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Double-diffusive rotatory convection coupled with cross-diffusions in viscoelastic fluid

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ABSTRACT

The present paper investigates the effect of a uniform vertical rotation on the physical problem of double-diffusive convection coupled with cross-diffusions in viscoelastic fluid. Some general qualitative results concerning the stability of oscillatory motions and limitations on the oscillatory motions of growing amplitude are derived. The results for the double- diffusive convection problems with or without the individual consideration of Dufour and Soret effects follow as a consequence.

Keywords: Double-diffusive convection, Dufour-Soret effects, Rivlin-Ericksen viscoelastic fluid, Rayleigh numbers, Prandtl number, Taylor number
MSC 2000 No: 76E99, 76E06

INTRODUCTION

Thermosolutal convection or more generally double-diffusive convection, like its classical counterpart, namely, single –diffusive convection, has carved a niche for itself in the domain of hydrodynamic stability on account of its interesting complexities as a double- diffusive phenomenon as well as its direct relevance in the fields of Oceanography, Astrophysics, Geophysics, Limnology and Chemical engineering etc. can be seen from the review articles by Turner [1] and Brandt and Fernando [2]. An interesting early experimental study is that of Caldwell [3]. The problem is more complex than that of a single - diffusive fluid because the gradient in the relative concentration of two components can contribute to a density gradient just as effectively as can a temperature gradient. Further, the presence of two diffusive modes allows either stationary or overstable flow states at the onset of convection depending on the magnitude of the fluid parameters, the boundary conditions and the competition between thermal expansion and the thermal diffusion. More complicated double- diffusive phenomenon appears if the destabilizing thermal/concentration gradient is opposed by the effect of a magnetic field or rotation. In the domain of linear stability theory the double- diffusive convection problems can be described by a set of linear ordinary differential equations with constant coefficient and homogeneous boundary conditions. The task of finding the explicit analytical solutions of these equations (especially when boundaries are rigid) and thereby characterizing the critical conditions at the threshold of instability are not entirely trivial since prohibitive amount of numerical work is required to affirm oscillatory or non- oscillatory motions as the eigen value equation involves all the parameters of the problem implicitly.

The stability properties of binary fluids are quite different from pure fluids because of Soret and Dufour effects [4], [5]. An externally imposed temperature gradient produces a chemical potential gradient and the phenomenon known as the Soret effect, arises when the mass flux contains a term that depends upon the temperature gradient. The analogous effect that arises from a concentration gradient dependent term in the heat flux is called the Dufour effect.

Although it is clear that the thermosolutal and Soret-Dufour problems are quite closely related, their relationship has never been carefully elucidated. They are in fact, formally identical and identification is done by means of a linear transformation that takes the equations and boundary conditions for the latter problem into those for the former. Mohan [6,7] mollified the nastily behaving governing equations of Dufour- driven thermosolutal convection and Soret – driven thermosolutal convection problems of the Veronis [8] type by the construction of a linear transformation and derived the desired results concerning the linear growth rate and the behavior of oscillatory motions on the lines suggested by Banerjee et. al. [9, 10]. The analysis of double diffusive convection becomes complicated in case when the diffusivity of one property is much greater than the other. Further, when two transport processes take place simultaneously, they interfere with each other and produce cross diffusion effect. The Soret and Dufour coefficients describe the flux of mass caused by temperature gradient and the flux of heat caused by concentration gradient respectively. The coupling of the fluxes of the stratifying agents is a prevalent feature in multicomponent fluid systems. In general, the stability of such systems is also affected by the cross-diffusion terms. Generally, it is assumed that the effect of cross diffusions on the stability criteria is negligible. However, there are liquid mixtures for which cross diffusions are of the same order of magnitude as the diffusivities. There are only few studies available on the effect of cross diffusion on double diffusion convection largely because of the complexity in determining these coefficients. Hurle and Jakeman [11] have studied the effect of Soret coefficient on the double–diffusive convection. They have reported that the magnitude and sign of the Soret coefficient were changed by varying the composition of the mixture. McDougall [12] has made an in depth study of double diffusive convection where in both Soret and Dufour effects are important.

In all the above studies, the fluid has been considered to be Newtonian. However, with the growing importance of non-Newtonian fluids in modern technology and industries, the investigations on such fluids are desirable. The Rivlin-Ericksen [13] fluid is such fluid. Many research workers have paid their attention towards the study of Rivlin-Ericksen fluid. Johri [14] has discussed the viscoelastic Rivlin-Ericksen incompressible fluid under time dependent pressure gradient. Sisodia and Gupta [15] and Srivastava and Singh [16] have studied the unsteady flow of a dusty elastico-viscous Rivlin-Ericksen fluid through channel of different cross-sections in the presence of the time dependent pressure gradient. Sharma and Kumar [17] have studied the thermal instability of a layer of Rivlin-Ericksen elastico-viscous fluid acted on by a uniform rotation and found that rotation has a stabilizing effect and introduces oscillatory modes in the system. Sharma and Kumar [18] have studied the thermal instability in Rivlin-Ericksen elastico-viscous fluid in hydromagnetics.

