krylov [5]. He used the method of expansion of Eigen function to obtain his results. Lowan [6] also considered the problem of transverse oscillations of beams under the action of moving loads for the general case of any arbitrarily prescribed law of motion. He obtained his solution using Green's functions.

More recently, the problem of the dynamic response of a non uniform beam resting on elastic foundation and under concentrated masses was tackled by Oni [9]. Analysis of his results show that the response amplitude of both moving force and moving mass decrease with increasing foundation moduli.

Oni [8] considered the response of a non uniform thin beam resting on a constant elastic foundation to several moving masses. For the solution of the problem, he used the versatile technique of Galerkin to reduce the complex governing fourth order partial differential equation with variable and singular coefficients to a set of ordinary differential equations. The set of ordinary differential equations was later simplified and solved using modified asymptotic of struble. Other studies on non-uniform beam include Doughlas etal [3] Awodola and Oni [1] and Oni and Omolafe [7].

THE GOVERNING EQUATION

The transverse displacement of the beam when it is under the action of a moving load is governed by the fourth order partial differential equation given by:

$$\frac{\partial^2}{\partial x^2}\left(EI(x)\frac{\partial^2 V(x,t)}{\partial x^2}\right)+\mu(x)\frac{\partial^2 V(x,t)}{\partial t^2}-\mu(x)R_O\frac{\partial^4 V(x,t)}{\partial x^2\partial t^2}+P_G(x,t)=P(x,t) \tag{2.1}$$

where x is the spatial coordinate, t is the time, $V(x,t)$ is the transverse displacement, is the young modulus, I is the moment of inertia, EI is the flexural rigidity of the structure, while I(x) and μ (x) are variable moment of inertia and mass per unit length of the beam respectively.
By substituting the moving load $P(x,t)$ of the form

$$P(x,t)=P_f(x,t)\left[1-\frac{1}{g}\frac{d^2V(x,t)}{dt^2}\right] \tag{2.2}$$

and convective acceleration operator $\frac{d^2}{dt^2}$ defined as

$$\frac{d^2}{dt^2}=\frac{\partial^2}{\partial t^2}+2c\frac{\partial^2}{\partial x\partial t^2}+c^2\frac{\partial^2}{\partial x^2} \tag{2.3}$$

into (2.1) we have:

$$\frac{\partial^2}{\partial x^2}\left(EI(x)\frac{\partial^2(x,t)}{\partial x^2}\right)+\mu(x)\frac{\partial^2(x,t)}{\partial t^2}-R_0\frac{\partial^2 V(x,t)}{\partial x^2}-G\frac{\partial^2 V(x,t)}{\partial x^2}+kV(x,t)+$$
$$M\delta(x-ct)\left[\frac{\partial^2 V(x,t)}{\partial t^2}+2c\frac{\partial^2 V(x,t)}{\partial x\partial t}+c^2\frac{\partial^2 V(x,t)}{\partial x^2}\right]=mg\delta(x-ct) \tag{2.4}$$

We adopt the example in Oni, S. T. [8] and take I(x) and $\mu(x)$ to be of the form

$$I(x)=I_0\left(1+\sin\frac{\pi x}{L}\right) \tag{2.5}$$

and

$$\mu(x)=\mu_0\left(1+\sin\frac{\pi x}{L}\right) \tag{2.6}$$

where μ_0 is the constant mass per unit length of the beam.
Using equations (2.5) and (2.6) in equation (2.4) and after some simplification and rearrangement one obtains

$$N\left\{\frac{\partial^4 V(x,t)}{\partial x^4}\left(\frac{5}{2}+\frac{15}{4}\sin\frac{\pi x}{L}-\frac{1}{4}\sin\frac{3\pi x}{L}-\frac{3}{2}\cos\frac{2\pi x}{L}\right)+\frac{\partial^2 V(x,t)}{\partial x^2}\left(\frac{9\pi^2}{4L^2}\sin\frac{3\pi x}{L}-\frac{15}{4L^2}\sin\frac{\pi x}{L}+\frac{6\pi^2}{L^2}\cos\frac{2\pi x}{L}\right)\right\}$$
$$+\left(1+\sin\frac{\pi x}{L}\right)\frac{\partial^2 V(x,t)}{\partial t^2}-\frac{1}{\mu_0}(R_0+G)\frac{\partial^2 V(x,t)}{\partial x^2}+\frac{K}{\mu_0}V(x,t)+$$
$$\frac{M}{\mu_0}\delta(x-ct)\left[\frac{\partial^2 V(x,t)}{\partial x^2}+2c\frac{\partial^2 V(x,t)}{\partial x\partial t}+c^2\frac{\partial^2 V(x,t)}{\partial x^2}\right]=\frac{Mg}{\mu_0}\delta(x-ct) \tag{2.7}$$

where

$$N=\frac{EI_0}{\mu_0} \tag{2.8}$$

SOLUTION PROCEDURES

Equation (2.7) cannot be solved by generalized finite integral transformation because the beam is non-uniform. The approach involves expressing the Dirac delta function as a Fourier cosine series and then reducing the fourth order partial differential equation (2.7) using Generalized Galerkin's method (GGM). The resulting transformed differential equation is then simplified using the modified struble's asymptotic technique. The generalized Galerkin's method is defined by

$$U_n(x,t)=\sum_{m=1}^{n}W_m(t)V_m(x) \tag{3.1}$$

where $V_m(x)$ is chosen such that the desired boundary conditions are satisfied.

Operation simplification

By applying the generalized Galerkin's method (3.1), equation (2.7) can be written as

$$\sum_{m=1}^{n}\left\{\left(V_m(x)+\sin\frac{\pi x}{L}V_m(x)\right)\ddot{W}_m(t)+N\left[\left(\frac{5}{2}+\frac{15}{4}\sin\frac{\pi x}{L}-\frac{1}{4}\sin\frac{3\pi x}{L}-\frac{3}{2}\cos\frac{2\pi x}{L}\right)V_m^{iv}(x)+\right.\right.$$
$$\left.\left(\frac{9\pi^2}{4L^2}\sin\frac{3\pi x}{L}-\frac{15}{4L^2}\sin\frac{\pi x}{L}+\frac{6\pi^2}{L^2}\cos\frac{2\pi x}{L}\right)V_m^{ii}(x)\right]W_m(t)-\frac{1}{\mu_0}(R_0+G)V_m^{ii}(x)W_m(t)+\frac{K}{\mu_0}W_m(t)V_m(x)+$$
$$\frac{M}{\mu_0}\delta(x-ct)\left[\ddot{w}_m(t)V_m(x)+2c\dot{w}_m(t)V_m^{i}(x)+c^2W_m(t)V_m^{ii}\right]-\frac{Mg}{\mu_0}\delta(x-ct)=0 \tag{3.2}$$

In order to determine $W_m(t)$, it is required that the expression on the left hand side of equation (3.2) be orthogonal to function $V_k(x)$. Hence

$$\int_0^L\left\{\sum_{m=1}^{n}\left\{\left(V_m(x)+\sin\frac{\pi x}{L}V_m(x)\right)\ddot{W}_m(t)+N\left[\left(\frac{5}{2}+\frac{15}{4}\sin\frac{\pi x}{L}-\frac{1}{4}\sin\frac{3\pi x}{L}-\frac{3}{2}\cos\frac{2\pi x}{L}\right)V_m^{iv}(x)+\right.\right.\right.$$
$$\left.\left(\frac{9\pi^2}{4L^2}\sin\frac{3\pi x}{L}-\frac{15}{4L^2}\sin\frac{\pi x}{L}+\frac{6\pi^2}{L^2}\cos\frac{2\pi x}{L}\right)V_m^{iv}(x)\right]W_m(t)-\frac{1}{\mu_0}(R_0+G)V_m^{ii}(x)W_m(t)+\frac{K}{\mu_0}W_m(t)V_m(x)+$$
$$\left.\frac{M}{\mu_0}\delta(x-ct)\left[\ddot{W}_m(t)V_m(x)+2c\dot{W}_m(t)V_m^{i}(x)+C^2W_m(t)V_m^{ii}\right]-\frac{Mg}{\mu_0}\delta(x-ct)\right\}V_k(x)dx=0 \tag{3.3}$$

