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Convective Couette flow of a Jeffrey fluid in an inclined channel when the walls are provided with porous lining

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ABSTRACT

The effect of thickness of porous lining on the Couette flow of a Jeffrey fluid in a parallel plate channel is investigated. The channel is inclined at an angle ϕ with the horizontal. The flow in the free flow region is governed by Jeffrey model whereas the flow in the porous region is described by Darcy law. The velocity field and the temperature distribution are obtained. The results are depicted graphically and discussed for various physical parameters.

Keywords: *Parallel Plate channel, Jeffrey fluid, porous lining, Heat transfer.*

INTRODUCTION

Convection in the presence of internal heat source/ sink has several applications in fields of geophysical science, safety engineering, nuclear science, chemical engineering, heat exchangers, petroleum reservoir etc. Prasada Rao and Krishna [1] made an analysis on the free and forced convective flow of a viscous conducting fluid in a rotating channel maintained at constant temperature gradient along the walls. Alam et al. [2] discussed the numerical study of the combined free-forced convection and mass transfer flow past a vertical porous plate in a porous medium with heat generation and thermal diffusion. Rafael Cortell [3] investigated the internal heat generation and radiation effects on a certain free convection flow. Singh et al [4] studied the effect of volumetric heat source/ sink on mixed convection stagnation point flow on an isothermal vertical plate in porous media. Sreenadh et al. [5] described the free convection flow of Jeffrey fluid through a vertical deformable porous stratum.

The study of porous flow is very useful in effective tapping of ground water which in turn solves the water scarcity problems in drought areas. The biologists are interested in water movement through plants which comprise of porous material. Channabasappa et al. [6] investigated analytically the effect of the thickness of the porous material on the parallel plate channel flow when the walls are provided with non- erodible porous lining. Rapits [7] has studied the flow of a micropolar fluid through a porous medium, taking angular velocity into account. Kumaraswamy Naidu et al. [8] investigated the effect of the thickness of the porous material on the parallel plate channel flow of Jeffrey fluid when the walls are provided with non-erodible porous lining.

The flow of non-Newtonian fluids is widely observed in industry and physiology. Most of the theoretical investigations have been carried out by assuming that blood and most of the physiological fluids behave like non-Newtonian fluids. Nadeem et al. [9] studied the effect of Jeffrey fluid with variable viscosity in the form of a well known Reynolds model of viscosity in an asymmetric channel. Srinivas et al. [10] investigated the peristaltic transport of a Jeffrey fluid under the effect of slip in an inclined asymmetric channel. Akbar et al. [11] studied Jeffery fluid model for blood flow through a tapered artery with a stenosis by assuming blood as Jeffrey fluid. Mekheimer et al. [12] studied the equations for the two dimensional incompressible fluid flow of an electrically conducting Jeffrey fluid. Lie group analysis was performed and group invariant solutions were obtained. The effects of Jeffrey fluid parameters λ_1 and λ_2 were noted. Hayat et al. [13] described the mixed convection stagnation point flow and heat transfer of a Jeffrey fluid toward a stretching surface.

The object of this paper is to develop a theoretical model for analyzing the effect of porous lining of on Couette flow of a Jeffery fluid in a parallel plate channel with heat transfer, when one of the parallel walls is provided with porous lining. The influence of the non dimensional parameters representing the thickness of the porous medium on the velocity field in the channel and the temperature has been studied. The results are discussed for various physical parameters.

Nomenclature

u	velocity of the fluid
λ_1	Jeffrey parameter
$\frac{\partial p}{\partial x}$	pressure gradient
μ	viscosity
Q	Darcy velocity
k	permeability of the porous medium
ρ	density
Gr	Grashof number
g	acceleration due to gravity
h	distance between porous layers
ϕ	angle of the inclination of the channel to the horizontal
T_0	temperatures of the lower plate
T_1	temperatures of the upper plate
β	the volumetric expansion coefficient
α	slip parameter
σ	permeability parameter
β_1	heat source parameter
Nu	Nusselt number

2. Mathematical Formulation of the Problem

Consider the Couette flow in an inclined channel where the lower end of the plate is lined with porous material (see Fig.1). The lower plate is at rest, whereas the upper one moves with a constant velocity u_0. Let the inclination of the channel with horizontal be ϕ . Let h be the width of the free flow region above the porous medium and δ be the width of the porous lining. The temperature at lower porous layer is T_0 whereas the temperature of the moving upper plate is T_1. Let the x -axis (the flow direction) be taken midway between the porous interface and upper plate of the channel.

