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Convective Heat Transfer Flow of a Viscous Electrically Conducting Fluid in a Non-Uniformily Heated Axially Varying Pipe

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ABSTRACT

We analyse the effect of radiation on combined heat transfer of an electrically conducting viscous fluid in a non-uniformly heated corrugated pipe in the presence of a constant heat source. A non –uniform temperature is maintained on the boundary. Taking the slope δ of the boundary of the pipe as perturbation parameter, the equations governing the flow , heat transfer and magnetic induction have been solved. The velocity and temperature have been evaluated for variations in the different governing parameters. The effect of the various governing parameters on flow, heat transfer has been exhibited through various profiles of velocity, temperature distributions.

Keywords : Wavy pipe, Heat Transfer, Non-uniform Temperature, Axial Magnetic Field

INTRODUCTION

Thermal convection problem in porous occurs in a broad spectrum of the disciplines, ranging from Chemical Engineering to Geophysics. Applications include heat insulations by fibrus materials, spreading of pollutants convection of Earth's mantle. Alarge cross-section of fundamental research has been carried out by several authors in the recent times. In most of the investigations the boundaries are uniform in cross-sections as well as the boundary temperatures. However, there are a few physical situations which warrant the assumption of non-uniformity in either the boundaries or the boundary temperatures. In a convection flow through a channel such a non-uniformity creates a secondary flow. This secondary flow is of vital importance to technological processes. For example, the process of modified chemical vapour deposition(MCVD[3,5])has been suggested in drawing optical glass fibkres of extremely low and wide band width. Performs from which these fibres are drawn are made by passing a gaseous mixture into a fused-silicon tube which is heated locally by an oxy-hydrogen flow. Particulates of Sio2-Geo2 composition are formed from the mixture and coillect on the interior of the tube. Subsequently ,these are fused to form a vitreous deposit as the flames traversed along the tube. The deposition is carried out in the radial direction through the secondary flow creates due to non-uniform wall temperature.

In most of the studies pertaining to convection flows through the pipes, the axial dependence of the flow variables [2,4,5,6,9,10,12] is neglected and either the temperature or its gradient in maintained uniform on the boundary .Also the heat transfer analysis is investigated in the absence of any internal heat sources in the flow filed. The heat transfer in a flow through a pipe in the presence of additional internal heat source has direct application to the modified chemical vapour deposition process. This MCVD process is being used to make high quality optical glass fibres [13,17,18].

In hydro magnetic case flow through channel with non-uniform gap has been considered by Mc Michael and Deutsch [11] in their paper on MHD laminar flow in the slowly varying tube in the presence of an axial magnetic filed. They considered a small parameter δ <<1 (given by the ratio of radial to axial length scaling) which characterises the wall slope of the regions of varying radius. The problem is analysed as a regular perturbation problem at finite magnetic Reynolds number and Hartman number as large as $o(\delta^{-1/2})$. It is observed that the onset of flow separation is associated with adverse axial gradients of wall pressure created by radial magnetic body forces.

These are produced by electric currents induced at first order by zeroth order stream lines crossing the uniform field, developing obviously large radial pressure gradients. The dimensionless current density is independent of the Hartmann and Reynolds numbers, so that the physical current density varies linearly with flow rate and applied field strength high current densities are localized in the vicinity of maximum wall slope near the half radius point. Magnetic effects, treated as first order perturbation to the basic local Hagen-poiseuville flow, lead to separation in both converging and diverging sections of the tube, while inertial effects promote separation only in diverging sections. The induced magnetic filed un affects the flow even at the first order as long as long as (δR_m) remains sufficiently small and thus the interactions between the induced field and the base flow are only of second order. This analysis has been extended by Deschikachar et al (3) to include the effect of unsteadiness on the flow. Such purely oscillatory flow over non-uniform surfaces that are not straight exhibits a steady stream caused by the non-linear equations governing the motion. This phenomena of steady streaming is of great mathematical and physical interest. The pressure and sheer stress on the wall for various parameters governing the flow are discussed.

Krishna et al[8] have analysed the combined free and forced convection flow through an axially varying vertical pipe in the presence of an internal heat source of constant strength. Murthy[12] has extended this to study the effects of a uniform axial magnetic filed. Costa[1] has analysed the viscous dissipation effects in natural and mixed convection heat transfer. Recently Basack et al[16] have discussed the natural convection flow in a square cavity filled with a porous heated bottom wall and adiabatic top wall maintaining constant temperature of cold vertical walls. Several authors [6a,11a-11d, 14a] have discussed the flow of the viscous through channels of variable cross section under different conditions.

In this paper, we investigate the effect of radiation on combined heat transfer of an electrically conducting viscous fluid in a non-uniformly heated corrugated pipe in the presence of a constant heat source. A non –uniform temperature is maintained on the boundary. Taking the slope δ of the boundary of the pipe as perturbation parameter, the equations governing the flow , heat transfer and magnetic induction have been solved. The velocity and temperature have been evaluated for variations in the different governing parameters. The effect of the various governing parameters on flow, heat transfer has been exhibited through various profiles of velocity, temperature distributions.

FORMULATION OF THE PROBLEM

We consider the steady axisymmetric flow of an incompressible ,viscous electrically conducting fluid in a vertical pipe of variable cross section maintained at non-uniform temperature $\gamma(\delta x/a)$.The Boussinesq approximation is used so that the density variations will be retained only in the buoyancy force. The viscous dissipation is neglected in comparison to the heat flow by convection. The concentration on these walls is taken to be constant. The cylindrical polar system(r,x) is chosen with x-axis along the axis of the pipe. The boundary of the pipe is assumed to be

$$r = af(\delta x/a)$$

where 'a ' is characteristic radial length, f is twice differentiable and 'δ'is a small parameter proportional to the boundary slope. The flow is maintained by a constant flow for which a characteristic velocity U is defined as

$$U = \left(\frac{2}{a^2}\right)\int_0^{af(\delta x/a)} urdr \tag{1}$$

The applied magnetic field Bo is uniform and directed along the axis of the pipe. No electric field is applied and there is no induced electric field for the constraints given(24).The electrical conductivity of the pipe walls remains arbitrary and without influence on the flow are