Rotation introduces a number of new elements into a hydrodynamic problem and the key to our understanding of the consequences of rotation, some of which might appear rather intriguing and unexpected at first sight, is best provided by an analysis of its effects on certain general theorems of Helmholtz and Kelvin relating vorticity. The destabilizing part played by viscosity through its introduction into nonviscous slow and steady problems in the presence of rotation which otherwise would remain stable in the absence of viscosity, and the stabilizing role of rotation through its introduction into nonviscous slow and steady problems which otherwise might remain unstable in the absence of rotation are consequences which have a common origin that lies in the Taylor-Proudman theorem. There is another important factor to remember when an externally imposed rotation is present in a hydrodynamic problem: namely, that it imparts to the fluid certain properties of elasticity which enable the fluid to transmit disturbances by new modes of wave propagation. For this reason, overstabilty as a means of developing instability may be anticipated and this may, under certain circumstances, play a crucial role in the problem.

Motivated by these considerations the present paper investigates the effect of rotation on the instability of double-diffusive convection problem coupled with cross-diffusions in viscoelastic fluid and derives some general qualitative results concerning the stability of oscillatory motions and limitations on the oscillatory motions of growing amplitude. The results for the double- diffusive convection problems with or without the individual consideration of Dufour and Soret effects follow as a consequence.

2. MATHEMATICAL FORMULATION OF THE PROBLEM

The relevant governing equations and boundary conditions of double – diffusive rotatory convection coupled with cross – diffusions in Rivlin –Ericksen voscoelastic fluid are:

$$\left(D^2 - a^2\right)\left((D^2 - a^2) - \frac{p}{\sigma}(1-F)\right)w = R_T\, a^2\theta - R_s a^2\phi - TD\zeta \quad , \tag{2.1}$$

$$\left(D^2 - a^2 - p\right)\theta + D_T (D^2 - a^2)\,\phi = -w \ , \tag{2.2}$$

$$\left(D^2 - a^2 - \frac{p}{\tau}\right)\phi + S_T (D^2 - a^2)\,\theta = -\frac{w}{\tau} \ , \tag{2.3}$$

$$\left(D^2 - a^2 - \frac{p}{\sigma}\right)\zeta = -Dw. \tag{2.4}$$

with

$$w = 0 = \theta = \phi = Dw = \zeta \quad \text{at z=0 and z=1} \quad \text{(on rigid boundaries)} \tag{2.5}$$

$$w = 0 = D^2 w = \theta = \phi = D\zeta \quad \text{at z=0 and z=1} \quad \text{(on a dynamical free boundaries)} \tag{2.6}.$$

In (2.1)–(2.6), z is real independent variable such that $0 \le z \le 1$, $D = \frac{d}{dz}$ is differentiation w.r.t z , $a^2 > 0$ is a constant, $\sigma > 0$ is a constant, $\tau > 0$ is a constant, $0 < F < 1$ is a constant, R_T and R_S are positive constants for the Veronis' configuration and negative constant for Stern's configuration, $T > 0$ is a constant, $p = p_r + ip_i$ is complex constant in general such that p_r and p_i are real constants and as a consequence the dependent variables $w(z) = w_r(z) + iw_i(z)$, $\theta(z) = \theta_r(z) + i\theta_i(z)$, $\phi(z) = \phi_r(z) + i\phi_i(z)$ and $\zeta(z) = \zeta_r(z) + i\zeta_i(z)$ are complex valued functions(and their real and imaginary parts are real valued). The meanings of symbols from the physical point of view are as follows; z is the vertical coordinate, d/dz is differentiation along the vertical direction, a^2 is square of horizontal wave number, $\sigma = \frac{\upsilon}{\kappa}$ is the thermal Prandtl number, $\tau = \frac{\eta_1}{\kappa}$ is the Lewis number, $F = \frac{\upsilon_0}{d^2}$ is the viscoelastic parameter, $R_T = \frac{g\alpha\beta_1 d^4}{\kappa\upsilon}$ is the thermal Rayleigh number, $R_S = \frac{g\alpha\beta_2 d^4}{\kappa\upsilon}$ is the concentration Rayleigh number, $T = \frac{4\Omega^2 d^4}{\nu^2}$ is the Taylor number , $D_T = \frac{\beta_2 D_f}{\beta_1 \kappa}$ is the Dufour number, $S_T = \frac{\beta_1 S_f}{\beta_2 \eta_1}$ is the Soret number, ϕ is the concentration, θ is the temperature, p is the complex growth rate, w is the vertical velocity and ζ is the vertical vorticity.