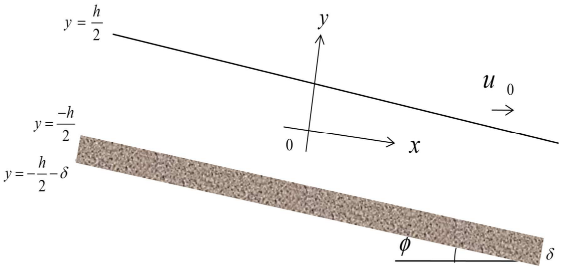

Fig.1. Physical model

The governing equations for the flow are

$$\frac{\partial u}{\partial x}=0 \tag{1}$$

$$\left(\frac{\mu}{1+\lambda_1}\right)\frac{d^2u}{dy^2}-G+\rho g\beta(T-T_0)\sin\phi=0 \tag{2}$$

$$k\frac{d^2T}{dy^2}+Q_0=0 \tag{3}$$

where $G=\frac{\partial p}{\partial x}$

The Darcy velocity is

$$Q=\frac{-k(1+\lambda_1)}{\mu}\frac{\partial p}{\partial x} \tag{4}$$

The boundary conditions of the problem are

$$u=u_0 \quad at \quad y=\tfrac{h}{2} \tag{5}$$

$$\frac{du}{dy}=\frac{\alpha}{\sqrt{k}}u \quad at \quad y=-\tfrac{h}{2} \tag{6}$$

$$T=T_0 \quad at \quad y=-\tfrac{h}{2} \tag{7}$$

$$T=T_1 \quad at \quad y=\tfrac{h}{2} \tag{8}$$

The non-dimensional quantities are given by

$$u^*=\frac{u}{U},\quad x^*=\frac{x}{h}\quad \eta=\frac{y}{h},\quad p^*=\frac{p}{\left(\mu U/h\right)},Gr=\frac{\beta g T_0 h^2}{U\nu},T^*=\frac{T-T_0}{T_0},$$

$$\sigma=\frac{h}{\sqrt{k}},\quad \nu=\frac{\mu}{\rho},\quad \beta_1=\frac{Q_0h^2}{kT_0},\quad Q^*=\frac{Q}{U},\quad G^*=\frac{G}{\mu U} \tag{9}$$

Substituting (9) into equations (1)-(8), and for simplicity dropping the asterisks, we get

$$\frac{\partial u}{\partial x} = 0 \quad (10)$$

$$\frac{d^2u}{d\eta^2} - G(1+\lambda_1) + Gr(1+\lambda_1)T\sin\phi = 0 \quad (11)$$

$$\frac{d^2T}{d\eta^2} + \beta_1 = 0 \quad (12)$$

The Darcy velocity is given by

$$Q = -\frac{G(1+\lambda_1)}{\sigma^2} \quad (13)$$

$$u = u_0 \quad at \quad \eta = \tfrac{1}{2} \quad (14)$$

$$\frac{du}{d\eta} = \alpha\sigma u \quad at \quad \eta = -\tfrac{1}{2} \quad (15)$$

$$T = 0 \quad at \quad \eta = -\tfrac{1}{2} \quad (16)$$

$$T = m-1 \quad at \quad \eta = \tfrac{1}{2} \quad (17)$$

3. Solution of the problem

Solving equation (12) subject to the boundary conditions (16) and (17), we get the temperature distribution as

$$T = \frac{-\beta_1\eta^2}{2} + A\eta + B \quad (18)$$

Solving (11) subject to the conditions (14) and (15), we get the velocity distribution in the form

$$u = G(1+\lambda_1)\frac{\eta^2}{2} + Gr(1+\lambda_1)\left[\frac{\beta_1\eta^4}{24} - \frac{A\eta^3}{6}\frac{B\eta^2}{2}\right]\sin\phi + C\eta + D \quad (19)$$

