

Creep Analysis of an Isotropic Linearly Decreasing Functionally Graded Rotating Disc at Linearly Increasing Temperature

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ABSTRACT

The present study investigates steady-state creep analysis of thermally graded isotropic rotating disc made of linearly varying particle reinforced functionally graded material using threshold stress based creep law. The stress and strain rate distributions have been calculated for the discs rotating at linearly increasing temperature from inner to outer radii with the help of von Mises' yield criterion. The results are displayed and compared graphically in designer friendly format for the said temperature profile with uniform temperature. A small variation is observed for radial and tangential stresses for said thermal gradations. However, the strain rates vary significantly in the presence of thermal gradation as compared to a disc having uniform temperature. A functionally graded rotating disc with linearly increasing thermal profile can be more efficient than those with uniform distribution as observed from the study.

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INTRODUCTION

Advancement in technology has made it possible to synthesize materials for components that exhibit graded-variation in their properties. Typically, under severe environments, such as high temperature or thermal gradient, the conventional materials (metals or ceramics) alone may not survive. Thus, a new material concept of functionally graded materials (FGMs) emerged and led to the development of superior heat resistant materials. In FGMs, the constituents or their contents vary in some direction, thus enabling these materials to provide unique performance. FGMs have been

developed as ultra high temperature resistant materials for potential applications in air-crafts, space vehicles, and other components working at elevated temperature. FGMs have high potential for applications in components subjected to severe mechanical and thermal loadings because of their unique performance due to spatial tailoring of properties at a microscopic level. Hirai and Chen [3] and Uemura [10] have given many applications of FGM. Suryanarayanan et al. [9] studied the potential use of Al-SiC metal matrix composite (MMC) with particular reference to the aerospace industry. Gupta et al.

[1] investigated the creep behavior of a rotating disc having thermal gradient in the radial direction made of isotropic functionally graded material (FGM) by Sherby's law. The analysis indicated that for the assumed linear particle distribution, the steady-state strain rates were significantly lower compared to that in an isotropic disc with uniform distribution.

MATHEMATICAL MODELING

Reinforcement Distribution in the Disc

In the present study, an isotropic functionally graded disc rotating with angular velocity ω is considered. The FGM disc has silicon carbide particles varying linearly from inner radius a to outer radius b ; as a result, the density and the creep constants will vary with radial distance. The material properties of the disc are assumed to be functions of the volume fraction of the constituent materials. The composition variation in terms of volume percent of silicon carbide, along the radial distance, $V(r)$, is given as:

$$V(r) = A - Br \quad a \leq r \leq b \quad (1)$$

Where

$$A = \frac{bV_{max} - aV_{min}}{b-a} \quad (2)$$

And

$$B = \frac{V_{max} - V_{min}}{b-a} \quad (3)$$

Here V_{max} and V_{min} are the particle content at the inner and outer radius respectively.

Now, using the law of mixtures, the density variation in the composite is expressed as

$$\rho(r) = \rho_m + (\rho_d - \rho_m) \frac{V(r)}{100} \quad (4)$$

where ρ_m and ρ_d are the densities of the matrix alloy and of the dispersed silicon carbide particles, respectively. Now substituting the value of $V(r)$ from eq (1) into eq (4), we get

$$\rho(r) = \rho_m + (\rho_d - \rho_m) \frac{A - Br}{100} \quad (5)$$

If the average particle content in the FGM disc is V_{avg} , and h is the uniform thickness of the disc,

$$\text{then } \int_a^b 2\pi r h V(r) dr = V_{avg} h (b^2 - a^2) \quad (6)$$

Putting the expression of $V(r)$ from eq (1) into eq (6), we get the following relation:

$$V_{avg} = A - \frac{2B(b^2 - a^2)}{3(b^2 - a^2)} \quad (7)$$

Thermal Gradient

For the present study, the following types of discs have been considered;

1. Disc D_1 operating under linearly increasing temperature distribution, $T(r)$, from inner to outer radius respectively, given as:

$$T(r) = C - Dr \quad a \leq r \leq b \quad (8)$$

$$\text{where } C = \frac{bT_a - aT_b}{b-a} \quad (9)$$

$$\text{and } D = \frac{T_a - T_b}{b-a} \quad (10)$$

Here T_a and T_b are the imposed temperatures at the inner and outer radius respectively.

