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Double-Diffusive convection in Walters' B' elastico-viscous fluid in the presence of rotation and magnetic field

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ABSTRACT

The effect of magnetic field and rotation on thermosolutal convection in Walters' B' elastico-viscous fluid is considered. For the case of stationary convection, the magnetic field, rotation and solute parameter postpone the onset of convection. The magnetic field, rotation and solute parameter introduce oscillatory modes in the system, which were non-existent in their absence. The case of overstability is also considered wherein the sufficient conditions for the non-existence of overstability are obtained.

Key words: Thermosolutal Instability; Walters' B' Fluid; Viscoelasticity; permeability.

INTRODUCTION

Chandrasekhar [4] has given a detailed account of thermal convection in Newtonian fluid layer in the presence of magnetic field and rotation. Veronis [14] has investigated the problem of thermohaline convection in a layer of fluid heated from below and subjected to a stable salinity gradient. The buoyancy forces can arise not only from density differences due to variations in temperature but also from those due to variations in solute concentration. Thermosolutal convection problems arise in oceanography, limnology and engineering. Examples of particular interest are provided by ponds built to trap solar heat (Tabor and Matz [13]) and some Antarctic lakes (Shirtcliffe [12]). Bhatia and Steiner [2] have studied the problem of thermal instability of a viscoelastic (Maxwell) fluid in the presence of rotation and have found that the rotation has a destabilizing influence in contrast to the stabilizing effect on an ordinary (Newtonian) fluid. Bhatia and Steiner [3] have also studied the thermal instability of a Maxwellian viscoelastic fluid in the presence of magnetic field while the thermal convection in Oldroydian viscoelastic fluid in hydromagnetics has been considered by Sharma [9].

There are many elastico-viscous fluids that cannot be characterized by Maxwell's constitutive relations or Oldroyd's constitutive relations. Two such classes of elastico-viscous fluids are

Rivlin-Ericksen fluid [8] and Walters' B' fluid [15]. Walters has proposed the constitutive equations of such elastico-viscous fluids. Walters [16] reported that the mixture of polymethyl methacrylate and pyridine at 25˚C containing 30.5g of polymer per litre behaves very nearly as the Waltres' B' elatico-viscous fluid. Rivlin-Ericksen [8] has proposed a theoretical model for such another elastico-viscous fluid. Such and other polymers are used in agriculture, communication appliances and in bio-medical applications. Sharma and Kumar [11] have studied the stability of two superposed Walters' B' elastico-viscous liquids. A study on thermal convection in Rivlin-Ericksen elastico-viscous fluid in porous medium in hydromagnetics has been made by Sharma and Kango [10]. Rana and Kango [6] have studied the thermal instability of compressible Walters' (model B') elastico-viscous fluid permeated with suspended in porous medium.

Gupta et al [5] have studied the effect of γ-irradiation on thermal stability of CR-39 polymer whereas the effect of thickness of the porous materials on the peristaltic pumping when the tube wall is provided with non-erodible porous lining has been investigated by Reddy et al [7]. The above studies were helpful in studying porous materials and thermal stability.

Keeping in mind the growing importance of non-Newtonian fluids in modern technology, industry, chemical technology and dynamics of geophysical fluids and considering the conflicted tendencies of magnetic field and rotation while acting together, our interest, in the present paper is to study the Double-Diffusive convection in Walters' B' elastico-viscous fluid in the presence of rotation and magnetic field.

EFFECT OF MAGNETIC FIELD

2.1 Perturbation Equations

Consider an infinite layer of Waltres' B' elastico-viscous electrically conducting fluid confined between the planes $z=0$ and $z=d$, acted on by a uniform vertical magnetic field $\vec{H}=(0,0,H)$ and gravity force $\vec{g}=(0,0,-g)$. This layer is heated and soluted from below such that a steady adverse temperature gradient β $(= \mathrm{dT/dz})$ and a solute concentration gradient β' $(= \mathrm{dC/dz})$ are maintained.