3. THE LINEAR TRANSFORMATION AND MATHEMATICAL ANALYSIS

The nature of the system (2.1)-(2.4) is clearly qualitatively different from those of double-diffusive convection problems ($D_T = 0 = S_T$) as now we have coupling between all the three eigen- functions $w, \theta, and\ \phi$ in all the three equations. Consequently, they behave nastily and obstruct any attempt for the elegant extension of the earlier results for the double-diffusive convection problems to the present generalized set up. The nasty behaviour of these equations is mollified by the linear transformations given by:

$$\begin{aligned}
\tilde{w} &= (S_T + B)\, w \\
\tilde{\theta} &= E\theta + F\phi \\
\tilde{\phi} &= S_T\theta + B\phi \\
\tilde{\zeta} &= (S_T + B)\, \zeta
\end{aligned} \tag{3.1}$$

where

$$B = -\frac{1}{\tau}A, \qquad E = \frac{S_T + B}{D_T + A}A, \qquad F = \frac{S_T + B}{D_T + A}D_T$$

and A is a positive root of the equation

$$A^2 + (\tau - 1)A - \tau\, S_T D_T = 0 .$$

The system of equations (2.1)-(2.4) together with boundary conditions (2.5)-(2.6), upon using the transformations (3.1) assumes the following form:

$$\left(D^2 - a^2\right)\left((D^2 - a^2) - \frac{p}{\sigma}(1-F)\right)w = R_T{}' \, a^2\theta - R_S{}' \, a^2\phi + TD\zeta \quad , \tag{3.2}$$

$$\left(k_1\left(D^2 - a^2\right) - p\right)\theta = -w , \tag{3.3}$$

$$\left(k_2(D^2 - a^2) - \frac{p}{\tau}\right)\phi = -\frac{w}{\tau} \quad , \tag{3.4}$$

$$\left(D^2 - a^2 - \frac{p}{\sigma}\right)\zeta = -Dw , \tag{3.5}$$

with

$$w = 0 = Dw = \theta = \phi = \zeta \qquad \text{at z =0 and z = 1} \tag{3.6}$$

or $$w = 0 = Dw = \theta = \phi = \zeta \qquad \text{at z =0 and z = 1} \tag{3.7}$$

or

$$\left.\begin{array}{l} w = 0 = Dw = \theta = \phi = \zeta \quad at\ z = 0 \\ w = 0 = D^2 w = \theta = \phi = D\zeta \ at\ z = 1 \end{array}\right\} \tag{3.8}$$

or

$$\left.\begin{array}{l} w = 0 = D^2 w = \theta = \phi = D\zeta \quad at\ z = 0 \\ w = 0 = Dw = \theta = \phi = \zeta \qquad at\ z = 1 \end{array}\right\}, \tag{3.9}$$

where

$$k_1 = 1 + \frac{\tau\, D_T S_T}{A}, \; k_2 = 1 - \frac{S_T D_T}{A} \, are \;\; positive\; cons\tan ts$$

$$and \;\; R_T' = \frac{(D_T + A)(R_T B + R_S S_T)}{BA - S_T D_T}, \; R_S' = \frac{(S_T + B)(R_S A + R_T D_{T)}}{BA - S_T D_T}$$

are respectively the modified thermal Rayleigh number and the modified concentration Rayleigh number.

The sign tilde has been omitted for simplicity.

The system (3.2)-(3.5) together with either of the boundary conditions (3.6)-(3.9) constitutes a characteristics value problem for p for given values of the other parameters namely, $R_T', R_S', a^2, \sigma, \tau, T$ and a given state of the system is stable, neutral or unstable according as p_r the real part of p, is negative, zero or positive. Further, if $p_r = 0 \Rightarrow p_i = 0$ for all wave numbers a^2, then the principal of exchange of stabilities (PES) is valid otherwise we have overstability at least when instability sets in as certain modes.

We now prove the following theorems:

Theorem 1: If (p, w, θ, ϕ, ζ), p = p_r + ip_i, $p_i \neq 0$, $F < 1$ is a non-trivial solution of (3.2) – (3.5) together with either of the boundary conditions (3.6)-(3.9) with, $R_T' > 0$ $R_S' > 0$ and $M \leq 1$ then $p_r < 0$, where

$$M = \frac{4R'_T}{27\pi^4(1+\lambda\langle 1-F \rangle)\, k_1},$$

$$\lambda = \min\left\{\frac{\tau\, k_2}{\sigma}, 1\right\}.$$

Proof: Multiplying (3.2) by w*(the complex conjugate of w) and integrating the resulting equation over the vertical range of z, we get

$$\int_0^1 w^*(D^2-a^2)\,(D^2-a^2-\frac{p}{\sigma}\langle\ 1-F\ \rangle)\,w\,dz = R'_T a^2 \int_0^1 \theta\, w^*\,dz - R'_S a^2 \int_0^1 \phi\, w^*\,dz + T\int_0^1 w^* D\zeta \tag{3.10}$$

Taking the complex conjugate of (3.3) and (3.4) and using the resulting equations in (3.10), we get

$$\int_0^1 w^*(D^2-a^2)\,(D^2-a^2-\frac{p}{\sigma}\langle 1-F\ \rangle)\,w\,dz = -R'_T a^2 \int_0^1 \theta\left[k_1(D^2-a^2)-p^*\right]\theta^*\,dz$$

$$+R'_S a^2 \tau \int_0^1 \phi\left[k_2(D^2-a^2)-\frac{p^*}{\tau}\right]\phi^*\,dz - \mathrm{T}\int_0^1 \left\{\zeta\left(D^2-a^2-\frac{p}{\sigma}\right)\zeta^*\right\}dz. \tag{3.11}$$