$$\rho_e(\bar{\varsigma}x\bar{q}) = -\nabla p + \mu\nabla^2\bar{q} + \mu_e(\bar{J}x\bar{B}) - \rho\bar{g} \tag{2}$$

$$\nabla.\bar{q} = 0 \tag{3}$$

$$\rho_e C_p(\bar{q}.\nabla)T = \lambda\nabla^2 T + Q - \frac{1}{r}\frac{\partial(rq_R)}{\partial r} \tag{4}$$

$$\rho - \rho_e = -\beta(T - T_e) \tag{5}$$

The Maxwell's equations related to the magnetic induction vector $\bar{B}$ are

$$\nabla.\bar{B} = 0 \tag{6}$$

$$\nabla X\bar{B} = 4\pi\bar{J}\ . \tag{7}$$

The Ohm's law gives

$$\bar{J} = \sigma\mu_e(\bar{q}\, x\, \bar{B}) \qquad (8)$$

Where ρe is the density of the fluid in the equilibrium state, $\bar{q}$ is the velocity,ζ is the viscosity p, is the pressure. T, is the temperature in the flow region,ρ is the density of the fluid ,k is the coefficient of permeability, Q is the strength of the heat source ,μ is the coefficient of viscosity, Cp is the specific heat at constant pressure,λ is the coefficient of thermal conductivity,β_1 is the coefficient of volume expansion, $\bar{B}$ is the magnetic induction vector, $\bar{J}$ is the current density vector,σ is the electrical conductivity of the fluid ,D is the molecular diffusivity,k_{11} is the cross diffusivity and μe is the magnetic permeability.

Invoking Rosseland approximation the radiative heat flux is given by

$$q_R = -\frac{4\sigma^{\bullet}}{\beta_R}\frac{\partial(T'^4)}{\partial r} \qquad (9)$$

and expanding T'^4 by Taylor's expansion after neglecting higher order terms we get

$$T'^4 \cong 4T_e^3 T' - 3T_e^4 \qquad (10)$$

In the equilibrium state

$$-\frac{\partial p_e}{\partial x} - \rho_e g = 0 \qquad (11)$$

Where $p = p_e + p_D$; p_D is the hydrodynamic pressure.

Introducing the non-dimensional variables

$$\bar{q}^* = q/U,\, p^* = p/\rho U^2,\, \bar{B}^* = B/B_0,\, \bar{J}^* = J/\sigma U B_0$$

$$\theta = \frac{T - T_e}{\Delta T_e},\, \gamma^* = \gamma/\Delta T$$

$$\Delta T_e = T_e(0) - T_e(a)$$

The equations (2)-(8) after using (9)&(10) reduce to (on dropping the asterisks)

$$R_e(\bar{\zeta} x \bar{q}) + \nabla(p + \frac{1}{2q^2}) + \nabla^2 q + M^2(\bar{J} x \bar{B}) - G(\theta) \qquad (12)$$

$$\nabla.\bar{q} = 0 \qquad (13)$$

$$P_e(\bar{q}.\nabla)\theta = \frac{\partial^2\theta}{\partial r^2} + \frac{1}{r}\frac{\partial\theta}{\partial r} + \frac{\partial^2\theta}{\partial x^2} + \frac{4}{3N_1}(\frac{\partial^2\theta}{\partial r^2} + \frac{1}{r}\frac{\partial\theta}{\partial r}) + \alpha \qquad (14)$$

$$\nabla.\bar{B} = 0 \qquad (15)$$

$$\nabla x \bar{B} = R_m \bar{J} \qquad (16)$$

$$\bar{J} = (\bar{q} x \bar{B}) \qquad (17)$$

where

$$R_e = \frac{Ua}{\nu} \quad (the\, \mathrm{Re}ynolds\, number),\; R_m = \sigma\mu_e u_c a \;\; (the magnetic\, \mathrm{Re}ynolds\, number)$$

$$M = aB_0(\frac{\sigma}{\rho\nu})^{1/2}(the Hartmann\, number),\, G = \frac{\beta_1 g \Delta T_e a^3}{\nu^2}(the Grashof\; number)$$

$$P_e = \frac{\mu_e U C_p a}{\lambda\nu} \quad (the Peclet\, number),\, D^{-1} = \frac{a^2}{k}\; (the Darcy\, Parameter)$$

$$N_1 = \frac{\beta_R T_e^3}{4\sigma^\bullet}(the\,Radiation\;parameter),\alpha = \frac{Qa^2}{\lambda C_p}$$ (Heat source parameter)

Under these constraints imposed $\bar{J}$ has only the azimuthal components J_θ while q and B have axial and radial components.

We assume $\bar{q} = (u,v)$, $\bar{B} = (f,g)$

The boundary conditions relevant to the problem are

$$v(r,x) = 0, \frac{\partial v}{\partial r} = 0, \frac{\partial \theta}{\partial r} = 0 \qquad on\;\; r = 0$$

$$u(r,x) = 0, T - T_e = \gamma(\delta x) \qquad on\;\; r = a \tag{18}$$

Equations (12)-(17) constitute a system of six equations for the seven unknowns u,v,f,g,J_0, and θ. These may be reduced to three equations for the Stoker's stream function ψ(r,x) and the magnetic stream function $\phi(r,x)$ given by

$$u = -\frac{1}{r}\frac{\partial \psi}{\partial r}\;,\;\; v = \frac{1}{r}\frac{\partial \psi}{\partial x} \qquad f = -\frac{1}{r}\frac{\partial \phi}{\partial r}\;,\;\; g = \frac{1}{r}\frac{\partial \phi}{\partial x}$$

(The subscripts x and r denote the respective partial derivatives).