The hydromagnetic equations (Chandrasekhar [4], Veronis [14], Walters' [15]), relevant to the problem, following Boussinesq approximation, are

$$\left[\frac{\partial \vec{q}}{\partial t}+(q.\nabla)\vec{q}\right]=-\frac{1}{\rho_0}\nabla p+\vec{g}\left(1+\frac{\delta\rho}{\rho_0}\right)+\frac{\mu_e}{4\pi\rho_0}(\nabla\times\vec{H})\times\vec{H}+\left(\nu-\nu'\frac{\partial}{\partial t}\right)\nabla^2\vec{q}, \qquad (2.1)$$

$$\nabla.\vec{q}=0, \qquad (2.2)$$

$$\nabla.\vec{H}=0, \qquad (2.3)$$

$$\frac{\partial \vec{H}}{\partial t}=(\vec{H}.\nabla)q+\eta\nabla^2\vec{H}, \qquad (2.4)$$

$$\frac{\partial T}{\partial t}+(\vec{q}.\nabla)T=\kappa\nabla^2 T, \qquad (2.5)$$

$$\frac{\partial C}{\partial t}+(\vec{q}.\nabla)C=\kappa'\nabla^2 C, \qquad (2.6)$$

where

$$\rho = \rho_0 \left[1 - \alpha (T - T_0) + \alpha' (C - C_0) \right]. \tag{2.7}$$

Equations (2.1), (2.2), (2.5) and (2.6) express the conservation of momentum, mass, heat and solute respectively. Equations (2.3) and (2.4) are the Maxwell's equations. The equation of state (2.7) contains a thermal coefficient of expansion α and an analogous solute coefficient α'. The suffix zero refers to the values at the reference level $z = 0$. Here $\rho, p, T, C, \mu_e, \vec{q}(u,v,w), \vec{H}(0,0,H)$ and $\vec{g} = (0,0,-g)$ stand for density, pressure, temperature, solute mass concentration, magnetic permeability, velocity, magnetic field and gravitational acceleration respectively. The kinematic viscosity ν, the kinematic viscoelasticity ν', the thermal diffusivity κ, the analogous solute diffusivity κ' and the electrical resistivity η are each assumed to be constant.

The steady state solution is

$$\vec{q} = (0,0,0), T = T_0 - \beta z, C = C_0 - \beta' z, \rho = \rho_0 (1 + \alpha\beta z - \alpha'\beta' z), \tag{2.8}$$

where $\beta = (T_0 - T_1)/d$ and $\beta' = (C_0 - C_1)/d$ are the magnitudes of uniform temperature and concentration gradients and are both positive as temperature and concentration decrease upwards.

Here we use linearized stability theory and normal mode analysis method. Consider a small perturbation on the steady state solution and let $\vec{h}(h_x, h_y, h_z), \vec{q}(u,v,w), \delta\rho, \delta p, \theta$ and γ denote respectively the perturbations in magnetic field, velocity, density, pressure, temperature and solute concentration. The change in density $\delta\rho$, caused by the perturbation θ and γ in temperature and concentration, is given by

$$\delta\rho = -\rho_0 (\alpha\theta - \alpha'\gamma). \tag{2.9}$$

Then the linearized hydromagnetic perturbation equations become

$$\frac{\partial \vec{q}}{\partial t} = -\frac{1}{\rho_0}\nabla \delta p - \vec{g}(\alpha\theta - \alpha'\gamma) + \frac{\mu_e}{4\pi\rho_0}(\nabla \times \vec{h}) \times \vec{H} + \left(\nu - \nu'\frac{\partial}{\partial t}\right)\nabla^2 \vec{q}, \tag{2.10}$$

$$\nabla.\vec{q} = 0, \tag{2.11}$$

$$\nabla.\vec{h} = 0, \tag{2.12}$$

$$\frac{\partial \vec{h}}{\partial t} = (\vec{H}.\nabla)\vec{q} + \eta\nabla^2 \vec{h}, \tag{2.13}$$

$$\frac{\partial \theta}{\partial t} = \beta w + \kappa\nabla^2\theta, \tag{2.14}$$

$$\frac{\partial \gamma}{\partial t} = \beta' w + \kappa'\nabla^2\gamma. \tag{2.15}$$

The boundaries are taken to be free as well as perfect conductors of both heat and solute concentration and the adjoining medium is electrically non-conducting. The case of two free surfaces is a little artificial except in the case of stellar atmospheres. However, this assumption allows us to obtain the analytical solution without affecting the essential features of the problem. The boundary conditions appropriate for the problem are

$$w=\frac{\partial^2 w}{\partial z^2}=0, \theta=0, \gamma=0 \text{ at } z=0 \text{ and } z=d$$

and $\vec{h}$ is continuous with an external vacuum field. (2.16)