Integrating (3.11) by parts a suitable number of times, using either of the boundary conditions (3.6)-(3.9) and one of the following inequalities

$$\int_0^1 \psi^* D^{2n}\psi\,dz = (-1)^n 1\int_0^1 \left|D^{2n}\psi\right|^2 dz, \tag{3.12}$$

where,

$\psi = \theta = \phi,$ for n = 0, 1 and $\psi = w,$ for n = 0, 1, 2,

we have

$$\int_0^1 \left(\left|D^2 w\right|^2 + 2a^2|Dw|^2 + a^4|w|^2\right)dz + \frac{p(1-F)}{\sigma}\int_0^1 \left(|Dw|^2 + a^2|w|^2\right)dz$$

$$= R'_T a^2 \int_0^1 [k_1\left(|D\theta|^2 + a^2|\theta|^2\right) + p^*|\theta|^2\ ]dz - R'_S a^2\,\tau \int_0^1 [k_2(\ |D\phi|^2 + a^2|\phi|^2) + \frac{p^*}{\tau}|\phi|^2\ ]dz$$

$$+T\int_0^1 \left(|D\zeta|^2 + a^2 + \frac{p^*}{\sigma}|\zeta|^2\right)dz. \tag{3.13}$$

Equating the real and imaginary parts of (3.13) equal to zero and using $p_i \neq 0$, we get

$$\int_0^1 \left(\left|D^2 w\right|^2 + 2a^2 |Dw|^2 + a^4 |w|^2 \right) dz + \frac{p_r(1-F)}{\sigma} \int_0^1 \left(|Dw|^2 + a^2 |w|^2 \right) dz$$

$$-R_T' a^2 \int_0^1 [k_1 \left(|D\theta|^2 + a^2 |\theta|^2 \right) + p_r |\theta|^2 \;] dz - R_S' a^2 \tau \int_0^1 [k_2 (\; |D\phi|^2 + a^2 |\phi|^2) + \frac{p_r}{\tau} |\phi|^2 \;] dz$$

$$+ \mathrm{T} \int_0^1 \left(\left| D\zeta \; \right|^2 + a^2 + \frac{p_r}{\sigma} |\zeta|^2 \right) \mathrm{dz} \tag{3.14}$$

and

$$\frac{(1-F)}{\sigma} \int_0^1 (|Dw|^2 + a^2 |w|^2) dz + R_T' a^2 \int_0^1 |\theta|^2 dz - R_S' a^2 \int_0^1 |\phi|^2 dz - \frac{T}{\sigma} \int_0^1 |\zeta|^2 = 0 \tag{3.15}$$

If permissible, let $p_r \geq 0$.

Now, multiplying (3.15) by p_r and adding the resulting equation to (3.14), we have

$$\int_0^1 \left(\left|D^2 w\right|^2 + 2a^2 |Dw|^2 + a^4 |w|^2 \right) dz$$

$$-R_T' a^2 \int_0^1 [k_1 \left(|D\theta|^2 + a^2 |\theta|^2 \right) \;] dz + R_S' a^2 \tau \int_0^1 [k_2 (\; |D\phi|^2 + a^2 |\phi|^2) \;] dz + \frac{2 p_r (1-F)}{\sigma} \int_0^1 |Dw|^2 + a^2 |w|^2) dz$$

$$+\mathrm{T} \int_0^1 \left(|D\zeta|^2 + a^2 |\zeta|^2 \right) \mathrm{dz=0} \tag{3.16}$$

Equation (3.13) implies that

$$\frac{(1-F)}{\sigma} \int_0^1 (|Dw|^2 + a^2 |w|^2) dz - \frac{T}{\sigma} \int_0^1 |\zeta|^2 dz < R_S' a^2 \int_0^1 |\phi|^2 dz \tag{3.17}$$

or

$$\frac{(1-F)}{\sigma} \int_0^1 (|Dw|^2 + a^2 |w|^2) dz \; - R_S' a^2 \int_0^1 |\phi|^2 dz \leq \frac{T}{\sigma} \int_0^1 |\zeta|^2 dz\,. \tag{3.18}$$