4. Rate of Heat Transfer

The rate of heat transfer at the wall $\eta = 1$ is given by

$$Nu = -\left(\frac{\partial T}{\partial \eta}\right)_{\eta=1} = \beta_1 - A \quad (20)$$

Table 1 : Rate of heat transfer Nu at $\eta = 1$ for different values of heat source parameter β_1

Nu	0.15	0.25	0.35	0.45	0.55	0.65	0.75	0.85	0.95
β_1	0.65	0.75	0.85	0.95	1.05	1.15	1.25	1.35	1.45

where,

$$m = \frac{T_1}{T_0},\ A = m-1,\ B = \frac{\beta_1}{8} + \frac{m-1}{2},\ A_1 = \frac{\beta}{384} - \frac{A}{48} - \frac{B}{8},\ A_2 = Gr(1+\lambda_1)A_1\sin\phi,$$

$A_3 = \frac{G(1+\lambda_1)}{8}$, $A_4 = \frac{-\beta_1}{48} - \frac{A}{8} + \frac{B}{2}$, $A_5 = \frac{\beta_1}{384} + \frac{A}{48} - \frac{B}{8}$, $A_6 = \frac{G(1+\lambda_1)(\alpha\sigma+4)}{8}$,

$A_7 = Gr(1+\lambda_1)(\alpha\sigma A_5 - A_4)\sin\phi$, $A_8 = \frac{\alpha\sigma}{2} + 1$, $A_9 = \frac{A_6 + A_7}{2A_8}$, $A_{10} = 1 + \frac{\alpha\sigma}{2A_8}$,

$A_{11} = A_2 + A_3 + A_9$, $D = \frac{u_0 - A_{11}}{A_{10}}$, $C = \frac{A_6 + A_7 + \alpha\sigma D}{A_8}$

RESULTS AND DISCUSSION

The numerical values of velocity are computed from equation (11) and are depicted in Figures 2 to 5 for flow in a channel with one side porous lining. We observe from Fig. 2 that the velocity decreases with the increase in the permeability parameter σ. From Fig. 3 we observe that the velocity increases with the increase in the Jeffrey parameter λ_1. From Fig.4 we observe that the velocity decreases with the increase slip parameter α. From Fig.5 we observe that the velocity decreases with the increase in Grashof number Gr .

The numerical values of the temperature are computed from equation (12) and are depicted in Fig. 6. We observe that the temperature increases with the increase in the heat source parameter β_1.

The Nusselt number Nu at the wall $\eta = 1$ is computed from equation (20) for different values of heat source parameter β_1 and is presented in Table1. It is observed that the Nusselt number Nu increases with increasing heat source parameter β_1.

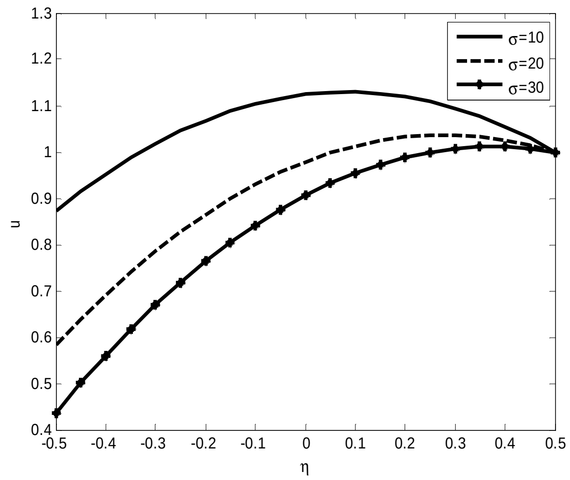

Fig.2 Velocity distribution for various values of σ for fixed λ_1 =0.5, α =0.1, ϕ=0.15, m=1, G=-1, Gr=0.5, u_0 =1, $\beta_1 = 0.1$