Writing equations (9)-(13) in scalar form, using (8) and eliminating $u, v, h_x, h_y, \delta p$ between them, we obtain

$$\frac{\partial}{\partial t}\nabla^2 w - g\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)(\alpha\theta-\alpha'\gamma)-\left(\frac{\mu_e H}{4\pi\rho_0}\right)\frac{\partial}{\partial z}\nabla^2 h_z-\left(\nu-\nu'\frac{\partial}{\partial t}\right)\nabla^4 w=0, \quad (2.17)$$

$$\left(\frac{\partial}{\partial t}-\eta\nabla^2\right)h_z = H\frac{\partial w}{\partial z}, \quad (2.18)$$

$$\left(\frac{\partial}{\partial t}-\kappa\nabla^2\right)\theta=\beta w, \quad (2.19)$$

$$\left(\frac{\partial}{\partial t}-\kappa'\nabla^2\right)\gamma=\beta' w, \quad (2.20)$$

where $\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}$.

2.2 Dispersion Relation and Discussion

Analyzing the disturbances into normal modes, we assume that the perturbation quantities are of the form

$$[w,\theta,h_z,\gamma]=[W(z),\Theta(z),K(z),\Gamma(z)]\exp\left(ik_x x+ik_y y+nt\right), \quad (2.21)$$

where k_x, k_y are wave numbers along the x and y directions, respectively, $k=\sqrt{k_x^{\,2}+k_y^{\,2}}$ is the resultant wave number and n is, in general, a complex constant.

Using expression (2.21), Equations (2.17) – (2.20), in non-dimensional form, become

$$\left(D^2-a^2\right)W+\frac{ga^2d^2}{\nu}(\alpha\Theta-\alpha'\Gamma)-\frac{\mu_e Hd}{4\pi\rho_0\nu}\left(D^2-a^2\right)DK-(1-F\sigma)\left(D^2-a^2\right)^2 W=0, \quad (2.22)$$

$$\left(D^2-a^2-p_2\sigma\right)K=-\left(\frac{Hd}{\eta}\right)DW, \quad (2.23)$$

$$\left(D^2-a^2-p_1\sigma\right)\Theta=-\left(\frac{\beta d^2}{\kappa}\right)W, \quad (2.24)$$

$$\left(D^2-a^2-q\sigma\right)\Gamma=-\left(\frac{\beta' d^2}{\kappa'}\right)W. \quad (2.25)$$

Here $p_1 = \nu / \kappa$ is the Prandtl number, $p_2 = \nu / \eta$ is the magnetic Prandtl number, $F = \nu' / d^2$ is the dimensionless kinematic viscoelasticity, $q = \nu / \kappa'$ is the Schmidt number, $a = kd$, $\sigma = nd^2 / \kappa'$ and we have put
$x^* = x / d, y^* = y / d, z^* = z / d$ and $D = d / dz^*$.

Eliminating Θ, Γ and K between equations (2.22) to (2.25), we obtain

$$\left(D^2 - a^2\right)\left(D^2 - a^2 - p_1\sigma\right)\left(D^2 - a^2 - q\sigma\right)[\sigma\left(D^2 - a^2 - p_2\sigma\right) +$$
$$+QD^2 - \left(1 - F\sigma\right)\left(D^2 - a^2\right)\left(D^2 - a^2 - p_2\sigma\right)]W$$
$$= \left(D^2 - a^2 - p_2\sigma\right)[Ra^2\left(D^2 - a^2 - q\sigma\right) - Sa^2\left(D^2 - a^2 - p_1\sigma\right)]W, \qquad (2.26)$$

where $R = \dfrac{g\alpha\beta d^4}{\nu\kappa}$ is the Rayleigh number, $S = \dfrac{g\alpha'\beta' d^4}{\nu\kappa'}$ is the analogous solute Rayleigh number and $Q = \dfrac{\mu_e H^2 d^2}{4\pi\rho_0 \nu\eta}$ is the Chandrasekhar number.

Now the boundary conditions (2.16) transform to (Chandrasekhar [1981]),

$$W = D^2W = 0, \Theta = D^2\Theta = 0, \Gamma = D^2\Gamma = 0, DK = D^3K = 0, \xi = 0 \text{ at } z^* = 0 \; \& \; z^* = 1, \qquad (2.27)$$

where $\xi = (curl \ \vec{h} \ )_z$ is the z-component of current density.

