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Constructing p-Groups from two permutation groups by wreath products method

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ABSTRACT
Let p be an arbitrary but fixed prime number. Let Ω be a finite set with n elements and G is a permutation group on Ω. We apply the concepts of p-groups and wreath products of two permutation groups to construct some groups of prime power order.

INTRODUCTION

The p-groups and its concepts are very important in the theory of groups. Thus, the study of finite groups is incomplete without p-groups. It is therefore, natural that mathematicians should want to study, quantify and exploit p-groups.

Definition 1.1
A group is a p-group if the order is a prime or prime power.

Definition 1.2
Direct product of a group is the Cartesian product of groups (considered as element of set) with component-wise multiplication.

Definition 1.3
The semi direct product of a group H with a group G acting on H via homomorphism μ from G into the automorphism group of H is the Cartesian product $G \times H$ with the multiplication

$$(g,n).(h,m) = (gh, n^{h\mu}m)$$

2.0 PRELIMINARIES

The following preliminary results will be required in the constructions of p-groups as proposed in this paper.

2.1 Group Action

Let G be a group and Ω be a non-empty set. We say that G act on the set Ω (or that G permutes Ω) if to each g in G and each α in Ω, there corresponds a unique point α g in Ω such that $\forall$ α in Ω and $g_1 g_2$ in G we have

1. $g_1(g_1 . \alpha) = (g_1 . g_2)\alpha$ and
2. $\alpha . 1 = \alpha$

Let H and K be two groups. We say that H act on K as a group if to each k in K and h in H there corresponds a unique element k^h in K such that for h_1, h_1, h_3 in H and $k_{1,} k_{2,} k_3$ in K

1. $(k^{h_1})^{h_2} = k^{h_1 h_2}$, $k^1 = k$ and
2. $(k_1 k_2)^h = k_1{}^h k_2{}^h$

2.2 Theorem

Let C and D be permutation group on Γ and Δ respectively. Let C^Δ be the set of all maps of Δ into the permutation group C. That is $C^\Delta = \{f: \Delta \longrightarrow C\}$ $\forall$ $f_1 f_2$ in C. Let $\forall$ $f_1 f_2$ in C^Δ be defined $\forall$ δ in Δ by

(f_1, f_2) $(\delta) = f_1(\delta)$ $f_1(\delta)$

With respect to this operation of multiplication, C^Δ acquire a structure of a group.

2.3 Lemma

Assume that D acts on P as follows

$f^d(\delta) = f^d(\delta d^{-1})$

for all $\delta \in \Delta, d \in D$. Then D acts on P as a group.

Proof:

Take $f, f_{1,} f_2 \in P$ and $d, d_{1,} d_2 \in D$ then

i. $(f^{d_1})^{d_2}(\delta) = f^{d_1}(\delta_2{}^{-1})$
$= f(\delta d_1{}^{-1} d_2{}^{-1})$
$= f^{d_1 d_2}(\delta)$

ii. $f^1(\delta) = f(\delta_2{}^{-1})$
$= f(\delta)$

iii. $(f_1 f_2)^d(\delta) = f_1 f_2(\delta d^{-1})$
$= f_1(d^{-1}) f_2(\delta d^{-1})$
$= f_1{}^d(\delta)$ $f_1{}^d(\delta)$

Thus D acts on P as a Group.

2.4 Theorem

Let D act on P as a group. Then the set of all ordered pairs (f, d) with $f \in P$, $d \in D$ is a group if we define for all $f_{1,} f_2 \in P$ and $d_{1,} d_2 \in D$ by

$$(f_1 d_1)(f_2 d_2) = \left(f_1 f_2{}^{d_1{}^{-1}}\right)(d_1 d_2)$$

Proof:

i Closure property follows from the definition of multiplication.
ii. Take $f, f_1, f_2 \in P$ and $d, d_1, d_2 \in D$ then

$$[(f_1 d_1)(f_2 d_2)](f_3 d_3) = (f_1\, f_2^{\,d_1^{-1}}, d_1 d_2)(f_3 d_3)$$
$$= (\, f_1\, f_2^{\,d_1^{-1}} f_3^{\ (d_1 d_2)^{-1}}, d_1 d_2 d_3)$$
$$= (\, f_1\, f_2^{\,d_1^{-1}} f_3^{\ d_1^{-1} d_2^{-1}}, d_1 d_2 d_3)$$