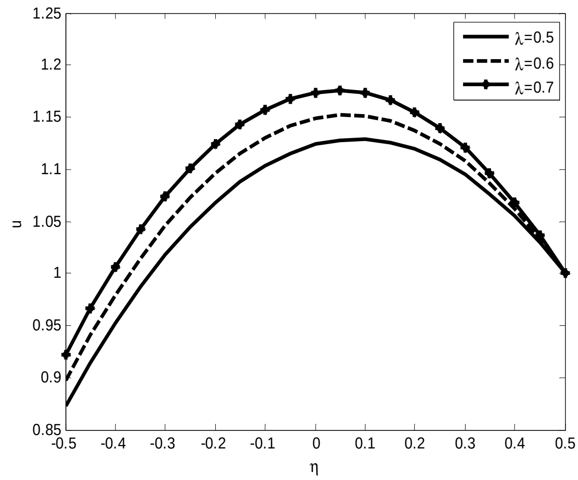

Fig.3 Velocity distribution for various values of λ_1 for fixed σ=10, α =0.1, ϕ=0.15, m=1, G=-1, Gr=0.5, u_0 =1, $\beta_1 = 0.1$

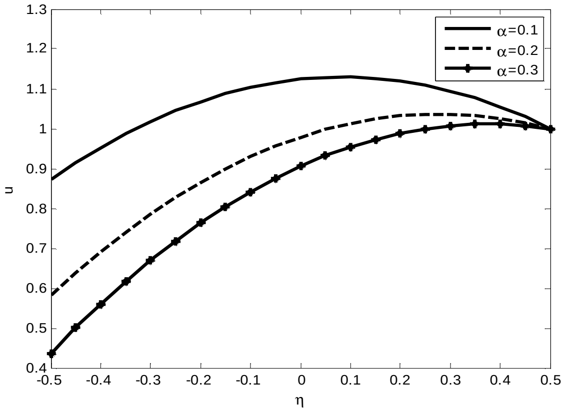

Fig.4 Velocity distribution for various values of α for fixed
σ=10, λ_1 =0.5, ϕ=0.15, m=1, G=-1, Gr=0.5, u_0 =1, $\beta_1 = 0.1$

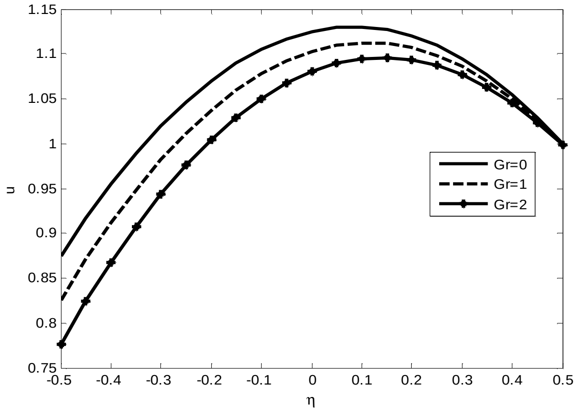

Fig.5 Velocity distribution for various values of Gr for fixed
σ=10, λ_1 = 0.5, α =0.1, ϕ=0.15, m=1, G=-1, u_0 =1, $\beta_1 = 0.9$

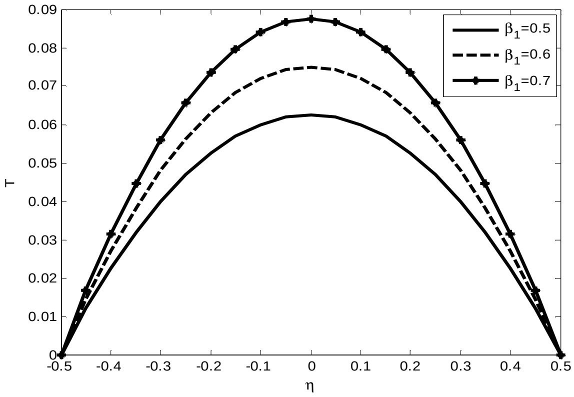

Fig.6 Velocity distribution for various values of β_1 for fixed
m=1, β_1 =0.7

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