Using the above boundary conditions, it can be shown that all the even order derivatives of W must vanish for $z = 0$ and $z = 1$ and hence the proper solution of (2.26) characterizing the lowest mode is

$$W = W_0 \sin \pi z, \qquad (2.28)$$

where W_0 is a constant. Substituting (2.28) in (2.26) and letting $a^2 = \pi^2 x$, $R_1 = R / \pi^4$, $S_1 = S / \pi^4, Q_1 = Q / \pi^2, i\sigma_1 = \sigma / \pi^2$, we obtain the dispersion relation

$$R_1 = \frac{(1+x)(1+x+ip_1\sigma_1)}{x}\left[(1+x)\left(1 - iF\sigma_1\pi^2\right) + i\sigma_1\right] +$$
$$+S_1 \frac{(1+x+ip_1\sigma_1)}{(1+x+iq\sigma_1)} + Q_1\left(\frac{1+x}{x}\right)\frac{(1+x+ip_1\sigma_1)}{(1+x+ip_2\sigma_1)}. \qquad (2.29)$$

It is being remembered that σ can be complex. Here we consider the overstable modes and so σ_1 is real in equation (2.29).

For the case of stationary convection, σ = 0 and Equation (2.29) reduces to

$$R_1 = \left(\frac{1+x}{x}\right)\left[(1+x)^2 + Q_1\right] + S_1. \qquad (2.30)$$

We thus find that for stationary convection, the Walters' B' elastico-viscous fluid behaves like an ordinary Newtonian fluid.

From (2.29), it follows that

$$\frac{dR_1}{dQ_1} = \left(\frac{1+x}{x}\right)\frac{(1+x+ip_1\sigma_1)}{(1+x+ip_2\sigma_1)}. \tag{2.31}$$

Rationalizing and equating the real and imaginary parts of (2.31), it follows that

$$\frac{dR_1}{dQ_1} = \frac{1+x}{x}, \tag{2.32}$$

which is always positive. The magnetic field thus has a stabilizing effect on the system. Similarly, it can be shown from equation (2.29) that

$$\frac{dR_1}{dS_1} = +1, \tag{2.33}$$

implying thereby the stabilizing effect of stable solute gradient. Equation (2.33) is identical with that of Aggarwal [1] in which effect of rotation on thermosolutal instability of Walters' (model B') fluid permeated with suspended particles in porous medium has been investigated.

2.3. Stability of the system and oscillatory modes

Here we examine the possibility of oscillatory modes, if any, in the stability problem due to the presence of magnetic field and stable solute gradient. Multiplying Eq. (2.22) by W*, the complex conjugate of W, integrating over the range of z and making use of Equation (2.23)- (2.25) together with the boundary conditions (2.27), we obtain

$$\sigma I_1 - \frac{g\alpha\kappa a^2}{\nu\beta}[I_2 + p_1\sigma^* I_3] + \frac{g\alpha'\kappa' a^2}{\nu\beta'}[I_4 + q\sigma^* I_5] + \left(\frac{\mu_e\eta}{4\pi\rho_0\nu}\right)[I_6 + p_2\sigma^* I_7] + (1 - F\sigma)I_8 = 0, \tag{2.34}$$

where

$$I_1 = \int_0^1 (|DW|^2 + a^2|W|^2)dz, I_2 = \int_0^1 (|D\Theta|^2 + a^2|\Theta|^2)dz, I_3 = \int_0^1 |\Theta|^2 dz,$$

$$I_4 = \int_0^1 (|D\Gamma|^2 + a^2|\Gamma|^2)dz, I_5 = \int_0^1 |\Gamma|^2 dz, I_6 = \int_0^1 (|D^2K|^2 + 2a^2|DK|^2 + a^4|K|^2)dz,$$

$$I_7 = \int_0^1 (|DK|^2 + a^2|K|^2)dz, I_8 = \int_0^1 (|D^2W|^2 + 2a^2|DW|^2 + a^4|W|^2)dz. \tag{2.35}$$