A rearrangement of equation (3.3) and ignoring the summation signs yields,

$$\left(U_1(m,k)+U_3(m,k)\right)\ddot{W}_m(t)+\left[N(T_0+T_1)-\frac{1}{\mu_0}(R_0+G)Z_2(m,K)+\frac{k}{\mu_0}U_1(m,k)\right]W_m(t)+$$
$$\frac{M}{\mu_0}\left[U_1(t)\ddot{W}_m(t)+2cU_2(t)\dot{W}_m(t)+C^2U_3(t)W_m(t)\right]=\frac{mg}{\mu_0}Z_4(t) \qquad (3.4)$$

where

$$T_0=\left(U_7(m,k)+U_8(m,k)\right)-\left(U_9(m,k)+U_{10}(m,k)\right)$$

$$T_1=\left(U_5(m,k)+U_6(m,k)\right)-\left(U_4(m,k)\right)$$

and

$$U_1(t)=\int_0^L \delta(x-t)V_m(x)V_k(x)dx$$
$$U_2(t)=\int_0^L \delta(x-t)V_m^1(x)V_k(x)dx$$
$$U_3(t)=\int_0^L \delta(x-t)V_m^{11}(x)V''_k(x)dx$$
$$U_4(t)=\int_0^L \delta(x-t)V_k(x)dx$$

while

$$U_1(m,k)=\int_0^L V_m(x)V_k(x)dx$$
$$U_2(m,k)=\int_0^L V_m^{11}(x)V_k(x)dx$$
$$U_3(m,k)=\int_0^L \frac{\sin \pi x}{L}V_m(x)V_k(x)dx$$
$$U_4(m,k)=\frac{15\pi^2}{4L^2}\int_0^L \frac{\sin \pi x}{L}V_m(x)V_k(x)dx$$
$$U_5(m,k)=\frac{9\pi^2}{4L^2}\int_0^L \frac{\sin \pi x}{L}V^{11}{}_m(x)V_k(x)dx$$
$$U_6(m,k)=\frac{6\pi^2}{L^2}\int_0^L \frac{\cos 2\pi x}{L}V^{11}{}_m(x)V_k(x)dx$$
$$U_7(m,k)=\frac{5}{2}\int_0^L V^{iv}{}_m(x)V_k(x)dx$$
$$U_8(m,k)=\frac{15}{4}\int_0^L \frac{\sin \pi x}{L}V^{iv}{}_m(x)V_k(x)dx$$
$$U_9(m,k)=\frac{1}{4}\int_0^L \frac{\sin 3\pi x}{L}V^{iv}{}_m(x)V_k(x)dx$$
$$U_{10}(m,k)=\frac{1}{4}\int_0^L \frac{\cos 2\pi x}{L}V^{iv}{}_m(x)V_k(x)dx$$

where

$$\left(U_1(m,k)+U_2(m,k)\right)\ddot{W}_m(t)+\left[N(T_0+T_1)-\frac{1}{\mu_0}(R_0+G)U_3(m,K)+\frac{k}{\mu_0}U_1(m,k)\right]W_m(t)+$$

$$\frac{M}{\mu_0}[U_{1A}(m,k)+\frac{2}{L}\sum_{n=0}^{\infty}\frac{\cos n\pi ct}{L}U_{1B}(m,k)\ddot{W}_m(t)$$
$$+2c[U_{2A}(m,k)+\frac{2}{L}\sum_{n=0}^{\infty}\frac{\cos n\pi ct}{L}U_{2B}(m,k)\dot{W}_m(t)+$$
$$c^2[U_{3A}(m,k)+\frac{2}{L}\sum_{n=0}^{\infty}\frac{\cos n\pi ct}{L}U_{3B}(m,k)W_m(t)=\frac{Mg}{\mu_0}V_k(ct) \tag{3.5}$$

$$U_{1A}(m,k)=\frac{1}{L}\int_0^L V_m(x)V_k(x)dx$$
$$U_{1B}(m,k)=\int_0^L\frac{\cos n\pi x}{L}V_m(x)V_k(x)dx$$
$$U_{2A}(m,k)=\frac{1}{L}\int_0^L V_m^1(x)V_k(x)dx$$
$$U_{2B}(m,k)=\int_0^L\frac{\cos n\pi x}{L}V_m^1(x)V_k(x)dx$$
$$U_{3A}(m,k)=\frac{1}{L}\int_0^L V_m^{11}(x)V_k(x)dx$$
$$U_{3B}(m,k)=\int_0^L\frac{\cos n\pi x}{L}V_m^{11}(x)V_k(x)dx$$
$$U_4(t)=V_k(ct)$$

In order to solve equation (3.5), the function $V_m(x)$ is chosen as the beam function

$$V_m(x)=\sin\left(\frac{\lambda_m x}{L}\right)+A_m\cos\left(\frac{\lambda_m x}{L}\right)+B_m\sinh\left(\frac{\lambda_m x}{L}\right)+C_m\cosh\left(\frac{\lambda_m x}{L}\right) \tag{3.6}$$

So that

$$V_k(x)=\sin\left(\frac{\lambda_k x}{L}\right)+A_k\cos\left(\frac{\lambda_k x}{L}\right)+B_k\sinh\left(\frac{\lambda_k x}{L}\right)+C_k\cosh\left(\frac{\lambda_k x}{L}\right) \tag{3.7}$$

The constants $A_m, A_k, B_m, B_k, C_m, C_k$ and the mode frequencies λ_m, λ_k can be determined by using the appropriate classical boundary conditions. Now, substituting (3.6) and (3.7) into (3.5) after some simplification and rearrangement, one obtains

$$E_1(m,k)\ddot{W}_m(t)+E_2(m,k)W_m(t)+\varepsilon_0 L\left[(U_{1A}(m,k)+\frac{2}{L}\sum_{n=0}^{\infty}\frac{\cos n\pi ct}{L}U_{1B}(n,m,k))\ddot{W}_m(t)+2c(U_{2A}(m,k)+\right]$$
$$\frac{\cos n\pi ct}{L}U_{2B}(n,m,k))\dot{W}_m(t)+C^2(U_{3A}(m,k)+\frac{2}{L}\sum_{n=0}^{\infty}\frac{\cos n\pi ct}{L}U_{3B}(n,m,k))W_m(t)$$
$$=\varepsilon_0 Lg(\sin\varphi t+A_k\cos\varphi t+B_k\sinh\varphi t+C_k\cosh\varphi t) \tag{3.8}$$

where

$$E_1(m,k)=U_1(m,k)+U_3(m,k)$$
$$E_2(m,k)=N(T_0+T_1)-S_1U_2(m,k)+S_2U_1(m,k)$$

$$S_1 = \frac{1}{\mu_0}(R_0 + G)$$

$$S_2 = \frac{k}{\mu_0}$$

$$\varphi = \frac{\lambda_k C}{L}$$

$$\varepsilon_0 = \frac{M}{\mu_0 L}$$

Equation (3.8) is now the fundamental equation governing the problem of the dynamic response to a moving concentrated load of non-uniform Rayleigh beam resting on Pasternak elastic foundation.
In what follows two cases of equation (3.8) are discussed.

Closed form solution
Case 1: The differential equation describing the response of a non uniform Rayleigh beam resting on Pasternak elastic foundation and subjected to a moving force may be obtained from equation (3.8) by setting $\varepsilon_o = 0$, in this case, we obtain:

$$\ddot{W}_m(t) + \psi_f^2 W_m(t) = \rho_f\left(\sin\varphi t + A_k\cos\varphi t + B_k\sin\varphi t + C_k\cosh\varphi t\right) \tag{3.9}$$

Where

$$\psi_f^2 = \frac{E_2(m,k)}{E_1(m,k)}$$

and

$$\rho_f = \frac{mg}{\mu_0 E_1(m,k)}$$

Solving equation (3.9) in conjunction with initial condition, the solution is given by

$$W_m(t) = \frac{p_f}{\psi_f(\psi_f^4 - \varphi^4)}\{(\psi_f^2 - \varphi^2)[C_k\psi_f(\cosh\varphi t - \cos\psi_f t) + B_k(\psi_f\sinh\varphi t - \varphi\sin\psi_f t)] +$$

$$(\psi_f + \varphi^2)[A_k\psi_f(\cos\varphi t - \cos\psi_f t) + (\psi_f\sin\varphi t - \varphi\sin\psi_f)\}\left[\sin\frac{\lambda_k x}{L} + A_m\cos\frac{\lambda_k x}{L} + B_m\sinh\frac{\lambda_k x}{L} + C_m\cosh\frac{\lambda_k x}{L}\right] \tag{3.10}$$

Thus, using (3.10) in (3.1), one obtains

$$U_n(x,t) = \sum_{m=1}^{\infty}\frac{p_f}{\psi_f(\psi_f^4 - \varphi^4)}\{(\psi_f^2 - \varphi^2)[C_k\psi_f(\cosh\varphi t - \cos\psi_f t) + B_k(\psi_f\sinh\varphi t - \varphi\sin\psi_f t)] +$$

$$(\psi_f + \varphi^2)[A_k\psi_f(\cos\varphi t - \cos\psi_f t) + (\psi_f\sin\varphi t - \varphi\sin\psi_f)\}\left[\sin\frac{\lambda_k x}{L} + A_m\cos\frac{\lambda_k x}{L} + B_m\sinh\frac{\lambda_k x}{L} + C_m\cosh\frac{\lambda_k x}{L}\right] \tag{3.11}$$

Equation (3.11) represents the response to a moving force for any classical Boundary conditions of a non uniform Rayleigh beam on Pasternak elastic foundation.