Combining (16) and (17)to eliminate J. We find

$$E^2\phi = R_m((\frac{1}{r})(\psi_x\phi_r - \psi_r\phi_x) \tag{19}$$

Where the operator E^2 is denoted by

$$E^2 = r\frac{\partial}{\partial r}(\frac{1}{r}\frac{\partial}{\partial r}) + \frac{\partial^2}{\partial x^2}$$

Eliminating J between (12) and (17) and taking the curl of the former to eliminate the pressure, we get

$$\begin{aligned} &R_e((\frac{1}{r}\frac{\partial \psi}{\partial x}\frac{\partial (E^2\psi)}{\partial r} - \frac{1}{r}\frac{\partial \psi}{\partial r}\frac{\partial (E^2\psi)}{\partial x} - \frac{2}{r^2}\frac{\partial \psi}{\partial x}E^2\psi) - E^2(E^2\psi) + \\ &+ M^2/R_m\{(1/r)\phi_x(E^2\phi)_r - (1/r)\phi_r(E^2\phi)_x - \\ &-(2/r^2)\phi_x\{E^2\phi)_r - \{G/R_e)(r(\frac{\partial \theta}{\partial r})) \end{aligned} \tag{20}$$

The energy equation is

$$P_1(\theta_r\psi_x - \theta_x\phi_r) = \theta_{rr} + 1/r)\theta_r + N_2\theta_{xx} + \alpha_1 \tag{21}$$

where

$$N_2 = \frac{3N_1}{4+3N_1}\;, \qquad P_1 = P_e N_2 \;\; \alpha_1 = \alpha N_2$$

The current density can be found once ψ and ϕ are known from equations (17) which reduce to

$$J_\theta = (1/r^2)(\psi_x\theta_r - \psi_r\phi) \tag{22}$$

These coupled equations (21)-(22) are to be solved subject to non dimensional boundary conditions.

$$\psi(r,x) = 0,\;\; \frac{\partial}{\partial r}((1/r)\frac{\partial \psi}{\partial r} = 0 \tag{24}$$

$$\frac{\partial \theta}{\partial r} = 0 \quad on \;\; r = 0 \tag{25}$$

$$\psi(r,x) = -1/2, \quad \frac{\partial \psi}{\partial r} = 0 \tag{26}$$

$$\theta(r,x) = \gamma(\delta x) \quad on\ r = f \tag{27}$$

The value of ψ on the boundary assures the constant volumetric flow in consistence with the hypothesis (1) and conditions (24) & (26) corresponds to axial symmetry of the flow.

Electric currents within the fluid induces a magnetic field exterior to the tube as well as within. This external field, $\hat{B} = (\hat{f}, \hat{g})$ is given by a potential $\hat{A} = \hat{\phi} e_\theta$, such that

$\hat{B} = \nabla x \hat{A}$, or

$$\hat{f} = -(1/r)\hat{\phi}_r, \quad \hat{g} = (1/r)\hat{\phi}_x$$

Since $R_m = 0$ in the exterior region equation (18) requires $E^2 \hat{\phi} = 0$

Both the potential and the field itself must be continuous at the wall (7). Hence we may write the matching condition are

$$\phi = \hat{\phi} \quad at \quad r = f(x)$$

$$\phi_r = \hat{\phi}_r \quad at \quad r = f(x)$$

The problem statement is completed by boundary conditions at r = 0 , $r \to \pm\infty$. Because of symmetry the radial field must vanish at the centre line.

$(1/r)\ \phi_x = 0$ at r = 0

and we must retrieve the uniform applied field both far from the tube,

$$\lim_{r\to\pm\infty}(1/r)\hat{\phi} = \lim_{x\to\pm\infty}(1/r)\phi_r = -1$$

On the boundary with variable cross-section $(1/r)\ (\partial\phi / \partial r)$ is either a constant or a function x. Supposing $\phi_r \neq 0$ then f is function of x along the tubs. In view of the continuity requirements it follows that in the neighbourhood of the tube at any case r.

$$\hat{f} = -(1/r)\hat{\phi}_r = -1$$

$$\underset{r\to\infty}{Lt}(1/r)\hat{\phi}_r = -1$$

Therefore $\hat{\phi}_r \neq 0$ on r = f(x) leads to a contradiction.

But $\phi_r = \hat{\phi}_r$ in view of the matching condition on r = f(x).

Therefore on r= f(x), $\phi_r = \hat{\phi}_r = 0$

ANALYSIS OF THE FLOW

We introduce the transformations [11] $\bar{x} = \delta x$

$we\ assume \dfrac{\partial}{\partial x} \approx O(\delta)\ such\ that\ \dfrac{\partial}{\partial \bar{x}} \approx O(1)$ for small values of δ ,the flow develops slowly along the axial direction with gradient $O(\delta)$ .Making use of the above transformation the equations(19)-(22) reduce to

$$E_1^2\phi = \delta R_m((\frac{1}{r})(\psi_x\phi_r - \psi_r\phi_x) \tag{28}$$

$$(\delta R_e)((\frac{1}{r}\frac{\partial\psi}{\partial\bar{x}}\frac{\partial(E_1^2\psi)}{\partial r}-\frac{1}{r}\frac{\partial\psi}{\partial r}\frac{\partial(E_1^2\psi)}{\partial\bar{x}}-\frac{2}{r^2}\frac{\partial\psi}{\partial\bar{x}}E_1^2\psi)-E_1^4\psi+$$

$$+M^2/R_m\{(1/r)\phi_{\bar{x}}(E_1^2\phi)_r-(1/r)\phi_r(E_1^2\phi)_{\bar{x}}--(2/r^2)\phi_{\bar{x}}\{E_1^2\phi)_r)-\{G/R_e)(r(\frac{\partial\theta}{\partial r}))$$

$$(\delta P_1)(\theta_r\psi_{\bar{x}}-\theta_x\phi_r)=\theta_{rr}+(1/r)\theta_r+\delta^2N_2\theta_{xx}+\alpha_1 \qquad (29)$$

where $E_1^2=r\frac{\partial}{\partial r}(\frac{1}{r}\frac{\partial}{\partial r})+\delta^2N_2\frac{\partial^2}{\partial\bar{x}^2}$