The integrals $I_1, ..., I_8$ are all positive definite. Putting $\sigma = \sigma_r + i\sigma_i$ and equating the real and imaginary parts of equation (2.34), we obtain

$$[I_1 - \frac{g\alpha\kappa a^2}{\nu\beta} p_1 I_3 + \frac{g\alpha'\kappa' a^2}{\nu\beta'} q I_5 + \frac{\mu_e\eta}{4\pi\rho_0\nu} p_2 I_7 - F I_8]\sigma_r$$

$$=-[I_8-\frac{g\alpha\kappa a^2}{\nu\beta}I_2+\frac{g\alpha'\kappa' a^2}{\nu\beta'}I_4+\frac{\mu_e\eta}{4\pi\rho_0\nu}p_2I_6], \qquad (2.36)$$

and

$$[I_1-FI_8+\frac{g\alpha\kappa a^2}{\nu\beta}p_1I_3-\frac{g\alpha'\kappa' a^2}{\nu\beta'}qI_5-\frac{\mu_e\eta}{4\pi\rho_0\nu}p_2I_7]\sigma_i=0. \qquad (2.37)$$

It follows from equation (2.36) that σ_r may be positive or negative which means that the system may be stable or unstable. It is clear from (2.37) that σ_i may be zero or non-zero, meaning that the modes may be non-oscillatory or oscillatory. The oscillatory modes are introduced due to the presence of magnetic field and stable solute gradient, which were non-existent in their absence.

2.4. The Case of Overstability

Here we discuss the possibility of whether instability may occur as overstability. Since we wish to determine the Rayleigh number for the onset of instability via a state of pure oscillations, it suffices to find conditions for which (2.29) will admit of solutions with σ_1 real.

If we equate real and imaginary parts of (2.29) and eliminate R_1 between them, we obtain

$$A'c_1^2+B'c_1+C'=0, \qquad (2.38)$$

where we have put $c_1=\sigma_1^2$, $b=1+x$ and

$$A'=p_2^2q^2b\left(1+p_1-F\pi^2b\right),$$
$$B'=\left(p_2^2+q^2\right)\left(1+p_1-F\pi^2b\right)b^3+Q_1q^2b\left(p_1-p_2\right)+S_1p_2^2\left(b-1\right)\left(p_1-q\right),$$
$$C'=b^5\left(1+p_1-F\pi^2b\right)+S_1b^2\left(b-1\right)\left(p_1-q\right)+Q_1b^3\left(p_1-p_2\right).$$

As σ_1 is real for instability, both the values of $c_1\left(=\sigma_1^2\right)$ must be positive. Equation (2.38) is quadratic in c_1 and does not involve any of its roots to be positive if

$$p_1>p_2,\quad p_1>q\quad \text{and}\quad p_1>F\pi^2b, \qquad (2.39)$$

which imply that

$$\kappa<\eta,\quad \kappa<\kappa'\quad \text{and}\quad \frac{\kappa\nu'}{d^2}\left(\pi^2+k^2d^2\right)<\nu \qquad (2.40)$$

Thus Equations (2.40) are, therefore, the necessary conditions for the non-existence of overstability, the violation of which does not necessarily imply the occurrence of overstability.

EFFECT OF ROTATION

3.1 Perturbation Equations and Dispersion Relation

Here we consider an infinite horizontal layer of Walters' B' elastico-viscous fluid of depth d heated from below and subjected to a stable solute gradient. The fluid is acted on by a uniform rotation $\vec{\Omega}=\left(0,0,\Omega\right)$ and gravity force $\vec{g}=\left(0,0,-g\right)$.

Then the linearized perturbation equations of motion are

$$\frac{\partial \vec{q}}{\partial t} = -\frac{1}{\rho_0}\nabla \delta p + \vec{g}\left(\frac{\delta \rho}{\rho_0}\right) + \left(\nu - \nu' \frac{\partial}{\partial t}\right)\nabla^2 \vec{q} + 2\left(\vec{q} \times \vec{\Omega}\right), \tag{2.41}$$

together with equations (2.11), (2.14) and (2.15).

Assuming $\zeta = \left(curl \vec{q}\right)_z = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$, the z -component of vorticity, of the form

$$\zeta = Z\left(z\right)\exp\left(ik_x + ik_y + nt\right), \tag{2.42}$$

And using expressions (2.21) and (2.42), equations (2.11), (2.14), (2.15) and (2.41), in dimensionless form, yield

$$\left[\left(1-F\sigma\right)\left(D^2-a^2\right)-\sigma\right]\left(D^2-a^2\right)W - \frac{ga^2d^2}{\nu}\left(\alpha\Theta - \alpha'\Gamma\right) - \frac{2\Omega d^3}{\nu}DZ = 0, \tag{2.43}$$

$$\left[\left(1-F\sigma\right)\left(D^2-a^2\right)-\sigma\right]Z = -\frac{2\Omega d}{\nu}DW, \tag{2.44}$$

together with (2.24) and (2.25).