Case II: If the inertial term is retained, then $\varepsilon_0 \neq 0$. This is termed the moving mass problem. In this case the solution to the entire equation (3.8) is required. As an exact solution to this problem is impossible, a modification of struble's technique is employed. To this end, equation (3.8) is simplified and rearranged to take the form

$$\ddot{W}_m(t)+\left[\frac{2C\varepsilon_0 Lq_{1b}(n,m,k)}{1+\varepsilon_0 Lq_{1a}(n,m,k)}\right]W_m(t)+\left[\frac{\psi_f^2+C^2\varepsilon_0 Lq_{1c}(n,m,k)}{1+\varepsilon_0 Lq_{1a}(n,m,k)}\right]W_m(t)$$
$$=\frac{\varepsilon_0 Lg}{E_1(m,k)}\left[\frac{\sin\varphi t+A_k\cos\varphi t+B_k\sinh\varphi t+C_k\cosh\varphi t}{1+\varepsilon_0 Lq_{1a}(n,m,k)}\right] \tag{3.12}$$

where

$$q_{1a}(n,m,k)=U_{1a}(m,k)+\frac{2}{L}\sum_{n=0}^{\infty}\frac{\cos n\pi ct}{L}U_{1b}(n,m,k)$$
$$q_{1b}(n,m,k)=U_{2a}(m,k)+\frac{2}{L}\sum_{n=0}^{\infty}\frac{\cos n\pi ct}{L}U_{2b}(n,m,k)$$
$$q_{1c}(n,m,k)=U_{3a}(m,k)+\frac{2}{L}\sum_{n=0}^{\infty}\frac{\cos n\pi ct}{L}U_{3b}(n,m,k)$$

By means of this technique, one seeks the modified frequency corresponding to the frequency of the, moving mass. An equivalent free system operator defined by modified frequency then replaces equation (3.11). Thus, the right-hand side of (3.11) is set to zero and a parameter $\epsilon_0 < 1$ is considered for any arbitrary ratio ϵ_0 defined as

$$\varepsilon_1=\frac{\varepsilon_0}{1+\varepsilon_0}$$

Evidently

$$\varepsilon_0=\varepsilon_1+0(\varepsilon_1^2) \tag{3.13}$$

Using (3.13), equation (3.12) becomes

$$\ddot{W}_m(t)+2c\varepsilon_1 Lq_{1b}(n,m,k)W_m(t)+\varepsilon_1 c^2 Lq_{1c}(n,m,k)W_m(t)+\psi_f^2\left[(1-\varepsilon_1 Lq_{1a}(n,m,k)\right]W_m(t)=$$
$$\frac{\varepsilon_0 Lg}{E_1(m,K)}\left[\sin\varphi t+Ak\cos\varphi t+Bk\sinh\varphi t+ck\cosh\varphi t\right] \tag{3.14}$$

Retaining terms to $O(\epsilon_1)$ only.

When we set ε_1=0 in (3.14), a case corresponding to the case when inertia effect to the mass of the system is neglected is obtained and the solution of (3.11) can be written as

$$W_m(t)=C_m Cos(\psi_f t-D_m)$$

where C_m and D_m are constant

Since ϵ_1< 1, an asymptotic solution of the homogenous part of (3.14) can be written as

$$W_m(t)=\delta(m,t)Cos(\psi_f t-\theta(m,t))+\varepsilon_1 W_1(m,t)+0(\varepsilon_1^{\ 2}) \tag{3.15}$$

where $\delta(m,t)$ and $\theta(m,t)$ are slowly time varying functions.

Substituting equation (3.15) and its derivatives into equation (3.14) and neglecting the terms in (ϵ_1^2) one obtains

$$-2\dot{\delta}(m,t)\psi_f \sin(\psi_f t-\theta(m,t))+2\delta(m,t)\dot{\theta}(m,t)\psi_f Cos(\psi_f t-\theta(m,t)$$
$$-2C\varepsilon_1 LU_{2a}(m,K)\delta(m,t)\psi_f \sin(\psi_f t-\theta(m,t))-2C\varepsilon_1\delta(m,t)\psi_f\sum_{n=0}^{\infty}\frac{\cos\pi nct}{L}\sin(\psi_f t-\theta(m,t))U_{2b}(n,m,k)$$
$$-4C\varepsilon_1\delta(m,t)\psi_f\sum_{n=0}^{\infty}\frac{\cos n\pi ct}{L}\sin(\psi_f t-\theta(m,t))U_{2b}(n,m,k)-\psi_f^2\varepsilon_2 LU_{1a}(m,k)\delta(m,t)Cos(\psi_f t-\theta(m,t))$$
$$-2\psi_f^2\varepsilon_1\delta(m,t)\sum_{n=0}^{\infty}\frac{\cos n\pi ct}{L}Cos(\psi_f t-\theta(m,t))U_{1b}(n,m,k)+C^2\varepsilon_1 LU_{3a}(m,k)\delta(m,t)Cos(\psi_f t-\theta(m,t))$$
$$+2C^2\varepsilon_1\delta(m,t)\sum_{n=0}^{\infty}\frac{\cos n\pi ct}{L}Cos(\psi_f t-\theta(m,t))U_{3b}(n,m,k)=0 \tag{3.16}$$

Retaining terms to (ε_1) only
The variational equations are obtained by equating the co-efficients of
$Sin(\psi_f t - \theta(m,t))$ and $Cos(\psi_f t) - \theta(m,t))$ terms on both side of the equation (60) to zero

Hence, noting the following trigonometric Identities:

$$\cos\frac{n\pi ct}{L}\sin(\psi_f t-\theta(m,t)=\frac{1}{2}Sin\frac{n\pi ct}{L}+\psi_f t-\theta(m,t)-\frac{1}{2}\sin(\frac{n\pi ct}{L}-\psi_f t-\theta(m,t)$$

$$\cos\frac{n\pi ct}{L}Cos(\psi_f t-\theta(m,t)=\frac{1}{2}\cos(\frac{n\pi ct}{L}+\psi_f t-\theta(m,t)+\frac{1}{2}\cos(\frac{n\pi ct}{L}-(\psi_f t-\theta(m,t)$$

and neglecting those terms that do not contribute to the variational equation, equation (3.16) reduces to

$$-2\dot{\delta}(m,t)\psi_f \sin(\psi_f t-\theta(m,t))+2\delta(m,t)\dot{\theta}(m,t)\psi_f Cos(\psi_f t-\theta(m,t))-2C\varepsilon_1 LU_{2a}(m,k)$$
$$\delta(m,t)\psi_f \sin(\psi_f t-\theta(m,t))-\psi_f^2\varepsilon_1 LU_{1a}(m,k)\delta(m,t)Cos(\psi_f t-\theta(m,t))+C^2\varepsilon_1 LU_{3a}(m,k)$$
$$\delta(m,t)Cos(\psi_f t-\theta(m,t))=0. \tag{3.17}$$

Then the vairational equations are respectively :

$$-2\dot{\delta}(m,t)\psi_f -2C\varepsilon_1 LU_{2a}(m,k)\delta(m,t)\Omega_f =0 \tag{3.18}$$

$$2\delta(m,t)\dot{\theta}(m,t)\psi_f -\psi^2\varepsilon_1 LU_{1a}(m,k)\delta(m,k)+C^2\varepsilon_1 LU_{3a}(m,k)\delta(m,t)=0 \tag{3.19}$$

Solving equation (3.18) and (3.19) respectively, one obtains

$$\delta(m,t)=C_m\ \ell^{-C\varepsilon_1 LZ_{2a}(m,k)t} \tag{3.20}$$

$$\theta(m,t)=\frac{\psi_f\varepsilon_1 u_{1a}(m,k)t}{2}-\frac{C^2\varepsilon_1 U_{3a}(m,k)t}{2\psi_f}+C_m \tag{3.21}$$

where C_m and D_m are constants

Therefore, when the effect of the mass of the particle is considered the first approximation to the homogenous system is

$$W_m = C_m \; \ell^{-C\,\varepsilon_1 L U_{2a}(m,k)t} Cos(\beta_m t - D_m \;)$$

where

$$\beta_m = \psi_f \left[1 - \frac{\varepsilon_1 L}{2}(U_{1a}(m,k) - \frac{C^2 U_{3a}(m,k)}{\psi_f^2}) \right]$$

is called the modified natural frequency representing the frequency of the free system due to the presence of the moving mass.

Thus, to solve the non homogenous equation (3.14), the differential operator which acts on the $W_m(t)$ is replaced by the equivalent free system operator defined by the modified frequency β_m

i.e. $\sin\varphi t + A_k \cos\varphi t + B_k \sinh\varphi t + C_k \cosh\varphi t$

$$\frac{d^2 w_m}{dt^2}(t) + \beta_m^2 \mathrm{W}_m(t) = \rho_f \left[\sin\varphi t + A_k \cos\varphi t + B_k \sinh\varphi t + C_k \cosh\varphi t \right] \tag{3.22}$$

where

$$\rho_f = \frac{\varepsilon_0 L g}{E_1(m,k)}$$

Solving equation (3.22) in conjunction with the initial condition, one obtains expression for W_m (t). Thus, in view of (3.1)

$$U_n(x,t) = \sum_{m=1}^{\infty} \frac{p_f}{\beta_m(\beta_m^4 - \varphi^4)} \;\; \{(\beta_m^2 - \varphi^2)[C_k \beta_m(\cosh\varphi t - \cos\beta_m t) + B_k(\beta_m \sinh\varphi t - \varphi \sin\beta_m t)] +$$

$$(\beta_m + \varphi^2)[A_k \beta_m(\cos\varphi t - \cos\beta_m t) + (\beta_m \sin\varphi t - \varphi \sin\beta_m)\} \left[\sin\frac{\lambda_m x}{L} + A_m \cos\frac{\lambda_m x}{L} + B_m \sinh\frac{\lambda_m x}{L} + C_m \cosh\frac{\lambda_m x}{L} \right] \tag{3.23}$$

Equation (3.23) represents the transverse displacement response to moving force of the simply supported non uniform Rayleigh beam on Pasternak elastic foundation.