Assume $R_e\approx O(1)\, in\, the\, \lim it\, \delta\to 0$ the inertial terms vanish in equations (29) leading the viscous terms and the magnetic terms. The order of the magnetic term depends on M as well as δ.From equation (28) we find that $E_1^2\phi\approx O(\delta)$ $for\ R_m\approx O(1)$. Thus we may consider the Hartmann number M as large as $O(\delta^{-1/2})$ and still retain only the viscous terms at the zeroth order with both inertial and magnetic perturbations appearing in first order. Thus we elevate magnetic effects to first order neglecting the second order inertial and viscous effects .
Taking the transformation

$$\eta=\frac{r}{f(\bar{x})}$$

the above equations reduce to

$$F^2\phi=(\delta\, f\, R_m)((\frac{1}{\eta})(\psi_x\phi_\eta-\psi_\eta\phi_x) \qquad (30)$$

$$(\delta\, f\, R_e)((\frac{1}{\eta}\frac{\partial\psi}{\partial\bar{x}}\frac{\partial(F^2\psi)}{\partial\eta}-\frac{1}{\eta}\frac{\partial\psi}{\partial\eta}\frac{\partial(F^2\psi)}{\partial\bar{x}}-\frac{2}{\eta^2}\frac{\partial\psi}{\partial\bar{x}}F^2\psi)-F^4\psi+$$

$$+(f\,\hat{M}^2/R_m)\{(1/\eta)\phi_{\bar{x}}F^2\phi)_\eta-(1/\eta)\phi_\eta(F^2\phi)_{\bar{x}}-$$

$$-(2/\eta^2)\phi_{\bar{x}}\{F\ \ \phi)_\eta)-\{Gf^4/R_e)(\eta(\frac{\partial\theta}{\partial\eta})) \qquad (31)$$

$$(\delta\, f\, P_1)(\theta_\eta\psi_{\bar{x}}-\theta_x\phi_\eta)=\theta_{\eta\eta}+(1/\eta)\theta_\eta+\delta^2f^2N_2\theta_{xx}+\alpha_1f^2 \qquad (32)$$

where

$$F^2=\eta\frac{\partial}{\partial\eta}(\frac{1}{\eta}\frac{\partial}{\partial\eta})$$

We use the asymptotic expansions

$$\psi(\eta,\bar{x})=\psi_0(\eta,\bar{x})+\delta\psi_1(\eta,\bar{x})+\delta^2\psi_2(\eta,\bar{x})+................$$

$$\theta(\eta,\bar{x})=\theta_0(\eta,\bar{x})+\delta\theta_1(\eta,\bar{x})+\delta^2\theta_2(\eta,\bar{x})+..............$$

$$\phi(\eta,\bar{x})=\phi_0(\eta,\bar{x})+\delta\phi_1(\eta,\bar{x})+\delta^2\phi_2(\eta,\bar{x})+.............. \qquad (33)$$

Substituting (33) in equations(30)-(31) and separating the like powers of δ,the equations corresponding to the zeroth order are

$$\theta_{0,\eta\eta}+\frac{1}{\eta}\theta_{0,\eta}+\alpha_1f^2=0 \qquad (34)$$

$$F^4\psi_0-h^2F^2\psi_0+\frac{Gf^4}{R_e}(\eta(\theta_{0,\eta}+NC_{0,\eta}))=0 \qquad (35)$$

$$F^2\phi_0=0 \qquad (36)$$

The corresponding conditions on ψ_0, θ0 , and ϕ_0 are

$$\psi_0(1,\bar{x}) = -1/2, \qquad (\psi_{0,\eta})_{\eta=1} = 0$$

$$(\eta\psi_{0,\eta\eta} - \psi_{0,\eta})_{\eta=0} = 0, \qquad \psi_0(0,\bar{x}) = 0 \tag{37}$$

$$\theta_0(1,\bar{x}) = \gamma(\bar{x}), \qquad (\theta_{0,\eta})_{\eta=0} = 0 \tag{38}$$

$$\phi_{0,x}(0,\bar{x}) = 0, \quad \underset{r\to\infty}{Lim}\left(\frac{1}{\eta}\phi_{0,\eta}\right) = -1 \tag{39}$$

The equations to the first order are

$$\eta\theta_{1.,\eta\eta} \qquad +\theta_{1,\eta} = P_1(\psi_{0,\bar{x}}\theta_{0,\eta} - \psi_{0,\eta}\theta_{0,\bar{x}}) \tag{40}$$

$$\begin{aligned} F^2(F^2-h^2)\psi_1 = {} & f^4 R_e(\psi_{0,\bar{x}}(F^2\psi_0)_\eta - \psi_{0,\eta}(F^2\psi_0)_{\bar{x}} - \\ & -(\frac{2}{r^2})\psi_{0,\bar{x}}(F^2\psi_0)) - \hat{M}^2 f^4/R_m)(\phi_{0,\bar{x}}(F^2\phi_0)_\eta \\ & -\phi_{0,\eta}(F^2\phi_0)_{\bar{x}} - (\frac{2}{r^2})\phi_{0,\bar{x}}(F^2\phi_0)) + \frac{Gf^4}{R_m}(\eta(\theta_{1,\eta})) \end{aligned} \tag{41}$$

$$F^2\phi_1 = R_m(\psi_{0,\bar{x}}\phi_{0,\eta} - \psi_{0,\eta}\phi_{0,\bar{x}}) \tag{42}$$

where

$$\hat{M}^2 = \delta M^2 \cong O(1), h^2 = \hat{M}^2 f^2$$

The corresponding conditions on ψ_1, θ_1 , and ϕ_1 are

$$\psi_1(1,\bar{x}) = -0, \qquad (\psi_{1,\eta})_{\eta=1} = 0$$

$$(\eta\psi_{1,\eta\eta} - \psi_{1,\eta})_{\eta=0} = 0, \qquad \psi_1(0,\bar{x}) = 0 \tag{43}$$

$$\theta_1(1,\bar{x}) = \gamma(\bar{x}), \qquad (\theta_{1,\eta})_{\eta=0} = 0 \tag{44}$$

$$\phi_{1,x}(0,\bar{x}) = 0, \quad \phi_{1,\eta} = \hat{\phi}_{1,\mu} = 0 \;\; on \quad \eta = 1 \tag{45}$$