Here also we consider the case of two free boundaries maintained at fixed temperatures and solute concentrations. The dimensionless boundary conditions appropriate for the problem are

$$W = D^2W = 0, \Theta = 0, \Gamma = 0, DZ = 0, \text{ at } z = 0 \text{ and } z = 1. \tag{2.45}$$

Eliminating Θ, Γ and Z between equations (2.24), (2.25), (2.43) and (2.44) and substituting the proper solution (2.28), we obtain the dispersion relation

$$R_1 = \frac{(1+x)(1+x+ip_1\sigma_1)}{x}\left[(1+x)\left(1-iF\sigma_1\pi^2\right)+i\sigma_1\right] + S_1\frac{(1+x+ip_1\sigma_1)}{(1+x+iq\sigma_1)} + T_{A_1}\frac{(1+x+ip_1\sigma_1)}{x\left[(1+x)\left(1-iF\sigma_1\pi^2\right)+i\sigma_1\right]}, \tag{2.46}$$

where

$T_{A_1} = \frac{4\Omega^2 d^4}{\nu^2\pi^4}$ and $i\sigma_1 = \frac{\sigma}{\pi^2}$, where σ_1 is real for overstable modes.

For the case of stationary convection, σ = 0 and Equation (2.46) reduces to

$$R_1 = \frac{(1+x)^3}{x} + S_1 + \frac{T_{A_1}}{x}. \tag{2.47}$$

We thus find that for stationary convection, the Walters' B' elastico-viscous fluid behaves like an ordinary Newtonian fluid.

From (2.46), it follows that

$$\frac{dR_1}{dT_{A_1}} = \frac{(1+x+ip_1\sigma_1)}{x\left[(1+x)\left(1-iF\sigma_1\pi^2\right)+i\sigma_1\right]}. \tag{2.48}$$

Rationalizing and equating the real and imaginary parts of (2.48), it follows that

$$\frac{dR_1}{dT_{A_1}} = \frac{\left[(1+x)^2 + p_1\sigma_1^2\left\{1-F\pi^2(1+x)\right\}\right]}{x\left[(1+x)^2 + \sigma_1^2\left\{1-F\pi^2(1+x)\right\}^2\right]}, \tag{2.49}$$

$$p_1 = 1 - F\pi^2(1+x). \tag{2.50}$$

Substituting (2.50) in (2.49), we obtain

$$\frac{dR_1}{dT_{A_1}} = \frac{1}{x}, \tag{2.51}$$

which is always positive.

$$\frac{dR_1}{dS_1} = +1. \tag{2.52}$$

The stable solute gradient and rotation, thus, have stabilizing effects on the thermosolutal convection in Walters' B' elastico-viscous fluid.

3.2 Stability of the system and oscillatory modes

Multiplying Equation (2.43) by W*, the complex conjugate of W, integrating over the range of z and making use of equations (2.24), (2.25) and (2.44) together with the boundary conditions (2.45), we obtain

$$\sigma I_1 - \frac{g\alpha\kappa a^2}{\nu\beta}\left[I_2 + p_1\sigma^* I_3\right] + \frac{g\alpha'\kappa' a^2}{\nu\beta'}\left[I_4 + q\sigma^* I_5\right] + \\ + d^2\left[(1-F\sigma^*)I_9 + \sigma^* I_{10}\right] + (1-F\sigma)I_8 = 0, \tag{2.53}$$

where

$$I_9 = \int_0^1 \left(|DZ|^2 + a^2|Z|^2\right)dz,\; I_{10} = \int_0^1 |Z|^2 dz, \tag{2.54}$$

and $I_1 - I_5$ are given in equation (2.35). The integrals $I_1 - I_5$ and $I_8 - I_{10}$ are all positive definite. Putting $\sigma = \sigma_r + i\sigma_i$ and equating the real and imaginary parts of equation (2.53), we obtain