2. Disc D_2 operating under uniform temperature of 623K.

Creep Law

The steady-state creep response of the Al-SiCp composite of varying composition has been described in terms of Sherbys' law [7] of the form:

$$\dot{\epsilon} = [M(\bar{\sigma} - \sigma_0)]^8 \quad (11)$$

where the symbol $\dot{\epsilon}$, $\bar{\sigma}$, σ_0 denote, respectively the effective strain rate under biaxial stress, the effective stress rate under biaxial stress and threshold stress.

Creep Parameter

The creep parameter is given by:

$$M = \frac{1}{E} \left[\frac{AD_L \lambda^8}{|B_r|^8} \right]^{1/8} \quad (12)$$

where the symbols A , D_L, λ , E , $[b_r]$ denote respectively, constant sensitive to microstructure, lattice diffusivity, subgrain size, Young's modulus of elasticity and magnitude of burger's vector.

In a particle-reinforced composite, the material parameters M and σ_0 depend on the particle size (p) and the percentage of dispersed particles ($V(r)$) apart from the temperature ($T(r)$). The value of M and σ_0 have been obtained from the creep results reported for Al-SiCp composite under uniaxial loading given by Pandey et al. [4] and are shown in Table 1.

The values of M and σ_0 as functions of T , p and V have been calculated in this study for $p = 1.7 \mu m$ by the following regression equations using data fit software:

$$\ln(M(r)) = 0.2112 \ln(p) + 4.89 \ln(T(r)) - 0.591 \ln(V(r)) - 34.91 \tag{13}$$

$$\sigma_0(r) = -0.02050(p) + 0.0378(T(r)) + 1.033(V(r)) - 4.9695 \tag{14}$$

Mathematical Formulation

From symmetry considerations, principal stresses are in the radial, tangential and axial directions.

For the purpose of modeling the following assumptions are made:

1. Steady state condition of stress is assumed.
2. Elastic deformations are small for the disc and can be neglected as compared to the creep deformations.
3. Biaxial state of stress exists at any point of the disc.
4. The composite shows a steady state creep behavior, which may be described by Sherby's constitutive model as given by equation (11).

Taking reference frame along the directions r , and z , the generalized constitutive equations for creep in an isotropic rotating disc takes the form:

$$\dot{\epsilon}_r = \frac{\dot{\bar{\epsilon}}}{2\bar{\sigma}} [2\sigma_r - (\sigma_\theta + \sigma_z)] \tag{15}$$

$$\dot{\epsilon}_\theta = \frac{\dot{\bar{\epsilon}}}{2\bar{\sigma}} [2\sigma_\theta - (\sigma_z + \sigma_r)] \tag{16}$$

$$\dot{\epsilon}_z = \frac{\dot{\bar{\epsilon}}}{2\bar{\sigma}} [2\sigma_z - (\sigma_r + \sigma_\theta)] \tag{17}$$

where the effective stress, $\bar{\sigma}$, is given by,

$$\bar{\sigma} = \frac{1}{\sqrt{2}} [(\sigma_r - \sigma_\theta)^2 + (\sigma_\theta - \sigma_z)^2 + (\sigma_r - \sigma_z)^2]^{1/2} \tag{18}$$

and $\dot{\epsilon}_r, \dot{\epsilon}_\theta, \dot{\epsilon}_z$ and $\sigma_r, \sigma_\theta, \sigma_z$ are the strain rates and stresses respectively in the directions indicated by the subscripts and $\dot{\bar{\epsilon}}$ is the effective strain rate. For biaxial state of stress ($\sigma_z = 0$), and the constitutive equations are,

$$\dot{\epsilon}_r = \frac{du_r}{dr} = \frac{[M(r)(\bar{\sigma} - \sigma_0(r))]^2 (2x-1)}{2(x^2-x+1)^{3/2}} \tag{19}$$

$$\dot{\epsilon}_\theta = \frac{u_r}{r} = \frac{[M(r)(\bar{\sigma} - \sigma_0(r))]^2 (2-x)}{2(x^2-x+1)^{3/2}} \tag{20}$$

$$\dot{\epsilon}_z = -(\dot{\epsilon}_r + \dot{\epsilon}_\theta) \tag{21}$$

where $x = \sigma_r(r)/\sigma_\theta(r)$ is the ratio of radial and tangential stress at any radius r and u_r is the radial displacement.