Since $w, \theta, \phi \; and \; \zeta$ vanish at z = 0 and z = 1, therefore Rayliegh-Ritz inequality [19] yields

$$\int_0^1 |Dw|^2 dz \geq \pi^2 \int_0^1 |w|^2 dz \tag{3.19}$$

$$\int_0^1 |D\theta|^2 dz \geq \pi^2 \int_0^1 |\theta|^2 dz \tag{3.20}$$

$$\int_0^1 |D\phi|^2 dz \geq \pi^2 \int_0^1 |\phi|^2 dz \tag{3.21}$$

$$\int_0^1 |D\zeta|^2 \, dz \geq \pi^2 \int_0^1 |\zeta|^2 \, dz \tag{3.22}$$

We note that when both the bounding surfaces are dynamically free, then the resulting eigen value problem described by (3.2) - (3.5) together with the boundary conditions (3.6) - (3.9) can be exactly solved with

$$\zeta = \frac{Q\pi \cos \pi z}{\left(\pi^2 + a^2 + \frac{p}{\sigma}\right)} \tag{3.23}$$

where Q is an arbitrary constant and therefore

$$\int_0^1 |D\zeta|^2 dz = \pi^2 \int_0^1 |\zeta|^2 dz. \tag{3.24}$$

Thus, from inequality (3.23) and (3.24), we have

$$\left(\pi^2 + a^2\right)\int_0^1 |\zeta|^2 dz \leq \int_0^1 \left(|D\zeta|^2 + a^2|\zeta|^2\right) dz. \tag{3.25}$$

Using inequality (3.19), inequalities (3.17) and (3.18) yield

$$\frac{(\pi^2 + a^2)(1-F)}{\sigma}\int_0^1 |w|^2 dz - \frac{T}{\sigma}\int_0^1 |\zeta|^2 dz \leq R_S' a^2 \int_0^1 |\phi|^2 dz$$

or (3.26)

$$\frac{(\pi^2 + a^2)(1-F)}{\sigma}\int_0^1 |w|^2 dz - R_S' a^2 \int_0^1 |\phi|^2 dz \leq \frac{T}{\sigma}\int_0^1 |\zeta|^2 dz \tag{3.27}$$

Further, utilizing Schwartz inequality, we have

$$\int_0^1 \left(|w^2|\right)^{\frac{1}{2}} \int_0^1 \left(|Dw|^2\right)^{\frac{1}{2}} \geq -\int_0^1 |w^* D^2 w| = \int_0^1 |Dw|^2 \geq \pi^2 \int_0^1 |w|^2$$

(using (3.19))

which on simplification yields

$$\int_0^1 \left(|D^2 w|^2\right) \geq \pi^4 \int_0^1 |w|^2 \tag{3.28}$$

Inequality (3.19) together with inequality (3.28) yields

$$\int_0^1 \left(|D^2 w|^2 + 2a^2|Dw|^2 + a^4|w|^2\right) dz \geq \left(\pi^2 + a^2\right)^2 \int_0^1 |w|^2 dz. \tag{3.29}$$

Multiplying (3.3) by its complex conjugate and integrating the resulting equation over the vertical range of z, we get

$$\int_0^1 \left[\left(k_1\left(D^2 - a^2\right) - p\right)\theta\left(k_1\left(D^2 - a^2\right) - p^*\right)\theta^*\right] dz = \int_0^1 ww^* dz.$$

Integrating the above equation by parts an appropriate number of times and using either of the boundary conditions, we get

$$\int_0^1 \left|k^2{}_1\left(D^2-a^2\right)\theta\right|^2 + 2p_r k\int_0^1 \left(\left|(D\theta)\right|^2 + a^2|\theta|^2\right)dz + |p|^2\int_0^1 |\theta|^2 dz = \int_0^1 |w|^2 dz. \tag{3.30}$$

Since $p_r \geq 0$, therefore from (3.30), we have

$$k^2{}_1\int_0^1 \left|\left(D^2-a^2\right)\theta\right|^2 dz \leq \int_0^1 |w|^2 dz \tag{3.31}$$

Also emulating the derivation of inequalities (3.28) and (3.29) we derive the following inequality

$$\int_0^1 \left|\left(D^2-a^2\right)\theta\right|^2 dz = \int_0^1 \left|D^2\theta\right|^2 + 2a^2|D\theta|^2 + a^4|\theta|^2 dz \geq \left(\pi^2+a^2\right)^2\int_0^1 |\theta|^2 dz \tag{3.32}$$

Combining inequalities (3.31) and (3.32), we get

$$\int_0^1 |w|^2 dz \geq \left(\pi^2+a^2\right)^2 k_1{}^2\int_0^1 |\theta|^2 dz \tag{3.33}$$

Also, we know

$$\int_0^1 |w|^2 = \int_0^1 \left(\left|w^2\right|\right)^{\frac{1}{2}}\int_0^1 \left(|w|^2\right)^{\frac{1}{2}}$$

which upon using inequalities (3.31) and (3.32) yields

$$\int_0^1 |w|^2 dz \geq k_1{}^2\left(\pi^2+a^2\right)\left\{\int_0^1 \left|\left(D^2-a^2\right)\theta\right|^2 dz\right\}^{\frac{1}{2}}\left\{\int_0^1 |\theta|^2\right\}^{\frac{1}{2}} dz \tag{3.34}$$

$$\geq k_1{}^2\left(\pi^2+a^2\right)\left|-\int_0^1 \theta^*\left(D^2-a^2\right)\theta\, dz\right| \qquad \text{(Using Schwartz inequality)}$$

$$\geq \left(\pi^2+a^2\right)k_1{}^2\int_0^1 \left\{|D\theta|^2 + a^2|\theta|^2\right\}dz \tag{3.35}$$