$$[I_1 - \frac{g\alpha\kappa a^2}{\nu\beta}p_1 I_3 + \frac{g\alpha'\kappa' a^2}{\nu\beta'}qI_5 + d^2(I_9 - FI_8) - FI_{10}]\sigma_r \\ = -[I_{10} - \frac{g\alpha\kappa a^2}{\nu\beta}I_2 + \frac{g\alpha'\kappa' a^2}{\nu\beta'}I_4 + d^2 I_8], \tag{2.55}$$

and

$$[I_1 - FI_{10} + \frac{g\alpha\kappa a^2}{\nu\beta}p_1 I_3 - \frac{g\alpha'\kappa' a^2}{\nu\beta'}qI_5 - d^2(I_9 - FI_8)]\sigma_i = 0. \tag{2.56}$$

It follows from equation (2.55) that σ_r may be positive or negative which means that the system may be stable or unstable. It is clear from (2.56) that σ_i may be zero or non-zero, meaning that the modes may be non-oscillatory or oscillatory. The oscillatory modes are introduced due to the presence of rotation and stable solute gradient, which were non-existent in their absence.

3.3. The Case of Overstability

Here we discuss the possibility of whether instability may occur as overstability. Since we wish to determine the Rayleigh number for the onset of instability via a state of pure oscillations, it suffices to find conditions for which (2.46) will admit of solutions with σ_1 real. Separating the real and imaginary parts of (2.46) and eliminate R_1 between them, we obtain

$$A_2 c_1^2 + A_1 c_1 + A_0 = 0, \tag{2.57}$$

where we have put $c_1 = \sigma_1^2$, $b = 1 + x$ and

$$A_2 = bq^2\left(1 - F\pi^2 b\right)^2\left(1 + p_1 - F\pi^2 b\right),$$

$$A_1 = b^3\left(1 - F\pi^2 b\right)\left[\left(1 - F\pi^2 b\right)^2 + p_1\left(1 - F\pi^2 b\right) + q^2\right] +$$

$$+ (b-1) S_1 (p_1 - q)\left(1 - F\pi^2 b\right)^2 + q^2\left[p_1 b^3 + T_{A_1}\left(p_1 - 1 + F\pi^2 b\right)\right],$$

$$A_0 = b^5\left(1 + p_1 - F\pi^2 b\right) + S_1 b^2 (b-1)(p_1 - q) + b^2 T_{A_1}\left(p_1 - 1 + F\pi^2 b\right).$$

As σ_1 is real for instability, both the values of $c_1\left(=\sigma_1^2\right)$ must be positive. Equation (2.57) is quadratic in c_1 and does not involve any of its roots to be positive if

$$p_1 > q, \quad p_1 > F\pi^2 b \quad \text{and} \quad p_1 > 1, \tag{2.58}$$

which imply that

$$\kappa < \kappa', \quad \frac{\kappa\nu'}{d^2}\left(\pi^2 + k^2 d^2\right) < \nu \quad \text{and} \quad \kappa < \nu. \tag{2.59}$$

Thus Equations (2.59) are, therefore, the necessary conditions for the non-existence of overstability, the violation of which does not necessarily imply the occurrence of overstability.

CONCLUSION

The study of viscoelastic fluids may find applications in geophysics and chemical technology. There are many elastico-viscous fluids that cannot be characterized by Maxwell's constitutive relations or Oldroyd's constitutive relations. Walters' B' is one such class of elastico-viscous fluids.

A layer of electrically conducting Walters' B' elastico-viscous fluid heated and soluted from below has been considered in the presence of a uniform horizontal magnetic field and uniform rotation. For stationary convection, the Walters' B' elastico-viscous fluid behaves like an ordinary (Newtonian) fluid. For stationary convection, the magnetic field and rotation have stabilizing effect on the thermoslolutal instability of Walters' B' elastico-viscous fluid. It is also found that the magnetic field and rotation introduce oscillatory modes in the system which were non-existent in their absence. The sufficient conditions for the non-existence of overstability for thermoslolutal instability in Walters' B' elastico-viscous fluid in the presence of magnetic field and rotation are, respectively, $\kappa < \eta, \kappa < \kappa', \frac{\kappa\nu'}{d^2}\left(\pi^2 + k^2 d^2\right) < \nu.$ and $\kappa < \kappa', \frac{\kappa\nu'}{d^2}\left(\pi^2 + k^2 d^2\right) < \nu, \kappa < \nu.$

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