The equations to the second order are

$$\eta\theta_{2.,\eta\eta} \qquad +\theta_{2,\eta} = P_1(\psi_{0,\bar{x}}\theta_{1,\eta} - \psi_{0,\eta}\theta_{1,\bar{x}} + \psi_{1,\bar{x}}\theta_{0,\eta} - \psi_{1,\eta}\theta_{0,\bar{x}}) \tag{46}$$

$$\begin{aligned} F^2(F^2-h^2)\psi_2 = {} & f^4 R_e(\psi_{1,\bar{x}}(F^2\psi_0)_\eta - \psi_{0,\eta}(F^2\psi_1)_{\bar{x}} - \\ & -(\frac{2}{r^2})(\psi_{0,\bar{x}}(F^2\psi_1) + \psi_{1,\bar{x}}(F^2\psi_0)) - \psi_{0,\bar{x}}(F^2\psi_1)_\eta - \\ & -\psi_{1,\eta}(F^2\psi_0)_{\bar{x}} - (\hat{M}^2 f^4/R_m)(\phi_{0,\bar{x}}(F^2\phi_0)_\eta - \phi_{0,\eta}(F^2\phi_0)_{\bar{x}} \\ & -(\frac{2}{r^2})\phi_{0,\bar{x}}(F^2\phi_0)) + \frac{Gf^4}{R_m}(\eta(\theta_{1,\eta})) \end{aligned} \tag{47}$$

$$F^2\phi_2 = fR_m(\psi_{0,\bar{x}}\phi_{1,\eta} - \psi_{0,\eta}\phi_{1,\bar{x}} + \psi_{1,\bar{x}}\phi_{0,\eta} - \psi_{1,\eta}\phi_{0,\bar{x}}) \tag{48}$$

The corresponding conditions on ψ_2, θ_2 , and ϕ_2 are

$$\psi_2(1,\bar{x}) = -0, \qquad (\psi_{2,\eta})_{\eta=1} = 0$$

$$(\eta\psi_{2,\eta\eta} - \psi_{2\eta})_{\eta=0} = 0, \qquad \psi_2(0,\bar{x}) = 0 \tag{49}$$

$$\theta_2(1,\bar{x}) = 0, \qquad (\theta_{2,\eta})_{\eta=0} = 0 \tag{50}$$

$$\phi_{2,x}(0,\bar{x}) = 0, \quad \phi_{2,\eta} = \hat{\phi}_{2,\mu} = 0 \;\; on \quad \eta = 1 \tag{51}$$

where

$$F^2 = \frac{\partial^2}{\partial \eta^2} - \frac{1}{\eta}\frac{\partial}{\partial \eta}$$

SOLUTION OF THE PROBLEM

Solving the coupled equations (34)-(36) subject to the corresponding boundary conditions (37)-(39), we get the expressions for zeroth order

$$\theta_0(\bar{x},\eta) = \gamma(\bar{x}) - f^2(1-\eta^2)/4$$

$$\psi_0 = a_5 + \frac{a_4}{4}\eta^2 - \frac{a_3}{360}\eta^6$$

$$\phi_0 = -\frac{\eta^2}{2}$$

Solving the coupled equations (40)–(42) subject to the corresponding conditions(43)-(45),the solution for $\theta_1, \psi_1 \; and \; \phi_1$ are

$$\theta_1 = \frac{a_9}{9}(\eta^3 - 1) + \frac{a_{10}}{25}(\eta^5 - 1) + \frac{a_{11}}{49}(\eta^7 - 1) - \frac{a_{12}}{81}(\eta^9 - 1)$$

$$\phi_1 = \frac{a_{21}}{4}\eta^2 + \frac{a_{22}}{9}\eta^3 + \frac{a_{33}}{25}\eta^5 + \frac{a_{24}}{64}\eta^8 + \frac{a_{25}}{81}\eta^9$$

$$\psi_1 = \frac{a_{61}}{9}\eta^3 + \frac{a_{62}}{180}\eta^5 + \frac{a_{63}}{576}\eta^6 + \frac{a_{64}}{1225}\eta^7 + \frac{a_{65}}{(36x64)}\eta^8 + \frac{a_{66}}{(49x81)}\eta^9$$
$$+ \frac{a_{67}}{6400}\eta^{10} + \frac{a_{68}}{(81x21)}\eta^{11} + \frac{a_{69}}{14400}\eta^{12} + \frac{a_{70}}{(160x121)}\eta^{13} + \frac{a_{71}}{(144x189)}\eta^{14}$$
$$+ \frac{B_1}{2}\eta^2 + B_2$$

where $a_1, a_2, \ldots\ldots\ldots\ldots, a_{71}$, B_1, B_2 are constants.

RATE OF HEAT TRANSFER

The local rate of heat transfer coefficient(Nusselt number) on the boundary of the pipe is calculated using the formula

$$Nu = \frac{1}{f(\theta_m - \theta_w)}\left(\frac{\partial \theta}{\partial \eta}\right)_{\eta=1}$$

where $$\theta_m = 2\int_0^1 \theta \, d\eta$$

and the corresponding expression is

$$Nu = \frac{B_{17} + \delta B_{18}}{f(B_{15} + \delta B_{16} - \gamma(\bar{x}))}$$

DISCUSSION OF THE NUMERICAL RESULTS

The aim of the analysis is to study of the effect of radiation on convective heat transfer flow in a non-uniform pipe which is maintained at non-uniform temperature in the presence of a constant heat source/sink. The coupled equations governing the flow and heat transfer have been solved using a perturbation technique. The velocity and the temperature distributions in the fluid region are analytically evaluated and their behaviour with reference to variations in the governing parameters M, β, α, α_1 and N_1 has been analyzed numerically. For computational purpose the geometry of the pipe wall in the non-dimensional form is assumed to be η = f $(\bar{x})$ = 1 +βexp(-x^2) and the prescribed wall temperature $\gamma(\bar{x})$ is chosen to be α_1 sin $(\bar{x})$. $\beta > 0$ corresponds to dilation and $\beta<0$ corresponds to constriction of the pipe. In this analysis we confine our study in a dilated pipe.