Since, $p_r \geq 0$, equation (3.16) together with inequalities (3.29),(3.35) and (3.25)-(3.27) yields

$$\left(\pi^2+a^2\right)^2\left(1+\frac{\tau k_2(1-F)}{\sigma}\right)\int_0^1 |w|^2 dz + T\left(1-\frac{\tau k_2}{\sigma}\right)\left(\pi^2+a^2\right)\int_0^1 |\zeta|^2 dz < \frac{R_T' a^2}{k_1\left(\pi^2+a^2\right)}\int_0^1 |w|^2 dz \tag{3.36}$$

or

$$(2-F)\left(\pi^2+a^2\right)^2\int_0^1 |w|^2 dz + R_S' a^2\left(\tau k_2-\sigma\right)\left(\pi^2+a^2\right)\int_0^1 |\phi|^2 dz < \frac{R_T' a^2}{k_1\left(\pi^2+a^2\right)}\int_0^1 |w|^2 dz \tag{3.37}$$

Now, if $\lambda = \min\left\{\frac{\tau k_2}{\sigma}, 1\right\}$ then it follows from either of the inequalities (3.36) and (3.37) that

$$\frac{k_1\left(\pi^2+a^2\right)^3}{a^2}\left(1+\lambda\langle 1-F\rangle\right)\int_0^1 |w|^2 dz < R_T'\int_0^1 |w|^2 dz \tag{3.38}$$

Since, minimum value of $\frac{\left(\pi^2+a^2\right)^3}{a^2}$ with respect a^2 is $\frac{27\pi^4}{4}$, it follows from inequality (3.38) that

$$\{ k_1 \frac{27\pi^4}{4}\left(1+\lambda\langle 1-F \rangle\right) - R'_T \}\int_0^1 |w|^2 dz < 0$$

which can be written as

$$(1-M)\int_0^1 |w|^2 dz < 0 , \qquad where\ M = \frac{4R'_T}{27\pi^4\left(1+\lambda\langle 1-F \rangle\right)k_1} . \tag{3.39}$$

Inequality (3.39) is clearly incompatible with the hypothesis of the theorem. Hence, we must have

$p_r < 0$

This completes the proof of the theorem.

Theorem 1 in the terminology of hydrodynamic stability implies that for the problem under consideration arbitrary oscillatory perturbations of growing amplitude are not allowed if $M \le 1$.

Corollary1. For the rotatory double-diffusive convection $(D_T = 0 = S_T)$, if $R_T > 0, R_S > 0, p_i \ne 0, F < 1\ and\ M' \le 1,\ then\ p_r < 0$

$$where\ M' = \frac{4R_T}{27\pi^4\left(1+\lambda_1\langle 1-F \rangle\right)},\quad \lambda_1 = \min\left\{\frac{\tau}{\sigma},1\right\}.$$

Corollary 2. For the rotatory Soret-driven double-diffusive convection ($D_T = 0$) if $R_T > 0, R_S > 0, p_i \ne 0, F < 1\ and\ M'' \le 1,\ then\ p_r < 0$

$$where\ M'' = \frac{4\{ R_T - \frac{\tau R_T S_T}{(1-\tau)} \}}{27\pi^4\left(1+\lambda_1\langle 1-F \rangle\right)} ,\quad \lambda_1 = \min\left\{\frac{\tau}{\sigma},1\right\}.$$

Corollary 3. For the rotatory Dufour-driven double-diffusive convection ($S_T = 0$) if $R_T > 0, R_S > 0, p_i \ne 0\ and\ M''' \le 1,\ then\ p_r < 0$

$$where\ M''' = \frac{4R_T\{1+\frac{D_T}{(1-\tau)} \}}{27\pi^4\left(1+\lambda_1\langle 1-F \rangle\right)} ,\quad \lambda_1 = \min\left\{\frac{\tau}{\sigma},1\right\}.$$

Remark: We note that if $M > 1$, then oscillatory modes of growing amplitudes can exist. Further, keeping in view Theorem 1 and the fact that the growth rate p has been intentionally avoided in the proof of this theorem, one strongly feels that a bound for the growth rate of oscillatory motions of growing amplitude in terms of the parameters of the problem specifically involving $(M-1)$ as factor must be derivable. The subsequent theorem justifies our intuition.

Theorem 2. If (p, w, θ, ϕ, ζ), p = p_r + ip_i, $p_i \ne 0, F > 1$ is a non-trivial solution of (3.2) – (3.5) together with either of the boundary conditions (3.6)-(3.9) with $R'_T > 0\ R'_S > 0$ then

$$|p| < \frac{R'_T\sqrt{M^2-1}}{4\pi^2(1+\lambda)},$$

where $M = \dfrac{4R'_T}{27\pi^4(1+\lambda)\,k_1}$ and $\lambda = \min\left\{\dfrac{\tau k_2}{\sigma},1\right\}$.