ILLUSTRATIVE EXAMPLES

For illustration of results in the foregoing analysis, we provide an example on simply supported uniform Rayleigh beam. In this case, the uniform Rayleigh beam has simple supports at ends X = 0 and X = L. The displacement and the bending moment vanish. Hence

$$V(0,t) = 0 = V(L,t), \frac{\partial^2 V(0,t)}{\partial x^2} = 0 = \frac{\partial^2 V(0,t)}{\partial x^2}$$

Consequently, for normal modes

$$V_m(0) = 0 = V_m(L) =$$

$$\frac{\partial^2 V_m(0)}{\partial x^2} = 0 = \frac{\partial^2 V_m(L)}{\partial x^2} \tag{4.1}$$

which implies

$$V_k(0) = 0 = V_k(L), \frac{\partial^2 V_k(0)}{\partial x^2} = 0 = \frac{\partial^2 V_k(t)}{\partial x^2} \tag{4.2}$$

In view of (4.1) and (4.2)

$A_m = 0,\ B_m = 0$ *and* $C_m = 0$

$$\lambda_m = m\pi$$

Similarly

$A_k = 0, B_k = 0$ *and* $C_k = 0$

$$\lambda_k = k\pi$$

Thus, the moving force problem is reduced to a non-homogeneous second order ordinary differential equation

$$W_m(t)+\beta_f^2 W_m(t)=\frac{\rho_f}{E_{11}}\sin\frac{K\pi ct}{L} \tag{4.3}$$

where

$$E_{11}=\frac{L}{2}+\frac{L}{4\pi}A_{11}$$

$$E_{12}=R_0\left[\frac{5m^4\pi^4}{4L^3}+\frac{15m^2\pi^3}{16L^3}(1+m^2)A_{11}-\frac{m^2\pi^4}{4L^4}(9+m^2)A_{11}\right]+S_1\frac{m^2\pi^2}{2L}+S_2\frac{L}{2}$$

Equation (4.3) when solved in conjunction with the initial conditions, one obtain an expression for $W_m(t)$.

Thus from

$$U(m,t)=\int_0^L V(x,t)V_m(x)dx \text{ and } V(x,t)=\sum_{k=1}^{\infty}\frac{\mu}{U_m(x)}U(m,t)V_m(x)$$

$$V(x,t)=\sum_{k=1}^{\infty}\frac{\mu}{U_m(x)}U(m,t)V_m(x)$$

we obtain

$$U_n(x,t)=\sum_{m=1}^{n}\frac{P}{2\mu_0 E_{11}}\left[\frac{\beta_f\sin\frac{k\pi ct}{L}-\frac{k\pi c}{L}\sin\beta_f t}{\beta_f\left(\beta_f^2-\left(k\pi c/L\right)^2\right)}\right]\sin\frac{m\pi x}{L} \tag{4.4}$$

Equation (4.4) represents the transverse displacement response to a moving force of the simply supported non uniform Rayleigh beam on Pasternak elastic foundation.

Following arguments similar to those in the previous section, use is made of the modified Struble's technique to obtain

$$\alpha_f=\beta_f-\frac{\left(C^2m^2\pi^2+\beta_f^2L^2\right)}{4E_{11}\beta_f L}\in_1$$

as the modified frequency corresponding to the free system due to the presence of the moving mass, thus, the moving mass problem takes the form:

$$\frac{d^2W_m(t)}{dt^2}+\alpha_f^2W_m(t)=\frac{\varepsilon_1 Lg}{E_{11}}\sin\frac{K\pi ct}{L} \tag{4.5}$$

In view of (3.1) the solution of (4.5) becomes

$$U_n(x,t)=\sum_{m=1}^{n}\frac{\in_1 Lg}{2\alpha_f E_{11}}\left[\frac{\alpha_f\sin\frac{k\pi ct}{L}-\frac{k\pi c}{L}\sin\alpha_f t}{\alpha_f^2-\left(k\pi c/L\right)^2}\right]\sin\frac{m\pi x}{L} \tag{4.6}$$

Equation (4.6) represents the transverse displacement response to a moving mass of the simply supported non uniform Rayleigh beam resting on Pasternak elastic foundation

DISCUSSION OF CLOSED FORM SOLUTION

The response amplitude of dynamical systems such as this may grow without bond. Condition under which this happens is termed resonance conditions. It is pertinent at this junction to establish conditions under which resonance occurs. This phenomenon in structural and highway engineering is of great concern to researchers or in particular, design engineers, because, for example, it causes cracks, permanent deformation and destruction in structures.

Bridges and other structures are known to have collapsed as a result of resonance occurring between the structure and some signals traversing them. Evidently a simply supported non- uniform Rayleigh beam resting on a Pasternak elastic foundation and traversed by a moving force will experience resonance when

$$\beta = \frac{k\pi c}{L} \tag{4.7}$$

while the same system traversed by a moving mass reaches the state of resonance whenever

$$\alpha_f = \frac{k\pi c}{L}$$

Evidently,

$$\frac{4E_{11}\beta_f^2 - (\beta_f^2 L^2 + c^2 m^2 \delta^2)E_1}{4E_{11}k\delta c} = \frac{k\delta c}{L} \tag{4.9}$$

Equations (4.7) and (4.9) show that for the same natural frequency, the critical speed for the same system consisting of a non uniform Rayleigh beam resting on elastic Pasternak foundation and traversed by a moving mass is smaller than that traversed by a moving force. Thus, resonance is reached earlier in the moving mass system than in the moving force system.

NUMERICAL RESULTS AND DISCUSSIONS

We shall illustrate the analysis proposed in this paper by considering a non uniform Rayleigh beam of modulus of elasticity E= 2.1×10^9 N/m^2, the moment of inertia I_0= 2.87698×10^{-3} m^4, the beam spam length L= 12.192 and the mass per unit length of the beam μ_0= 2758.27kg/m. the value of the foundation modulus is varied between 0n/m^2 and 4000000n/m^3, the values of Rotatory inertia R_0 is varied between 0m and 4.5m, the values of the shear modulus varied between 0n/m^3 and 9000000N/m^3 , the results are as shown on the various graphs below for the simply supported boundary condition so far considered .

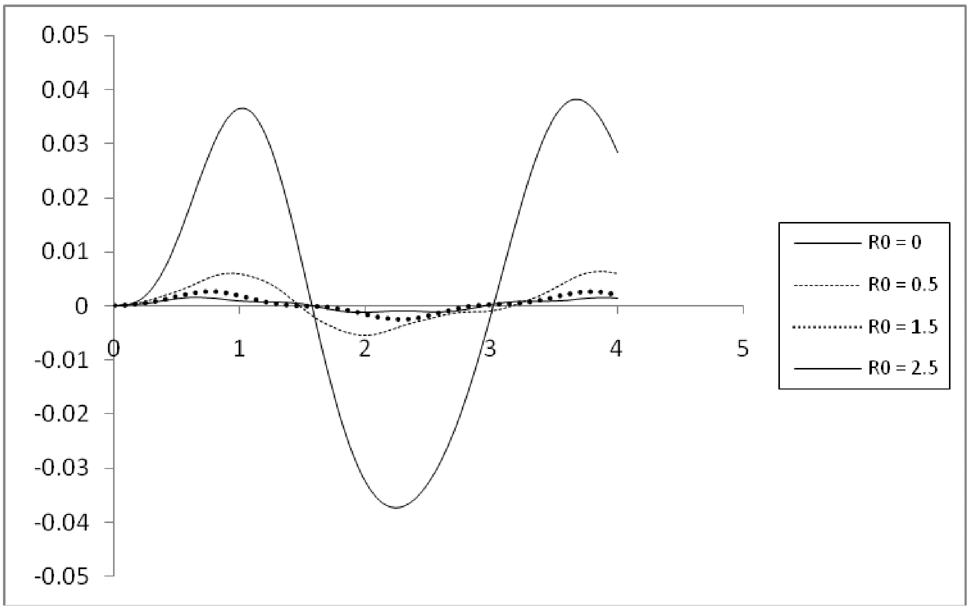

Fig 1: Transverse displacement of a simply supported non uniform Rayleigh beam under the actions of the concentrated forces travelling at constant velocity for various values of Rotatory inertia R_0 and for fixed values of foundation modulus k= 40000 and shear modulus G= 90000

From the graphs above, Figures (1) and (4) displays the effect of Rotatory inertia (R_0) on the transverse deflection of the simply supported non uniform Rayleigh beam in both cases of moving force and moving mass respectively. The graphs show that the response amplitude increases as the value of the Rotatory inertia decreases.

Figures (2) and (5) display the effect of foundation modulus (K) on the transverse deflection of simply supported non uniform Rayleigh beam in both cases of moving force and moving mass respectively. The graphs show that an increase in the Rotatory inertia resulted to decrease in the amplitude of vibration

Figures (3) and (6) shows the influence of shear modulus (G) on the deflection profile of simply supported non uniform Rayleigh beam in both cases of moving force and moving mass respectively. The graphs show that higher values of shear modulus decrease the vibration of the beams.