The equation of motion for a rotating disc of uniform thickness h may be obtained by considering the equilibrium of an element in the composite disc confined between radial distances r and $r + dr$ and an interval of angle between θ and $\theta + d\theta$. The equilibrium of forces in the radial direction of the element implies that

$$\frac{d}{dr} [r\sigma_r(r)] - \sigma_\theta(r) + \rho(r)\omega^2 r^2 = 0 \quad (22)$$

Where ρ is the density of the composite.

Equations (19) and (20) can be solved to obtain $\sigma_\theta(r)$ as given below:

$$\sigma_\theta(r) = \frac{(u_a)^{1/2}}{M(r)} \psi_1(r) + \psi_2(r) \quad (23)$$

where

$$\psi_1(r) = \frac{\psi(r)}{(x^2 - x + 1)^{1/2}} \quad (24)$$

$$\psi_2(r) = \frac{\sigma_a(r)}{(x^2 - x + 1)^{1/2}} \quad (25)$$

$$\psi(r) = \left[\frac{2(x^2 - x + 1)^{1/2}}{r(2-x)} \exp \int_a^r \frac{\varphi(r)}{r} dr \right]^{1/8} \quad (26)$$

and

$$\varphi(r) = \frac{(2x-1)}{(2-x)} \quad (27)$$

Knowing the tangential stress distribution $\sigma_\theta(r)$, values of $\sigma_r(r)$ can be obtained from (22) as follows:

$$\sigma_r(r) = \frac{1}{r} \int_a^r \sigma_\theta(r) dr - \frac{\omega^2}{r} \left[\frac{(r^2 - a^2)}{3} (\rho_m + (\rho_d - \rho_m) \frac{A}{100}) - \frac{(\rho_d - \rho_m) B (r^4 - a^4)}{100 \cdot 4} \right] \quad (28)$$

As the tangential stress, σ_θ , and the radial stress, σ_r , are determined by equations (23) and (28) at any point within the composite disc. Then the strain rates $\dot{\epsilon}_r, \dot{\epsilon}_\theta, \dot{\epsilon}_z$ are calculated from equations (19), (20) and (21) respectively.

Numerical Computation

The stress distribution is evaluated from the above analysis by iterative numerical

scheme of computations. The iteration is continued till the process converges yielding the values of stresses at different points of the radius grid. For rapid convergence 75% of the value of $\sigma_\theta(r)$ obtained in the current iteration has been mixed with 25% of the value of $\sigma_\theta(r)$ obtained in the last iteration for the use in the next iteration, i.e.

$$\sigma_{\theta_{next}} = 0.25\sigma_{\theta_{previous}} + 0.75\sigma_{\theta_{current}} \quad (29)$$

RESULTS AND DISCUSSION

The effect of imposing thermal and particle gradient on the creep behavior of the composite disc is investigated by determining the stress and strain distribution in the discs mentioned above.

Figure 1 shows variation of radial stress along radial distance developed in the rotating disc under the effect of thermal gradation. As one moves radially outwards from inner radius, the radial stress increases from zero, reaches a maximum value near middle of the disc and then decreases to zero towards the outer radius. The maximum value of radial stress for discs D₁ and D₂ are 38.92MPa and 39.48MPa, respectively, near the middle of the disc at radius 0.08001m. Thus, there is a minor change in radial stress for disc operating at linearly increasing temperature near the middle of the disc and decreases to zero towards the inner and outer radii.

The variation of tangential stress along radial distance under the effect of thermal gradation is shown graphically in Figure 2. The graph shows that the tangential stress increases from the inner radius, attain maximum value of 89.93MPa and 91.13MPa at radius 0.0559m for discs D₁ and D₂ and then decreases to 69.31MPa as one approaches outer radius. The tangential stress is lowest for disc D₁ in comparison to disc D₂.

Figure 3 shows graphically the effect of thermal gradation on radial strain rates

developed in rotating discs. With the change in temperature distribution the radial strain rate is affected significantly as seen from the graph. The radial strain rates are compressive in nature. For discs D_1 and D_2 , the radial strain rate attains $-9.88 \times 10^{-8} s^{-1}$ and $-9.17 \times 10^{-7} s^{-1}$ at the inner radius; and then decreases to $-4.05 \times 10^{-2} s^{-1}$ and $-2.18 \times 10^{-3} s^{-1}$, respectively at the outer radius. At the inner radius, disc operating at uniform temperature has lowest strain rate but as one moves toward the outer radius disc operating at linearly increasing temperature shows lowest strain rate.