Proof. Proceeding exactly as in theorem1, utilizing the fact that $p_r \geq 0$, we have from (3.16)

$$\int_0^1 \left(\left|D^2 w\right|^2 + 2a^2 \left|Dw\right|^2 + a^4 \left|w\right|^2 \right) dz + R_S' a^2 \tau \int_0^1 [k_2 (\left|D\phi\right|^2 + a^2 \left|\phi\right|^2)]dz$$

$$+ \mathrm{T} \int_0^1 \left(\left|D\zeta\right|^2 + a^2 \left|\zeta\right|^2 \right) < R_T' a^2 \int_0^1 [k_1 \left(\left|D\theta\right|^2 + a^2 \left|\theta\right|^2 \right)]dz \tag{3.40}$$

From (3.30) it follows that

$$\int_0^1 \left| k^2{}_1 \left(D^2 - a^2\right)\theta \right|^2 + \left|p\right|^2 \int_0^1 \left|\theta\right|^2 dz \leq \int_0^1 \left|w\right|^2 dz \tag{3.41}$$

Using inequality (3.30) in inequality (3.31), we get

$$\int_0^1 \left|w\right|^2 dz \geq \left(\pi^2 + a^2\right)^2 k_1{}^2 \left\{ 1 + \frac{\left|p\right|^2}{k_1{}^2 \left(\pi^2 + a^2\right)^2} \right\} \int_0^1 \left|\theta\right|^2 dz \tag{3.42}$$

Now,

$$\int_0^1 \left\{ \left|D\theta\right|^2 + a^2 \left|\theta\right|^2 \right\} dz = \left| - \int_0^1 \theta^* \left(D^2 - a^2\right)\theta dz \right|$$

$$\leq \int_0^1 \left|\theta\right| \left|\left(D^2 - a^2\right)\theta\right| dz$$

$$\leq \left\{ \int_0^1 \left|\left(D^2 - a^2 \right)\theta\right|^2 dz \right\}^{\frac{1}{2}} \left\{ \int_0^1 \left|\theta\right|^2 \right\}^{\frac{1}{2}}$$

(Using Schwartz inequality)

$$\leq \frac{1}{k_1{}^2 \left(\pi^2 + a^2\right)^2} \left\{ 1 + \frac{\left|p\right|^2}{k_1{}^2 \left(\pi^2 + a^2\right)^2} \right\}^{\frac{-1}{2}} \left\{ \int_0^1 \left|w\right|^2 dz \right\} \tag{3.43}$$

(using inequalities (3.41) and (3.42))

Making use of inequalities (3.17) or (3.18), (3.21), (3.43) in inequality (3.40), we have

$$\left(\pi^2 + a^2\right)^2 \left(1 + \frac{\tau k_2 \langle 1 - F \rangle}{\sigma} \right) \int_0^1 \left|w\right|^2 dz + T\left(\pi^2 + a^2\right) \left(1 - \frac{\tau k_2}{\sigma} \right) \int_0^1 \left|\zeta\right|^2 dz \leq \frac{R_T' a^2}{k_1 \left(\pi^2 + a^2\right) \left[1 + \frac{\left|p\right|^2}{k_1{}^2 \left(\pi^2 + a^2\right)^2} \right]^{\frac{1}{2}}} \int_0^1 \left|w\right|^2 dz \tag{3.44}$$

or

$$\langle\ 2-F\ \rangle\left(\pi^2+a^2\right)\int_0^1|w|^2\,dz+R_S' a^2\left(\pi^2+a^2\right)\left(\tau k_2-\sigma\right)\int_0^1|\phi|^2\,dz$$

$$\leq\quad \frac{R_T' a^2}{k_1\left(\pi^2+a^2\right)\left[1+\dfrac{|p|^2}{k_1^2\left(\pi^2+a^2\right)^2}\right]^{\frac{1}{2}}} \qquad (3.45)$$

Now, let

$$\lambda=\min\left\{\frac{\tau k_2}{\sigma},1\right\}$$

then it follows from either of the inequalities (3.44) and (3.45) that

$$k_1\left(1+\lambda\langle 1-F\ \rangle\right)\frac{\left(\pi^2+a^2\right)^3}{a^2}\int_0^1|w|^2<R_T'\left[1+\frac{|p|^2}{k_1^2\left(\pi^2+a^2\right)^2}\right]^{-\frac{1}{2}}\int_0^1|w|^2\,dz\,. \qquad (3.46)$$

Since, minimum value of $\dfrac{\left(\pi^2+a^2\right)^3}{a^2}$ with respect a^2 is $\dfrac{27\pi^4}{4}$, it follows from inequality (3.46) that

$$\left[\frac{27\pi^4\left(1+\lambda\langle 1-F\ \rangle\right)k_1}{4}-\frac{R_T'}{\left\{1+\dfrac{|p|^2}{k_1^2\left(\pi^2+a^2\right)^2}\right\}^{1/2}}\right]\int_0^1|w|^2\,dz<0\ . \qquad (3.47)$$