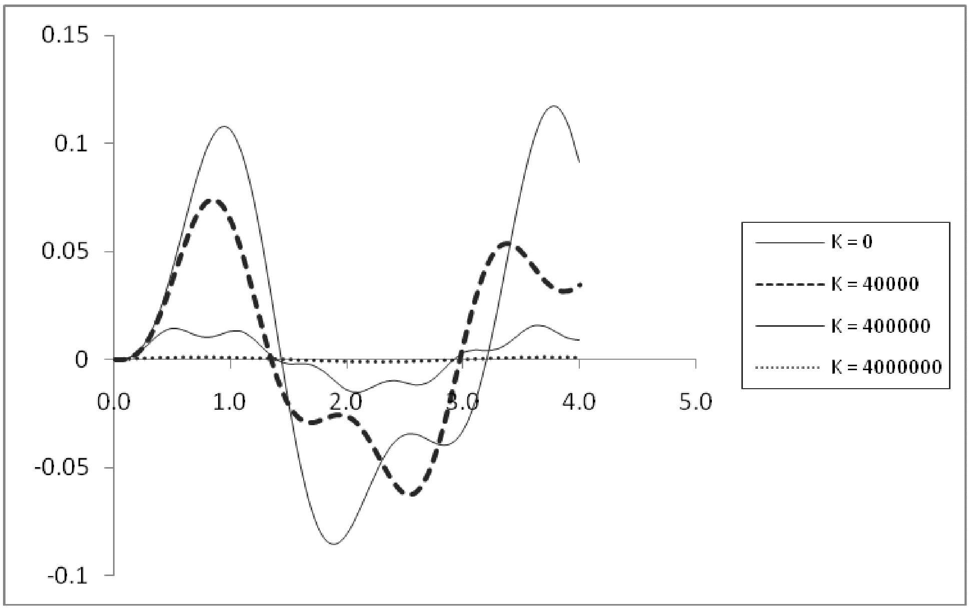

Fig 2: Deflection profile of a simply supported non- uniform Rayleigh beam under the actions of concentrated forces travelling at constant velocity for various values of foundation modulus K and fixed values of Rotatory inertia R_0= 2.5, and shear modulus G= 90000

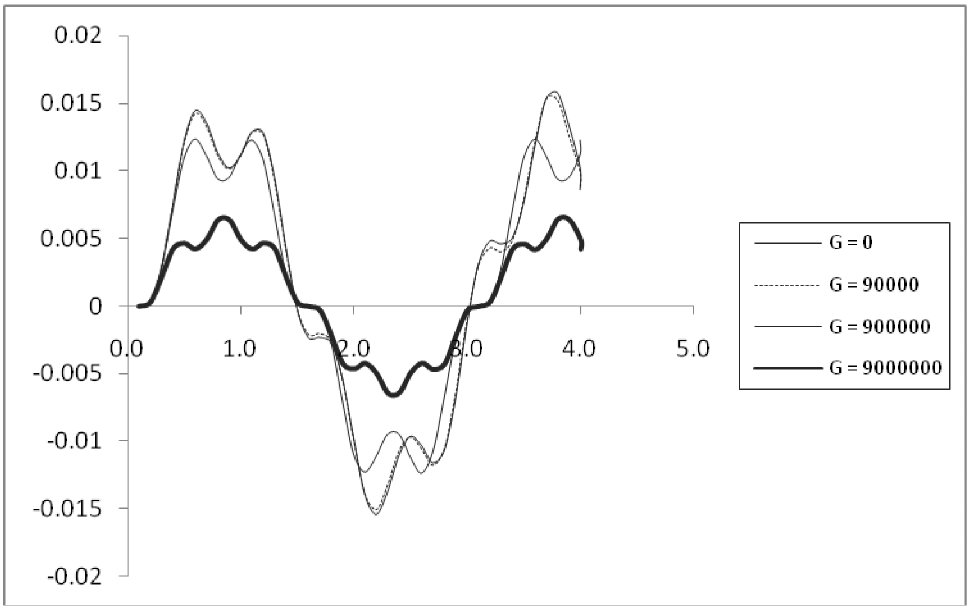

Fig 3: Response amplitude of a simply supported non uniform Rayleigh beam under the actions of concentrated forces travelling at constant velocity for various values of shear modulus G and for fixed values of Rotatory inertia $R_0 = 2.5$ and foundation modulus k= 400000

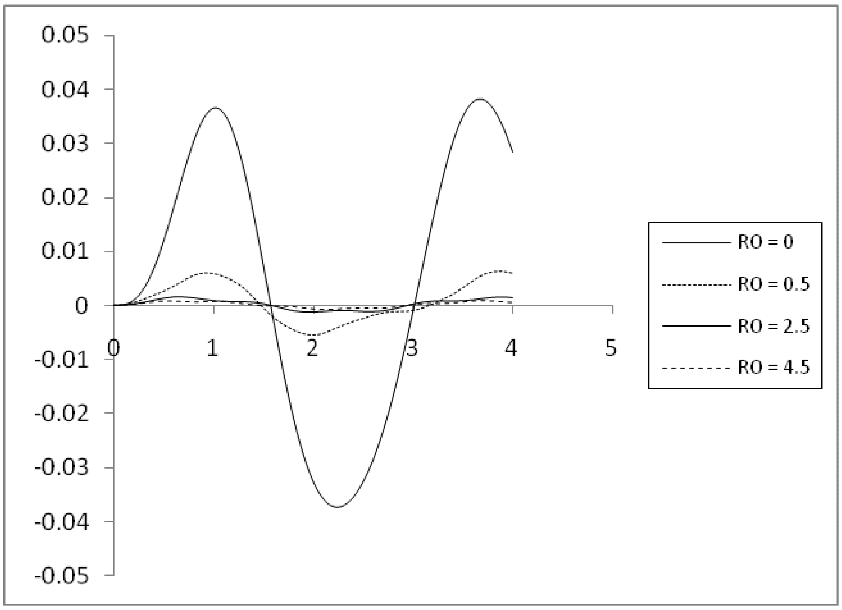

Fig 4: Response amplitude of a simply supported non uniform Rayleigh beam under the action of concentrated mass travelling at constant velocity for various values of Rotatory inertia R_0 and for fixed values of shear modulus G = 90,000 and foundation modulus k= 40,000

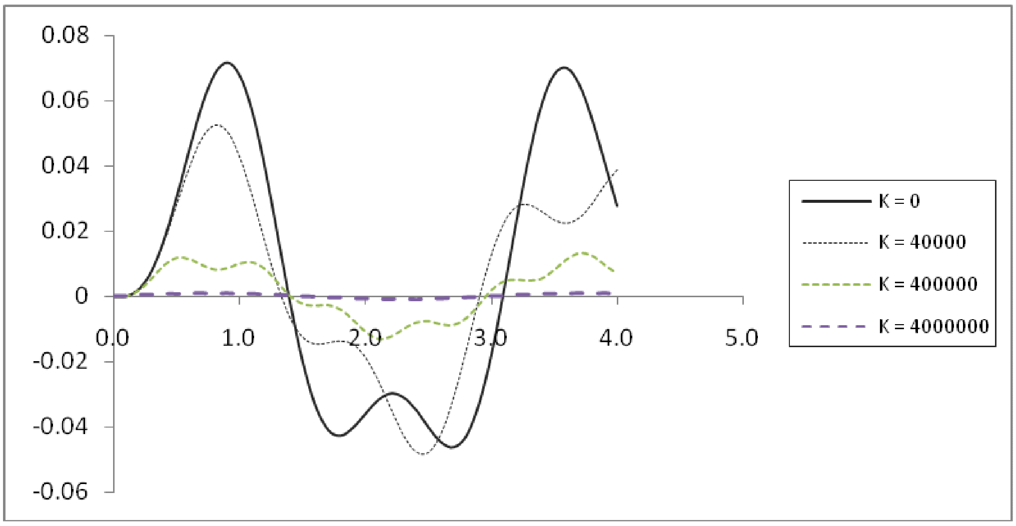

Fig 5: Response amplitude of a simply supported non uniform Rayleigh beam under the action of concentrated mass travelling at constant velocity for various values of foundation modulus K and for fixed values of shear modulus G = 90,000 Rotatory inertia $R_0 = 2.5$

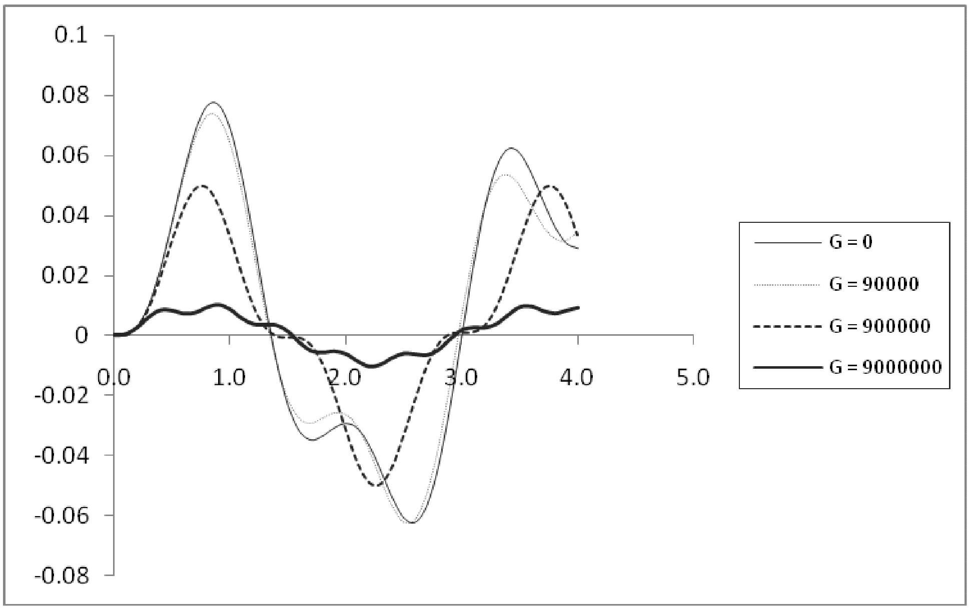

Fig 6: Response amplitude of a simply supported non uniform Rayleigh beam under the action of concentrated forces travelling at constant velocity for various values of shear modulus G and for fixed values of foundation modulus K= 40,000 and Rotatory inertia R_0 = 2.5