Figs. 1-10 give the profiles of u and v for different parametric values in a dilated pipe. We may note that the axial flow due to imposed flux and the pressure gradient is positive along the x-axis and hence negative axial velocity corresponds to the reversal flow. Such reversal flows are consequences of thermal buoyancy and give rise to convection cells. It is interesting to observe that the geometry of the boundary has direct influence on the occurrence of these convection cells. Fig.1 shows that the axial flow continuously positive in both heating and cooling cases of the pipe. It follows that the reversal flow appears in the case of higher $|G|\geq 3x10^3$ and no such reversal flow occurs in the case of G > 0.

Fig. 6 shows that this secondary velocity v in a dilated pipe is towards the boundary for all G. An increase in the strength of the magnetic field $M \leq 4$ decreases |u| and enhances with $M \geq 6$. Also |v| enhances in the fluid region with M. Fig 2 and 7 represents u and v with heat source parameter ‘α’. It is found that u enhances with increase in $\alpha \leq 4$ and depreciates with $\alpha \geq 6$ and it reduces with $|\alpha|$. v reduces with increase in $\alpha > 0$ and enhances with $|\alpha|$. From fig. 4 & 9, we find that ‘u’ enhances $\beta \leq 0.5$ and decreases with $\beta \geq 0.7$ and again at $\beta = 0.9$, u enhances in the central region and reduces in region adjacent. Also v enhances with increase in $\beta \geq 0.5$ (fig.9).The influence of the non-uniform temperature shows that |u| decreases with increase in the amplitude of the boundary temperature (fig. 5) while |v| enhances with α_1 (fig. 10). The variation of u and v with radiation parameter N_1 is sown in figs. 3 and 8. An increase in N_1 enhance u and reduces v in the entire flow region.