Similarly

$$(f_1, d_1)\, [(f_2 d_2)(f_3 d_3)] = ((f_1, d_1)\, (f_2\, f_3^{\,d_1^{-1}}, d_1 d_2)$$
$$= (f_1 (f_2\, f_3^{\ d_2^{-1}}), d_1 d_2 d_3)$$
$$= (\, f_1\, f_2^{\,d_1^{-1}} f_3^{\ d_1^{-1} d_2^{-1}}, d_1 d_2 d_3)$$

Hence multiplication is associative.
iii. For every $f \in P$, $f^1 = f$. Now for every $d \in D$,
the map $f \longrightarrow f^d$ is an automorphism of P. Also if e is the identity element of P, then $e^d = e$. Also, $(f^{-1})^d = f^{d^{-1}}$. Now

$$(f, d)(e, 1) = (f e^{d^{-1}}, d1)\ (f e^{d^{-1}}, d)$$
$$(f(e^{-1}), d) = \ (f, d)$$

Thus identity element exists.
iv.

$$((f, d)(f^{-1})^d, d^{-1}) = ((f^{-1})^d)^{-d}, dd^{-1})$$
$$= (f(f^{-1})^{dd^{-1}}, dd^{-1})$$
$$= (f(f^{-1})^1, dd^{-1}) = (e, 1)$$

Thus when D acts on P, the set of all ordered pairs (f, d) with $f \in P$ and $d \in D$ is a group if we define

$$(f_1 d_1)(f_2 d_2) = \left(f_1 f_2^{\,d_1^{-1}}\right)(d_1 d_2)$$

3.0 WREATH PRODUCTS

The Wreath product of C and D denoted by $W = C\ wr\ D$ is the semi-product of P by D, so that $W = \{(f, d) |\ f \in P, \delta \in \Delta\}$, with multiplication in W defined as

$$(f_1 d_1)(f_2 d_2) = \left(f_1 f_2^{\,d_1^{-1}}\right)(d_1 d_2)$$

For all $f_1 f_2 \in P$ and,$d_1 d_2 \in D$.

3.1 Theorem

Let D act on P as $f^d(\delta_1) = f\ (\delta d^{-1})$ where $f \in P$, $d \in D$ and $\delta \in \Delta$. Let W be the group of all juxtaposed symbols fd, with $f \in P$ and $d \in D$ and multiplication given by

$$(f_1 d_1)(f_2 d_2) = \left(f_1 f_2^{\,d_1^{-1}}\right)(d_1 d_2)$$

Then W is a group called the semi-direct product of P by D with the defined action.

3.2 Remarks

1. If C and D are finite groups, then the wreath product W determined by an action of D on a finite set is a finite group of order $|\,W\,| = |C|^{|\Delta|}$

2. P is a normal subgroup of W and D is a subgroup of N.
3. The action of W on $\Gamma \times \Delta$ is given by
$$(\beta, \delta)fd = (\beta f(\delta), d\delta) \text{ where } \beta \in \Gamma \text{ and } \delta \in \Delta.$$

RESULTS

We shall now construct some p-groups by means of wreath product of two permutation groups as introduced above.