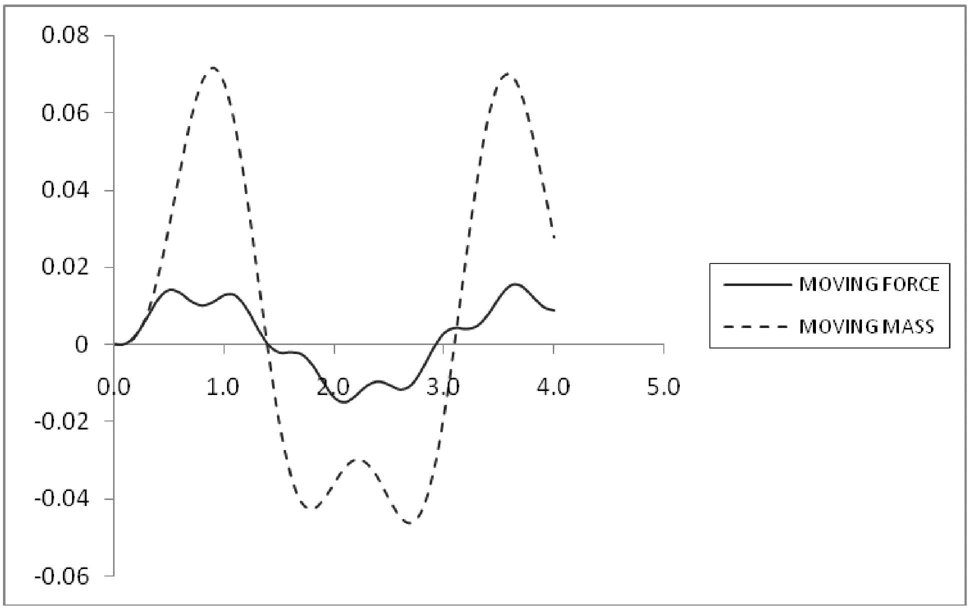

Fig 5.7: Comparison of the displacement response of moving force and moving mass cases of a non uniform simply supported Rayleigh beam for fixed values of R_0 = 2.5, K= 400000 and G=90000

Table 1: Results for various values of rotatory inertia R_O, with fixed values of shear modulus G_O = 900,000 and foundation modulus K_O = 400,000 for both cases of moving force and moving mass

	MOVING FORCE				MOVING MASS			
T(sec)	R0 = 0	R0 = 0.5	R0 = 1.5	R0=2.5	R0 = 0	R0 = 0.5	R0 = 2.5	R0 = 4.5
0	0	0	0	0	0	0	0	0
0.1	1.27E-04	1.02E-04	7.28E-05	5.77E-05	1.27E-04	1.02E-04	5.77E-05	4.44E-05
0.2	9.80E-04	4.96E-04	3.04E-04	2.27E-04	9.80E-04	4.96E-04	2.27E-04	1.70E-04
0.3	3.12E-03	1.11E-03	6.20E-04	5.08E-04	3.12E-03	1.11E-03	5.08E-04	4.04E-04
0.4	6.83E-03	1.87E-03	1.12E-03	9.14E-04	6.83E-03	1.87E-03	9.14E-04	6.27E-04
0.5	1.20E-02	2.60E-03	1.72E-03	1.25E-03	1.20E-02	2.60E-03	1.25E-03	7.55E-04
0.6	0.018166	3.55E-03	2.18E-03	1.49E-03	0.018166	3.55E-03	1.49E-03	7.42E-04
0.7	2.46E-02	4.58E-03	2.49E-03	1.48E-03	2.46E-02	4.58E-03	1.48E-03	6.43E-04
0.8	3.04E-02	5.53E-03	2.54E-03	1.32E-03	3.04E-02	5.53E-03	1.32E-03	5.94E-04
0.9	3.46E-02	5.96E-03	2.24E-03	1.04E-03	3.46E-02	5.96E-03	1.04E-03	6.13E-04
1.0	0.036451	5.85E-03	1.78E-03	8.08E-04	0.036451	5.85E-03	8.08E-04	6.74E-04
1.1	3.55E-02	5.33E-03	1.22E-03	6.79E-04	3.55E-02	5.33E-03	6.79E-04	6.69E-04
1.2	3.17E-02	4.45E-03	6.31E-04	6.41E-04	3.17E-02	4.45E-03	6.41E-04	5.13E-04
1.3	2.53E-02	3.17E-03	2.66E-04	6.43E-04	2.53E-02	3.17E-03	6.43E-04	2.73E-04
1.4	1.69E-02	1.38E-03	3.67E-05	5.21E-04	1.69E-02	1.38E-03	5.21E-04	2.87E-05
1.5	7.20E-03	-5.28E-04	-1.06E-04	2.80E-04	7.20E-03	-5.28E-04	2.80E-04	-1.12E-04
1.6	-2.83E-03	-2.29E-03	-1.46E-04	-1.53E-04	-2.83E-03	-2.29E-03	-1.53E-04	-1.50E-04
1.7	-1.24E-02	-3.58E-03	-3.15E-04	-5.97E-04	-1.24E-02	-3.58E-03	-5.97E-04	-1.92E-04
1.8	-2.08E-02	-4.59E-03	-6.36E-04	-1.01E-03	-2.08E-02	-4.59E-03	-1.01E-03	-3.01E-04
1.9	-2.76E-02	-5.26E-03	-1.05E-03	-1.21E-03	-2.76E-02	-5.26E-03	-1.21E-03	-5.13E-04
2.0	-3.26E-02	-5.51E-03	-1.62E-03	-1.24E-03	-3.26E-02	-5.51E-03	-1.24E-03	-7.19E-04
2.1	-3.58E-02	-5.19E-03	-2.14E-03	-1.13E-03	-3.58E-02	-5.19E-03	-1.13E-03	-8.13E-04
2.2	-3.72E-02	-4.44E-03	-2.44E-03	-1.00E-03	-3.72E-02	-4.44E-03	-1.00E-03	-7.77E-04
2.3	-3.70E-02	-3.68E-03	-2.55E-03	-9.69E-04	-3.70E-02	-3.68E-03	-9.69E-04	-6.45E-04
2.4	-3.55E-02	-3.00E-03	-2.32E-03	-1.01E-03	-3.55E-02	-3.00E-03	-1.01E-03	-5.58E-04
2.5	-3.26E-02	-2.46E-03	-1.84E-03	-1.13E-03	-3.26E-02	-2.46E-03	-1.13E-03	-5.54E-04
2.6	-2.85E-02	-1.84E-03	-1.29E-03	-1.14E-03	-2.85E-02	-1.84E-03	-1.14E-03	-5.79E-04
2.7	-0.02321	-1.39E-03	-6.93E-04	-1.04E-03	-0.02321	-1.39E-03	-1.04E-03	-5.56E-04
2.8	-1.68E-02	-1.16E-03	-2.23E-04	-6.96E-04	-1.68E-02	-1.16E-03	-6.96E-04	-3.85E-04
2.9	-9.30E-03	-1.15E-03	9.41E-06	-2.59E-04	-9.30E-03	-1.15E-03	-2.59E-04	-1.26E-04
3.0	-1.08E-03	-9.73E-04	1.73E-04	2.11E-04	-1.08E-03	-9.73E-04	2.11E-04	1.18E-04
3.1	7.52E-03	-4.71E-04	2.53E-04	5.57E-04	7.52E-03	-4.71E-04	5.57E-04	2.67E-04
3.2	1.60E-02	2.61E-04	3.40E-04	7.31E-04	1.60E-02	2.61E-04	7.31E-04	2.96E-04
3.3	2.37E-02	1.11E-03	6.29E-04	7.76E-04	2.37E-02	1.11E-03	7.76E-04	3.23E-04
3.4	3.02E-02	2.03E-03	1.02E-03	7.48E-04	3.02E-02	2.03E-03	7.48E-04	4.17E-04
3.5	3.49E-02	3.25E-03	1.52E-03	8.14E-04	3.49E-02	3.25E-03	8.14E-04	5.96E-04
3.6	3.76E-02	4.52E-03	2.07E-03	9.48E-04	3.76E-02	4.52E-03	9.48E-04	7.83E-04
3.7	3.81E-02	5.64E-03	2.42E-03	1.21E-03	3.81E-02	5.64E-03	1.21E-03	8.44E-04
3.8	3.65E-02	6.18E-03	2.52E-03	1.40E-03	3.65E-02	6.18E-03	1.40E-03	7.76E-04
3.9	0.033083	6.26E-03	2.39E-03	1.48E-03	0.033083	6.26E-03	1.48E-03	6.19E-04
4.0	2.83E-02	5.92E-03	1.92E-03	1.34E-03	2.83E-02	5.92E-03	1.34E-03	4.91E-04