Figure 4 depicts variation of tangential strain rate along radial distance under the effect of thermal gradation. The tangential strain rates are also significantly affected with the change in temperature distribution of disc. The graph clearly shows that the tangential strain rate decreases for disc D_1 in the region near the inner radius and increases as one moves toward the region near the outer radii in comparison to disc D_2 . At the inner radius, disc D_1 has lowest tangential strain rate of $1.97 \times 10^{-7} s^{-1}$ but at the outer radius disc D_2 has lowest rate of $4.36 \times 10^{-3} s^{-1}$.

CONCLUSIONS

The present study reveals the following conclusions:

1. The radial stress traces a parabolic path as the radial distance varies from inner to outer radius with the temperature (whether uniform or gradient). However, the disc operating at linear thermal profile shows lower stress value.

2. The tangential stress increases near the inner radius and decreases towards the outer radius. But, the disc operating at linearly increasing temperature shows less tangential stress value.

3. The strain rates are significantly affected with thermal gradation of the disc. When the thermal gradation increases from inner to outer radii, the radial strain rate is more till the region near the middle of the disc and then becomes less as one moves towards the outer radius in comparison to the disc operating at uniform temperature.

4. The tangential strain rate is less for disc operating at linearly increasing temperature till the region near the middle of the disc and then increases as one moves towards the outer radius in comparison to the disc operating at uniform temperature.

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Table 1: Material parameters from experimental study of Pandey [4]

Particle Size	Temperature	Particle Content	Creep Parameter	Creep Parameter
p	T	V	M	σ_c
(m)	(K)	(vol%)	($s^{-1}/E/MPa$)	(MPa)
1.7	623	10	0.00963	15.24
14.5	623	10	0.01444	11.46
45.9	623	10	0.01897	13.65
1.7	623	10	0.00963	15.24
1.7	623	20	0.00594	24.83
1.7	623	30	0.00518	34.32
1.7	623	20	0.00594	24.83
1.7	673	20	0.00897	24.74
1.7	723	20	0.01295	25.72