4.1 Consider the permutation groups C_1 and D_1

C_1= {(1),(12345),(13524),(14253),(15432)}
D_1={(1),(6,7)} acting on the sets S_1 ={1,2,3,4,5} and Δ_1={6,7} respectively.
Let $P = C_1^{\Delta_1}= \{f: \Delta_1 \longrightarrow C_1\}$. Then $|P| = |C_1|^{|\Delta_1|}=5^2=25$
The mappings are
F_1: 6—› (1), 7—›(1)
F_2: 6—›(12345), 7—›(12345)
F_3: 6—›(13524), 7—›(13524)
F_4: 6—›(14253), 7—›(14253)
F_5: 6—›(15432), 7—›(15432)
F_6: 6—›(1), 7—›(12345)
F_7: 6—›(1), 7—›(13524)
F_8: 6—›(1), 7—›(14253)
F_9: 6—›(1), 7—›(15432)
F_{10}: 6—›(12345), 7—›(1)
F_{11}: 6—›(12345), 7—›(13524)
F_{12}: 6—›(12345), 7—›(14253)
F_{13}: 6—›(12345), 7—›(15432)
F_{14}: 6—›(13524), 7—›(1)
F_{15}: 6—›(13524), 7—›(12345)
F_{16}: 6—›(13524), 7—›(14253)
F_{17}: 6—›(13524), 7—›(15432)
F_{18}: 6—›(14253), 7—›(1)
F_{19}: 6—›(14253), 7—›(12345)
F_{20}: 6—›(14253), 7—›(13524)
F_{21}: 6—›(14253), 7—›(15432)
F_{22}: 6—›(15432), 7—›(1)
F_{23}: 6—›(15432), 7—›(12345)
F_{24}: 6—›(15432), 7—›(13524)
F_{25}: 6—›(15432), 7—›(14253)

We can easily verify that P is a group with respect to the operations
(f_1, f_2) (δ) = $f_1(\delta_1)$ $f_1(\delta_1)$, where $\delta_1 \in \Delta_1$.

We recall the definition of the action of D_1 on P as $f^d(\delta_1) = f\ (\delta_1 d^{-1})$ where $f \in P$, $d \epsilon D_1$ and $\delta_1 \in \Delta_1$, then D_1 acts on P as a groups.

We also recall the definition $W = C_1 wr\ D_1$, the semi-direct product of P by D_1 in that order; i.e.

$W = \{(f,d) \mid f \in P, \delta_1 \in \Delta_1\}$.

Now, W is a group with respect to the operation;
$(f_1 d_1)(f_2 d_2) = \left(f_1 {f_2}^{{d_1}^{-1}}\right)(d_1 d_2)$, and
Accordingly, d_1= (1), d_2=(6,7). Then the elements of W are

$(f_1 d_1)$, $(f_2 d_1)$, $(f_3 d_1)$, $(f_4 d_1)$, $(f_5 d_1)$, $(f_6 d_1)$, $(f_7 d_1)$, $(f_8 d_1)$, $(f_9 d_1)$, $(f_{10} d_1)$, $(f_{11} d_1)$, $(f_{12} d_1)$, $(f_{13} d_1)$, $(f_{14} d_1)$, $(f_{15} d_1)$, $(f_{16} d_1)$, $(f_{17} d_1)$, $(f_{18} d_1)$, $(f_{19} d_1)$, $(f_{20} d_1)$, $(f_{21} d_1)$, $(f_{22} d_1)$, $(f_{23} d_1)$, $(f_{24} d_1)$, $(f_{25} d_1)$, $(f_1 d_1)$, $(f_2 d_2)$, $(f_3 d_2)$, $(f_4 d_2)$, $(f_5 d_2)$, $(f_6 d_2)$, $(f_7 d_2)$, $(f_8 d_2)$, $(f_9 d_2)$, $(f_{10} d_2)$, $(f_{11} d_2)$, $(f_{12} d_2)$, $(f_{13} d_2)$, $(f_{14} d_2)$, $(f_{15} d_2)$, $(f_{16} d_2)$, $(f_{17} d_2)$, $(f_{18} d_2)$, $(f_{19} d_2)$, $(f_{20} d_2)$, $(f_{21} d_2)$, $(f_{22} d_2)$, $(f_{23} d_2)$, $(f_{24} d_2)$, $(f_{25} d_2)$
Now, define action of W on $\Gamma \times \Delta$ as
$(\beta, \delta) f d = (\beta f(\delta), d\delta)$ where $\beta \in \Gamma$ and $\delta \in \Delta$..
Further, $\Gamma \times \Delta$ ={(1,6),(1,7),(2, 6),(2,7),(3,6),(3,7),(4,6),(4,7),(5,6),(5,7)}
We obtain the following permutation by action of W on $\Gamma \times \Delta$