Table 2: Results for various values of foundation modulus K_O, with fixed values of shear modulus G_O =900,000 and rotatory inertia R_O for both cases of moving force and moving mass

	MOVING FORCE				MOVING MASS			
T(sec)	K = 0	K = 40000	K = 400000	K = 4000000	K = 0	K = 40000	K = 400000	K = 4000000
0	0	0	0	0	0	0	0	0
0.1	2.24E-04	2.23E-04	2.16E-04	1.56E-04	1.61E-04	1.60E-04	1.56E-04	1.18E-04
0.2	3.19E-03	3.13E-03	2.68E-03	4.85E-04	2.49E-03	2.46E-03	2.14E-03	4.67E-04
0.3	1.06E-02	1.02E-02	7.23E-03	5.39E-04	8.23E-03	7.96E-03	5.84E-03	4.55E-04
0.4	2.34E-02	2.19E-02	1.19E-02	9.15E-04	1.82E-02	1.71E-02	9.89E-03	7.91E-04
0.5	4.11E-02	3.70E-02	1.42E-02	7.76E-04	3.12E-02	2.84E-02	1.20E-02	6.84E-04
0.6	0.0613656	5.26E-02	1.33E-02	1.09E-03	4.57E-02	3.98E-02	1.15E-02	9.35E-04
0.7	8.15E-02	6.57E-02	1.11E-02	9.37E-04	5.90E-02	4.87E-02	9.68E-03	9.37E-04
0.8	9.77E-02	7.31E-02	1.01E-02	1.15E-03	6.83E-02	5.26E-02	8.34E-03	8.57E-04
0.9	0.1066552	0.0725247	1.11E-02	9.62E-04	7.16E-02	5.06E-02	8.84E-03	9.42E-04
1.0	0.1062352	0.063943	1.28E-02	9.02E-04	6.74E-02	0.0426257	1.03E-02	6.99E-04
1.1	9.51E-02	4.84E-02	1.27E-02	7.81E-04	5.59E-02	3.04E-02	1.05E-02	8.16E-04
1.2	0.0741809	2.87E-02	9.34E-03	5.64E-04	3.88E-02	1.64E-02	8.46E-03	4.34E-04
1.3	4.57E-02	8.58E-03	4.24E-03	5.39E-04	1.83E-02	3.21E-03	4.25E-03	4.08E-04
1.4	1.30E-02	-8.95E-03	-1.61E-04	1.11E-04	-2.32E-03	-6.73E-03	5.87E-05	2.04E-04
1.5	-1.96E-02	-2.12E-02	-2.11E-03	9.58E-05	-2.04E-02	-1.27E-02	-2.27E-03	-3.10E-05
1.6	-4.82E-02	-2.76E-02	-2.04E-03	-3.55E-04	-3.35E-02	-1.47E-02	-2.72E-03	-1.25E-04
1.7	-6.98E-02	-2.91E-02	-2.57E-03	-3.01E-04	-4.08E-02	-1.43E-02	-2.59E-03	-5.01E-04
1.8	-8.22E-02	-2.74E-02	-5.41E-03	-6.84E-04	-4.26E-02	-1.38E-02	-3.98E-03	-4.09E-04
1.9	-8.56E-02	-2.57E-02	-9.92E-03	-7.27E-04	-4.01E-02	-1.48E-02	-7.25E-03	-7.66E-04
2.0	-0.0809889	-2.62E-02	-1.39E-02	-9.60E-04	-0.0357606	-1.90E-02	-1.09E-02	-7.66E-04
2.1	-7.09E-02	-3.03E-02	-1.51E-02	-1.02E-03	-3.16E-02	-2.60E-02	-1.30E-02	-8.91E-04
2.2	-5.88E-02	-3.82E-02	-1.32E-02	-9.72E-04	-2.97E-02	-0.0346683	-1.23E-02	-9.37E-04
2.3	-4.73E-02	-4.79E-02	-1.07E-02	-1.11E-03	-0.030874	-4.29E-02	-9.97E-03	-8.23E-04
2.4	-3.89E-02	-5.68E-02	-9.69E-03	-9.08E-04	-3.47E-02	-0.0478374	-7.98E-03	-1.01E-03
2.5	-3.49E-02	-0.0620812	-1.06E-02	-1.05E-03	-4.01E-02	-4.79E-02	-7.57E-03	-6.88E-04
2.6	-3.48E-02	-6.08E-02	-1.18E-02	-6.51E-04	-4.46E-02	-4.20E-02	-8.53E-03	-7.52E-04
2.7	-3.71E-02	-5.20E-02	-1.06E-02	-7.04E-04	-4.61E-02	-3.07E-02	-8.70E-03	-4.83E-04
2.8	-3.95E-02	-3.62E-02	-6.37E-03	-3.19E-04	-4.26E-02	-1.61E-02	-6.61E-03	-4.04E-04
2.9	-3.88E-02	-0.0152765	-1.05E-03	-3.08E-04	-3.30E-02	-4.15E-04	-2.50E-03	-2.31E-04
3.0	-3.31E-02	7.20E-03	2.99E-03	2.21E-05	-1.77E-02	1.31E-02	2.04E-03	1.12E-04
3.1	-2.04E-02	2.77E-02	4.29E-03	2.34E-04	1.92E-03	2.28E-02	4.72E-03	8.05E-05
3.2	-8.87E-04	4.33E-02	4.01E-03	4.17E-04	2.32E-02	2.76E-02	5.23E-03	4.66E-04
3.3	2.40E-02	5.18E-02	4.83E-03	6.85E-04	4.31E-02	2.80E-02	4.93E-03	4.56E-04
3.4	5.16E-02	5.34E-02	7.89E-03	6.59E-04	5.88E-02	2.60E-02	5.59E-03	7.67E-04
3.5	7.83E-02	4.96E-02	1.22E-02	1.01E-03	6.80E-02	2.33E-02	8.26E-03	7.81E-04
3.6	0.1000384	4.28E-02	0.015339	8.97E-04	6.99E-02	2.24E-02	1.15E-02	8.13E-04
3.7	0.1137604	3.63E-02	1.52E-02	1.18E-03	0.0647916	2.42E-02	1.34E-02	1.01E-03
3.8	0.1169195	3.22E-02	1.25E-02	9.26E-04	5.44E-02	2.86E-02	1.27E-02	8.28E-04
3.9	0.1090217	3.17E-02	9.80E-03	1.05E-03	4.12E-02	3.44E-02	9.82E-03	9.74E-04
4.0	9.12E-02	3.45E-02	8.82E-03	8.58E-04	2.77E-02	3.91E-02	7.14E-03	6.81E-04

Table 3: Results for various values of shear modulus G_O, with fixed values of foundation modulus K_O = 400,000 and rotatory inertia R_O = 2.5 for both cases of moving force and moving mass