Inequality (3.47) clearly implies that

$$|p|<k_1\left(\pi^2+a^2\right)\sqrt{M^2-1}\ , \qquad (3.48)$$

where

$$M=\frac{4R_T'}{27\pi^4\left(1+\lambda\langle 1-F\ \rangle\right)k_1}.$$

Now, from inequality (3.46), we can have

$$\frac{\left(\pi^2+a^2\right)^2}{a^2}k_1\left(1+\lambda\langle 1-F\ \rangle\right)<R_T' \qquad (3.49)$$

Since, minimum value of $\dfrac{\left(\pi^2+a^2\right)^2}{a^2}$ with respect to a^2 is $4\pi^2$, therefore inequality (3.49) yields

$$\left(\pi^2+a^2\right)<\frac{R_T'}{4\pi^2(1+\lambda\langle 1-F\ \rangle)\,k_1}. \qquad (3.50)$$

Using inequality (3.50), inequality (3.48) yields

$$|p| < \frac{R_T'}{4\pi^2\left(1+\lambda\langle 1-F\rangle\right)}\sqrt{M^2-1}$$

This completes the proof of the theorem.

Theorem 2 from the point of view of hydrodynamic stability theory may be stated as:

The complex growth rate $p = p_r + ip_i$ of an arbitrary oscillatory ($p_i \neq 0$) perturbation of growing amplitude ($p_r \geq 0$) for the problem under consideration lies inside a semi- circle in the right-half of the $p_r p_i$ - plane whose centre is at the origin and whose radius is

$$\frac{R_T'\sqrt{M^2-1}}{4\pi^2\left(1+\lambda\langle 1-F\rangle\right)}.$$

Corollary 4. For the rotatory double-diffusive convection $(D_T = 0 = S_T)$, the complex growth rate $p = p_r + ip_i$ of an arbitrary oscillatory ($p_i \neq 0$) perturbation of growing amplitude ($p_r \geq 0$) lies inside a semi-circle in the right-half of the $p_r p_i$ - plane whose centre is at the origin and whose radius is

$$\frac{R_T\sqrt{M'^2-1}}{4\pi^2\left(1+\lambda\langle 1-F\rangle\right)},$$

$$\text{where } M' = \frac{4R_T}{27\pi^4\left(1+\lambda_1\langle 1-F\rangle\right)}, \quad \lambda_1 = \min\left\{\frac{\tau}{\sigma}, 1\right\}.$$

Corollary 5. For the rotatory Soret -driven double-diffusive convection ($D_T = 0$), the complex growth rate $p = p_r + ip_i$ of an arbitrary oscillatory ($p_i \neq 0$) perturbation of growing amplitude ($p_r \geq 0$) lies inside a semi-circle in the right-half of the $p_r p_i$ - plane whose centre is at the origin and whose radius is

$$\frac{\left\{R_T - \frac{\tau R_T S_T}{(1-\tau)}\right\}}{4\pi^2\left(1+\lambda_1\langle 1-F\rangle\right)}\sqrt{M''^2-1} \ ,$$

where

$$M'' = \frac{4\left\{R_T - \frac{\tau R_T S_T}{(1-\tau)}\right\}}{27\pi^4\left(1+\lambda_1\langle 1-F\rangle\right)}, \quad \lambda_1 = \min\left\{\frac{\tau}{\sigma}, 1\right\}.$$

Corollary 6. For the rotatory Dufour-driven double-diffusive convection ($S_T = 0$) the complex growth rate $p = p_r + ip_i$ of an arbitrary oscillatory ($p_i \neq 0$) perturbation of growing amplitude ($p_r \geq 0$) lies inside a semi-circle in the right-half of the $p_r p_i$ - plane whose centre is at the origin and whose radius is

$$\frac{R_T\left(1+\frac{D_T}{(1-\tau)}\right)\sqrt{M'''^2-1}}{4\pi^2\left(1+\lambda_1\langle 1-F\rangle\right)},$$

$$where\ M''' = \frac{4R_T\{1+\dfrac{D_T}{(1-\tau)}\ \}}{27\pi^4\left(1+\lambda_1\langle 1-F\ \rangle\right)}, \lambda_1 = \min\left\{\frac{\tau}{\sigma},1\right\}.$$

CONCLUSION

The effect of a uniform vertical rotation on the physical problem of double-diffusive convection coupled with cross-diffusions in viscoelastic fluid is considered. The principal conclusions from the analysis of this study are:

i) In the terminology of hydrodynamic stability, for the problem Double-Diffusive Rotatory convection coupled with cross-diffusions in viscoelastic fluid, an arbitrary oscillatory perturbations of growing amplitude are not allowed if $M \leq 1$.

ii) The complex growth rate $p = p_r + ip_i$ of an arbitrary oscillatory ($p_i \neq 0$) perturbation of growing amplitude ($p_r \geq 0$) for the problem under consideration lies inside a semi- circle in the right-half of the $p_r p_i$ - plane whose centre is at the origin and whose radius is

$$\frac{R'_T\sqrt{M^2-1}}{4\pi^2\left(1+\lambda\langle 1-F\ \rangle\right)}.$$

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