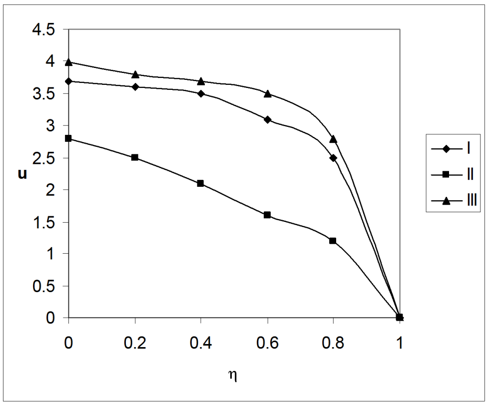

Fig. 1 : Variation of u with M

	I	II	III
M	2	4	6

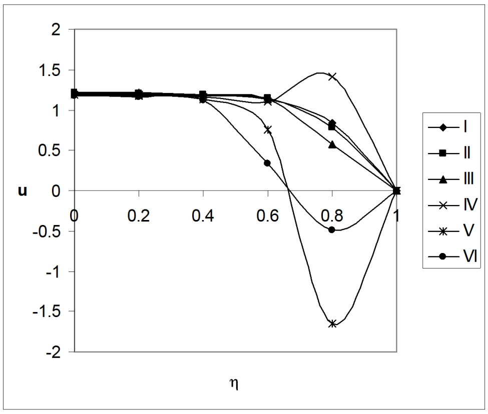

Fig. 2 : Variation of u with α

	I	II	III	IV	V	VI
α	2	4	6	-2	-4	-6

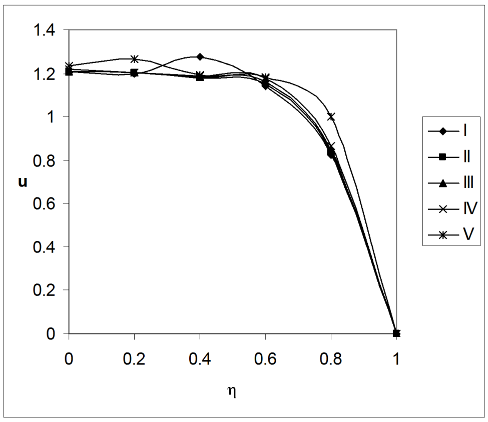

Fig. 3 : Variation of u with N

	I	II	III	IV	V
N_1	1.5	2.5	5	10	100

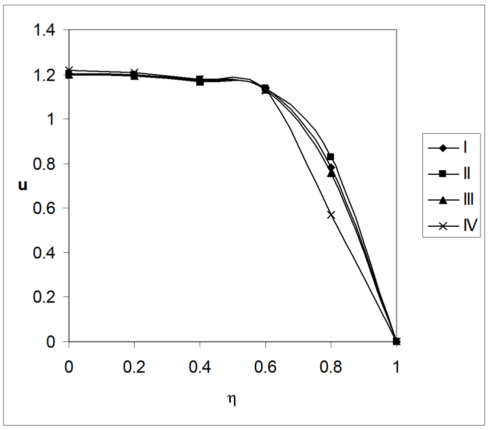

Fig. 4 : Variation of u with β

	I	II	III	IV
β	0.3	0.5	0.7	0.3

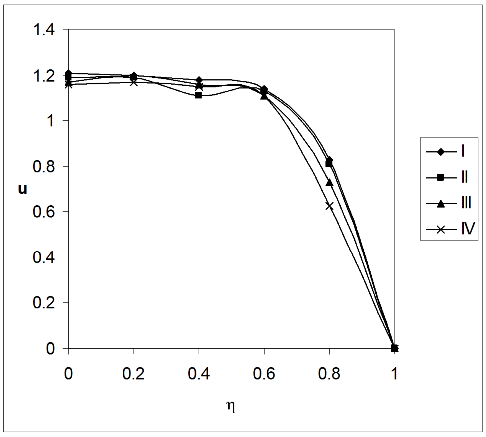

Fig. 5 : Variation of u with α_1

	I	II	III	IV
α_1	0.3	0.5	0.7	0.9

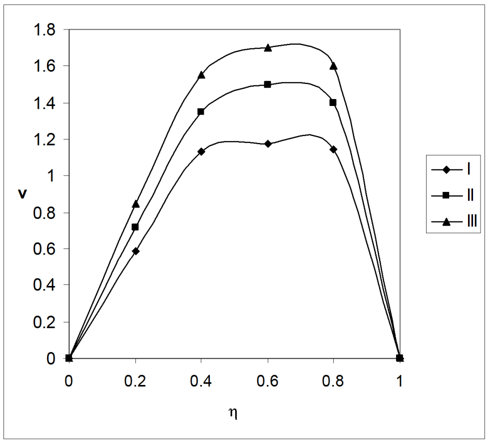

Fig. 6 : Variation of v with M

	I	II	III
M	2	4	6

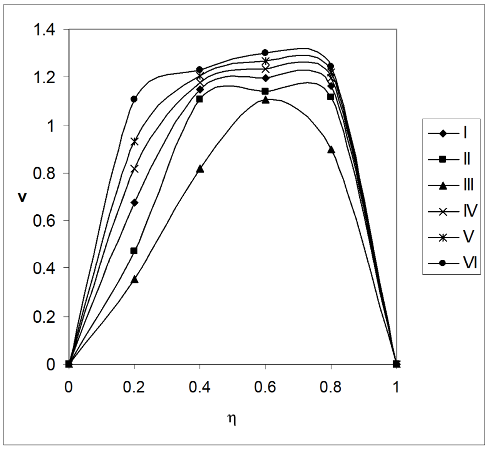

Fig. 7 : Variation of v with α

	I	II	III	IV	V	VI
α	2	4	6	-2	-4	-6

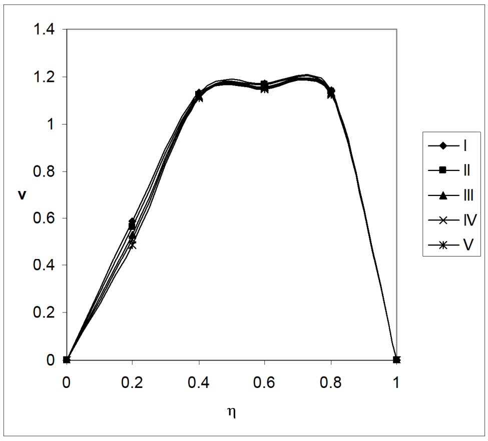

Fig. 8 : Variation of v with N

	I	II	III	IV	V
N_1	1.5	2.5	5	10	100

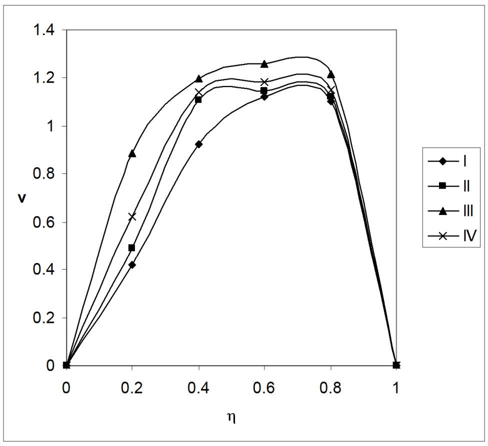

Fig. 9 : Variation of v with β

	I	II	III	IV
β	0.3	0.5	0.7	0.9

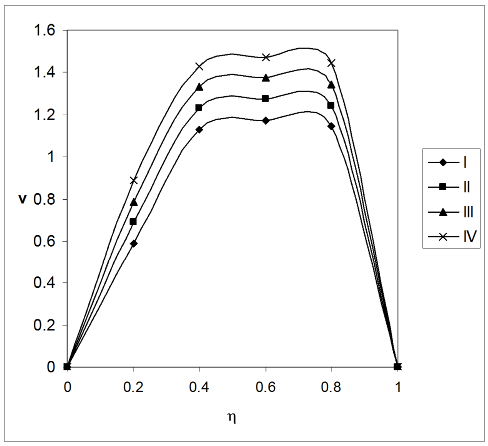

Fig. 10 : Variation of v with α_1

	I	II	III	IV
α_1	0.3	0.5	0.7	0.9

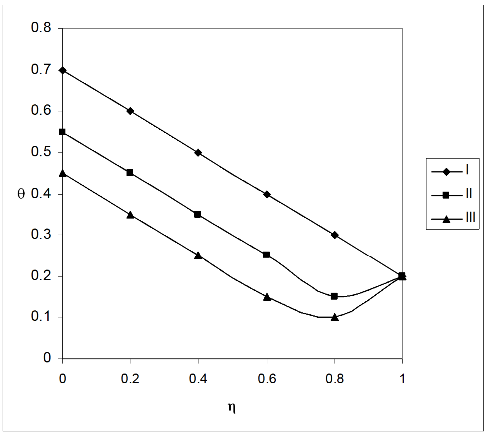

Fig. 11 : Variation of θ with M

	I	II	III
M	2	4	6

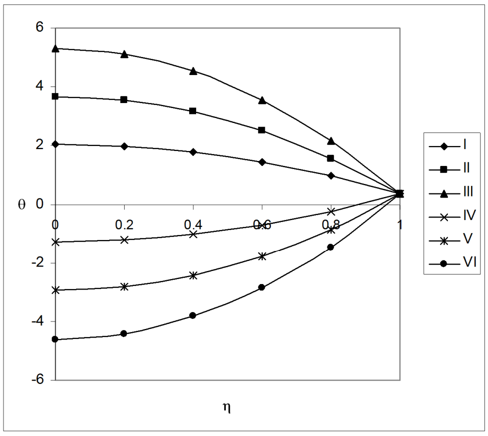

Fig. 12 : Variation of θ with α

	I	II	III	IV	V	VI
α	2	4	6	-2	-4	-6

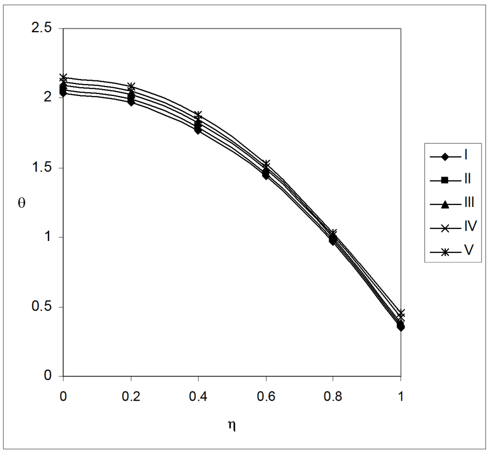

Fig. 13 : Variation of θ with N

	I	II	III	IV	V
N_1	1.5	2.5	5	10	100

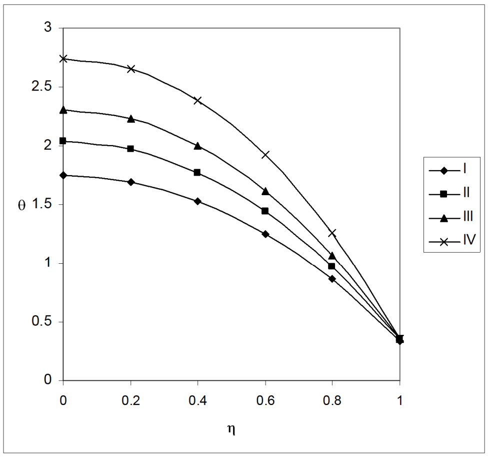

Fig. 14 : Variation of θ with β

	I	II	III	IV
β	0.3	0.5	0.7	0.3

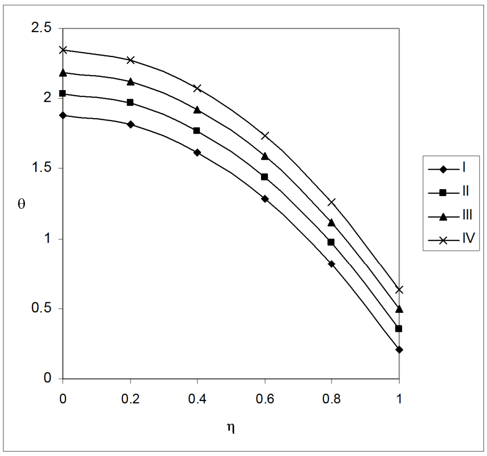

Fig. 15 : Variation of θ with α_1

	I	II	III	IV
α_1	0.3	0.5	0.7	0.9

The temperature distribution (θ) is exhibited is figures 11-15 for different variations in governing parameters M, N_1, α, α_1, β. It is found that θ is positive for all variations. The profile for 'θ' gradually falls from its maximum at the mid region to attain its prescribed value on the boundary η = 1. The variation of θ with Hartman number M reveals that the axial temperature depreciates with increase in M everywhere in the flow region (fig. 11). Fig. 12 represents the variation of θ with heat source parameter α. It is found that the axial temperature enhances with increase in α < 0 and reduces with |α|. An increase in N_1 leads to an enhancement in the