	MOVING FORCE				MOVING MASS			
T(sec)	G = 0	G = 90000	G = 900000	G = 9000000	G = 0	G = 90000	G = 900000	G = 9000000
0	0	0	0	0	0	0	0	0
0.1	2.15E-04	2.16E-04	2.25E-04	2.88E-04	2.22E-04	2.23E-04	2.33E-04	2.98E-04
0.2	2.69E-03	2.68E-03	2.63E-03	2.14E-03	3.14E-03	3.13E-03	3.08E-03	2.50E-03
0.3	7.27E-03	7.23E-03	6.91E-03	4.24E-03	1.02E-02	1.02E-02	9.72E-03	5.98E-03
0.4	1.20E-02	1.19E-02	1.09E-02	4.66E-03	2.21E-02	2.19E-02	2.01E-02	8.28E-03
0.5	1.44E-02	1.42E-02	1.24E-02	4.23E-03	3.76E-02	3.70E-02	3.22E-02	8.18E-03
0.6	1.36E-02	1.33E-02	1.12E-02	4.97E-03	5.38E-02	5.26E-02	4.28E-02	7.15E-03
0.7	1.14E-02	1.11E-02	9.46E-03	6.39E-03	6.79E-02	6.57E-02	4.91E-02	7.46E-03
0.8	1.02E-02	1.01E-02	9.54E-03	6.37E-03	7.63E-02	7.31E-02	4.93E-02	9.17E-03
0.9	1.11E-02	1.11E-02	1.12E-02	4.93E-03	0.0768529	0.0725247	4.32E-02	1.01E-02
1.0	0.0128663	1.28E-02	1.23E-02	4.22E-03	0.0690185	0.063943	3.26E-02	8.57E-03
1.1	1.29E-02	1.27E-02	1.09E-02	4.68E-03	5.35E-02	4.84E-02	2.05E-02	5.62E-03
1.2	9.63E-03	9.34E-03	6.89E-03	4.25E-03	3.32E-02	2.87E-02	9.82E-03	3.66E-03
1.3	4.51E-03	4.24E-03	2.54E-03	2.14E-03	1.15E-02	8.58E-03	2.85E-03	3.48E-03
1.4	-1.31E-04	-1.61E-04	2.88E-04	2.83E-04	-8.31E-03	-8.95E-03	-1.34E-04	3.32E-03
1.5	-2.33E-03	-2.11E-03	1.14E-05	-8.43E-06	-2.31E-02	-2.12E-02	-5.82E-04	1.17E-03
1.6	-2.33E-03	-2.04E-03	-3.07E-04	-3.16E-04	-3.18E-02	-2.76E-02	-7.08E-04	-2.49E-03
1.7	-2.71E-03	-2.57E-03	-2.59E-03	-2.17E-03	-3.46E-02	-2.91E-02	-3.21E-03	-5.26E-03
1.8	-5.28E-03	-5.41E-03	-6.93E-03	-4.23E-03	-3.33E-02	-2.74E-02	-9.68E-03	-5.79E-03
1.9	-9.71E-03	-9.92E-03	-1.10E-02	-4.62E-03	-3.06E-02	-2.57E-02	-1.97E-02	-5.40E-03
2.0	-0.0139621	-1.39E-02	-1.23E-02	-4.22E-03	-2.92E-02	-2.62E-02	-3.16E-02	-6.30E-03
2.1	-1.54E-02	-1.51E-02	-1.11E-02	-5.00E-03	-3.08E-02	-3.03E-02	-4.25E-02	-8.70E-03
2.2	-1.38E-02	-1.32E-02	-9.53E-03	-6.43E-03	-3.64E-02	-3.82E-02	-0.0489589	-1.04E-02
2.3	-1.11E-02	-1.07E-02	-9.48E-03	-6.37E-03	-4.48E-02	-4.79E-02	-4.94E-02	-9.78E-03
2.4	-9.67E-03	-9.69E-03	-1.12E-02	-4.91E-03	-5.39E-02	-5.68E-02	-4.36E-02	-7.54E-03
2.5	-1.03E-02	-1.06E-02	-1.24E-02	-4.21E-03	-6.06E-02	-0.0620812	-3.30E-02	-6.12E-03
2.6	-1.16E-02	-1.18E-02	-1.08E-02	-4.67E-03	-6.20E-02	-6.08E-02	-2.08E-02	-6.40E-03
2.7	-1.08E-02	-1.06E-02	-6.83E-03	-4.24E-03	-5.62E-02	-5.20E-02	-1.00E-02	-6.67E-03
2.8	-6.91E-03	-6.37E-03	-2.60E-03	-2.13E-03	-4.29E-02	-3.62E-02	-2.63E-03	-4.85E-03
2.9	-1.50E-03	-1.05E-03	-2.25E-04	-3.00E-04	-2.31E-02	-0.0152765	5.85E-04	-1.28E-03
3.0	3.03E-03	2.99E-03	8.85E-06	-7.79E-06	1.88E-04	7.20E-03	1.05E-03	1.67E-03
3.1	4.78E-03	4.29E-03	2.31E-04	3.37E-04	2.35E-02	2.77E-02	1.23E-03	2.51E-03
3.2	4.59E-03	4.01E-03	2.67E-03	2.22E-03	0.0432153	4.33E-02	3.44E-03	2.45E-03
3.3	5.01E-03	4.83E-03	6.99E-03	4.26E-03	5.64E-02	5.18E-02	9.53E-03	3.76E-03
3.4	7.53E-03	7.89E-03	1.09E-02	4.60E-03	0.0621357	5.34E-02	1.94E-02	6.76E-03
3.5	1.18E-02	1.22E-02	1.24E-02	4.19E-03	6.06E-02	4.96E-02	3.12E-02	9.32E-03
3.6	1.53E-02	0.015339	1.12E-02	5.00E-03	5.38E-02	4.28E-02	4.20E-02	9.52E-03
3.7	1.58E-02	1.52E-02	9.44E-03	6.45E-03	4.48E-02	3.63E-02	4.88E-02	8.02E-03
3.8	1.34E-02	1.25E-02	9.54E-03	6.38E-03	3.63E-02	3.22E-02	4.95E-02	7.21E-03
3.9	1.03E-02	9.80E-03	1.12E-02	4.91E-03	3.07E-02	3.17E-02	4.39E-02	8.12E-03
4.0	8.59E-03	8.82E-03	1.23E-02	4.21E-03	2.91E-02	3.45E-02	3.36E-02	9.10E-03

Table 4: comparism of the displacement response of moving force and moving mass of non uniform simply supported Rayleigh beam for fixed values of Rotatory inertia R_0 = 2.5, K_0 = 400000 and G_0 =90000.

T(sec)	MOVING FORCE	MOVING MASS
0	2.16E-04	1.61E-04
0.1	2.68E-03	2.49E-03
0.2	7.23E-03	8.23E-03
0.3	1.19E-02	1.82E-02
0.4	1.42E-02	3.12E-02
0.5	1.33E-02	4.57E-02
0.6	1.11E-02	5.90E-02
0.7	1.01E-02	6.83E-02
0.8	1.11E-02	7.16E-02
0.9	1.28E-02	6.74E-02
1.0	1.27E-02	5.59E-02
1.1	9.34E-03	3.88E-02
1.2	4.24E-03	1.83E-02
1.3	-1.61E-04	-2.32E-03
1.4	-2.11E-03	-2.04E-02
1.5	-2.04E-03	-3.35E-02
1.6	-2.57E-03	-4.08E-02
1.7	-5.41E-03	-4.26E-02
1.8	-9.92E-03	-4.01E-02
1.9	-1.39E-02	-0.03576
2.0	-1.51E-02	-3.16E-02
2.1	-1.32E-02	-2.97E-02
2.2	-1.07E-02	-0.03087
2.3	-9.69E-03	-3.47E-02
2.4	-1.06E-02	-4.01E-02
2.5	-1.18E-02	-4.46E-02
2.6	-1.06E-02	-4.61E-02
2.7	-6.37E-03	-4.26E-02
2.8	-1.05E-03	-3.30E-02
2.9	2.99E-03	-1.77E-02
3.0	4.29E-03	1.92E-03
3.1	4.01E-03	2.32E-02
3.2	4.83E-03	4.31E-02
3.3	7.89E-03	5.88E-02
3.4	1.22E-02	6.80E-02
3.5	0.015339	6.99E-02
3.6	1.52E-02	0.064792
3.7	1.25E-02	5.44E-02
3.8	9.80E-03	4.12E-02
3.9	8.82E-03	2.77E-02
4.0	2.16E-04	1.61E-04

CONCLUSION

The problem of vibrations of non uniform Rayleigh beam resting on elastic Pasternak foundation and transverse by concentrated masses travelling at constant velocity has been investigated. Illustrative example involving simply supported boundary condition was presented. The solutions hitherto obtained are analyzed and resonance conditions for the various problems are established. Results show that:

Resonance is reached earlier in a system traversed by moving mass than in that under the action of a moving force.

(a) As the shear modulus (G), Rotatory inertia (R_0) and foundation modulus (K) increases, the amplitude of non uniform Rayleigh beam under the action of moving loads moving at constant velocity decreases.

(b) When the values of the shear modulus (G) and Rotatory inertia (R_0) are fixed, the displacement of non uniform Rayleigh beam resting on elastic Pasternak foundation and traversed by masses travelling with constant velocity.
(c) For fixed value of axial force, shear modulus and foundation modulus, the response amplitude for the moving mass problem is greater than that of the moving force problem for the illustrated end condition considered.

(d) It has been established that, the moving force solution is not an upper bound for accurate solution of the moving mass in uniform Rayleigh beams under accelerating loads. Hence, the non- reliability of moving force solution as a safe approximation to the moving mass problem is confirmed.

(e) In the illustrated examples, for the same natural frequency, the critical velocity for moving mass problem is smaller than that of the moving force problem. Hence, resonance is reached earlier in the moving mass problem.

REFERENCES

[1] Awodola T.O. and Oni, S.T (**2003**): *Journal of the Nigerian association of Mathematical Physics*, vol 7, pp 191-206

[2] Ayre, R. S; Jacobsen, L. S; Hsu, R. S (**1951**): Transverse vibration of one and of two-span beams under the action of moving mass load. Proceedings of the first U.S. National Congress of applied, mechanics, Illinois Institute of technology, Chicago, Illinois, pp. 81-89.

[3] Douglas R, Charles C, and Frederic Y.M (**2002***): Q.JI Mech. Appl. Math*., 55(1), pp 29-48.

[4] Fryba, L (**2005**): Vibrations of solid structures under moving loads. *Journals of Nigerian Association of Mathematical Physics* vol. 9.

[5] Krylov, A.N (**1905**): Mathematical collection of papers of the Academy of sciences, vol. 61, Petersburg.

[6] Lowan, A.N (**1935**): *Phil. Mag. Ser.* 7, vol. 19, No. 127, pp. 708 – 715.

[7] Oni, S.T and Omolofe (**2005**): *ABACUS- Journal of the Mathematical association of Nigeria*, vol 32, No. 2A, pp 165-191.

[8] Oni, S.T (**1996**): *Abacus Journal of mathematical association of Nigeria* vol. 24, no 2, pp 72-88

[9] Oni, S.T (**2000**): *Journal of the Nigerian Society of Engineers*, 35(1), pp. 18 -27.

[10] Pestel, E (**1951**): *Ingenieur Archive*. Vol. 19, pp. 378 – 383.

[11] Sadiku, S and Leipholz, H.H.E (**1987**): *Ingenieur Archive*, Vol. 57, pp 223-242.

[12] Stanistic, M.M; Euler, J.A; Montgomery, S.T (**1974**): *Ingenieur Archive*, vol. 19, pp 295.

[13] Stokes, G (**1849**): Discussion of a differential equation relating to the breaking of railway bridges (Translations of the bridge philosophical society) 8(5), pp 707 – 735.