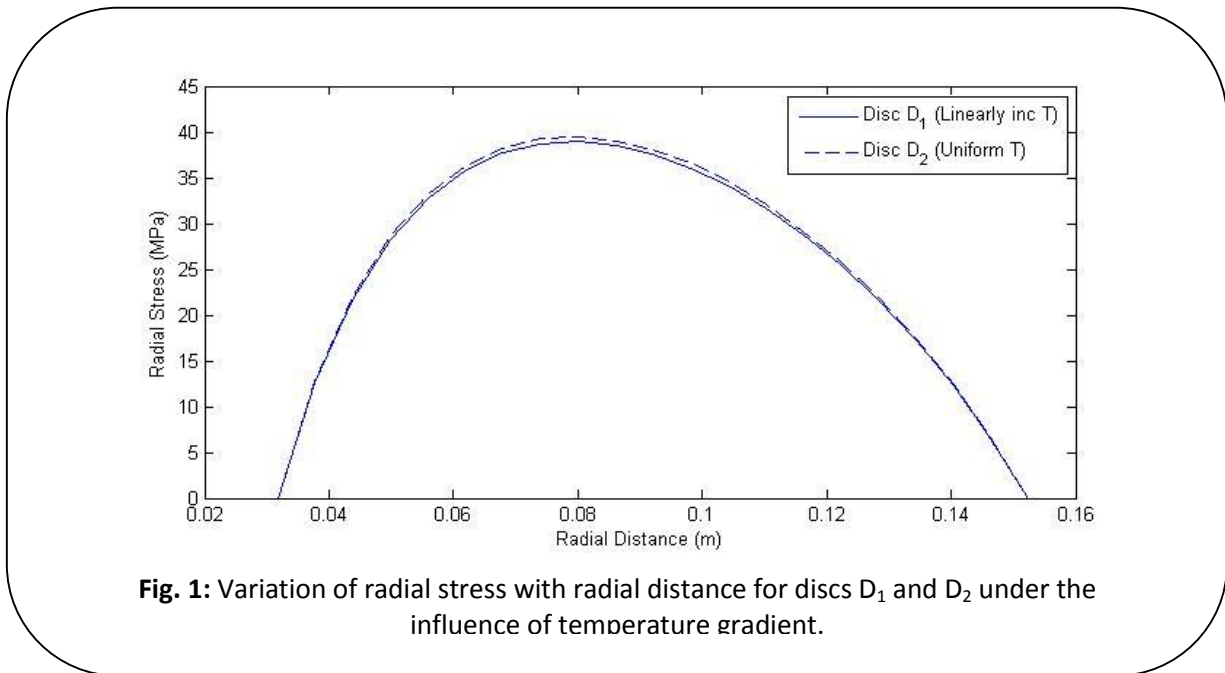


Fig. 1: Variation of radial stress with radial distance for discs D_1 and D_2 under the influence of temperature gradient.

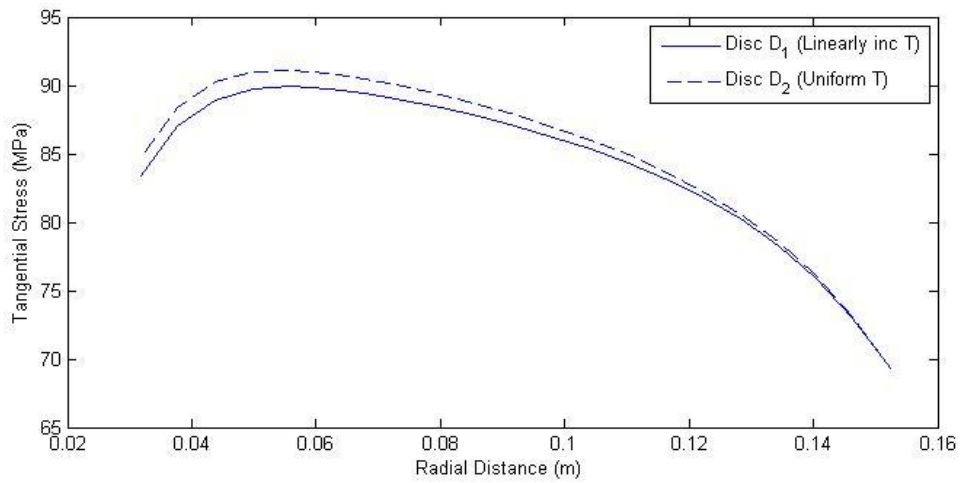


Fig. 2: Variation of tangential stress with radial distance for discs D₁ and D₂ under the influence of temperature gradient.

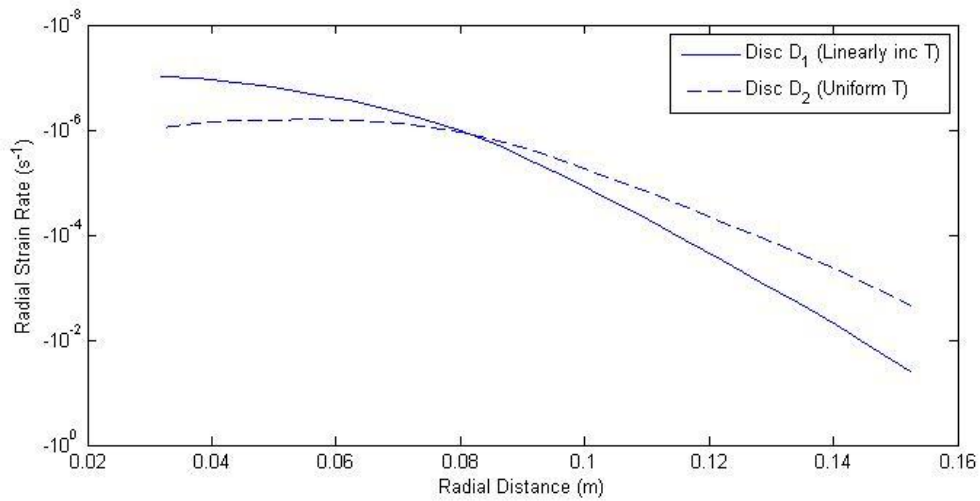


Fig. 3: Variation of radial strain rate with radial distance for discs D₁ and D₂ under the influence of temperature gradient.