axial temperature in the entire flow region (fig. 13). An increase in p results in an enhancement in the axial temperature. Thus greater the dilation of the pipe larger the axial temperature (fig. 14). We find from (fig. 15) that the axial temperature experiences an enhancement with increase in the amplitude α_1 of the boundary temperature.

The average Nusselt number (Nu) which measures the rate of heat transfer at the boundary is shown in tables (1 & 2) for different parametric values. An increase in the strength of heat source depreciates the rate of heat transfer for all G, while an increase in the strength heat source sink leads to the depreciation for |Nu| for G > 0 and enhances for G < 0. The variation of Nu with M indicates that higher the Loretz force larger the heat transfer at the boundary. The variation of Nu with β shows that greater the dilation of the pipe, smaller |Nu| for G > 0 and larger |Nu| for G < 0. Also an increase in the amplitude 'α'of the boundary temperature leads to an enhancement in the rate of heat transfer. From table 1 we find that an increase in the radiation parameter $N_1 \leq 2.5$ reduces the rate of heat transfer at boundary while for higher $N_1 \geq 5$. From table 2, we find that the rate of heat transfer depreciates in the heating case and enhances in the cooling case with increasing in β. An increase the amplitude α_1 of the boundary temperature results in an enhancement in the rate of heat transfer on the boundary.

Table-1 Nusselt Number (Nu) at η = 1

G	I	II	III	IV	V	VI	VII	VIII	IX
10^3	0.4507	0.311	0.274	-0.274	-0.101	-0.057	0.318	0.204	0.142
$3X10^3$	0.147	0.040	0.032	-0.572	-0.434	-0.413	0.104	-0.170	-0.253
-10^3	0.833	0.645	0.565	0.138	0.368	0.458	0.766	0.719	0.698
$-3X10^3$	1.329	1.065	0.918	0.743	1.078	1.267	1.281	1.471	1.541
α	2	4	6	-2	-4	-6	2	2	2
M	2	2	2	2	2	2	2	2	2
N_1	0.5	0.5	0.5	0.5	0.5	0.5	2.5	5	10

Table-2 Nusselt Number (Nu) at η = 1

G	I	II	III	IV	V	VI	VII
10^3	0.714	0.329	0.192	-0.069	0.451	0.565	0.674
$3X10^3$	0.557	0.337	0.265	0.144	0.447	0.550	0.647
-10^3	0.887	1.096	1.201	1.348	0.833	1.264	1.489
$-3X10^3$	1.080	1.166	2.149	2.448	1.329	1.485	1.637
β	0.3	0.5	0.7	0.9	0.5	0.5	0.5
α_1	0.3	0.3	0.3	0.3	0.5	0.7	0.9

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