$$(1,6) f_1 d_1 = (1 f_1(6), d_1) = \big(1(1), 6(1)\big) = (1,6)$$
$$(1,7) f_1 d_1 = (1 f_1(7), d_1) = \big(1(1), 7(1)\big) = (1,7)$$
$$(2,6) f_1 d_1 = (2 f_1(6), d_1) = \big(2(1), 6(1)\big) = (2,6)$$
$$(2,7) f_1 d_1 = (2 f_1(7), d_1) = \big(2(1), 7(1)\big) = (2,7)$$
$$(3,6) f_1 d_1 = (3 f_1(6), d_1) = \big(3(1), 6(1)\big) = (3,6)$$
$$(3,7) f_1 d_1 = (3 f_1(7), d_1) = \big(3(1), 7(1)\big) = (3,7)$$
$$(4,6) f_1 d_1 = (4 f_1(6), d_1) = \big(4(1), 6(1)\big) = (4,6)$$
$$(4,7) f_1 d_1 = (4 f_1(7), d_1) = \big(4(1), 7(1)\big) = (4,7)$$
$$(5,6)^{f_1 d_1} = (5 f_1(6), d_1) = \big(5(1), 6(1)\big) = (5,6)$$
$$(5,7)^{f_1 d_1} = (5 f_1(7), d_1) = \big(5(1), 7(1)\big) = (5,7)$$

And in summary,

$$(\Gamma \times \Delta) f_1 d_1 = \begin{pmatrix} (1,6)(1,7)(2,6)(2,7)(3,6)(3,7)(4,6)(4,7)(5,6)(5,7) \\ (1,6)(1,7)(2,6)(2,7)(3,6)(3,7)(4,6)(4,7)(5,6)(5,7) \end{pmatrix}$$
$$(\Gamma \times \Delta) f_2 d_1 = \begin{pmatrix} (1,6)(1,7)(2,6)(2,7)(3,6)(3,7)(4,6)(4,7)(5,6)(5,7) \\ (2,6)(2,7)(3,6)(3,7)(4,6)(4,7)(5,6)(5,7)(1,6)(1,7) \end{pmatrix}$$
$$(\Gamma \times \Delta) f_3 d_1 = \begin{pmatrix} (1,6)(1,7)(2,6)(2,7)(3,6)(3,7)(4,6)(4,7)(5,6)(5,7) \\ (3,6)(3,7)(4,6)(4,7)(5,6)(5,7)(1,6)(1,7)(2,6)(2,7) \end{pmatrix}$$
$$(\Gamma \times \Delta) f_4 d_1 = \begin{pmatrix} (1,6)(1,7)(2,6)(2,7)(3,6)(3,7)(4,6)(4,7)(5,6)(5,7) \\ (4,6)(4,7)(5,6)(5,7)(1,6)(1,7)(2,6)(2,7)(3,6)(3,7) \end{pmatrix}$$
$$(\Gamma \times \Delta) f_5 d_1 = \begin{pmatrix} (1,6)(1,7)(2,6)(2,7)(3,6)(3,7)(4,6)(4,7)(5,6)(5,7) \\ (5,6)(5,7)(1,6)(1,7)(2,6)(2,7)(3,6)(3,7)(4,6)(4,7) \end{pmatrix}$$
$$(\Gamma \times \Delta) f_6 d_1 = \begin{pmatrix} (1,6)(1,7)(2,6)(2,7)(3,6)(3,7)(4,6)(4,7)(5,6)(5,7) \\ (1,6)(2,7)(2,6)(3,7)(3,6)(4,7)(4,6)(5,7)(5,6)(1,7) \end{pmatrix}$$
$$(\Gamma \times \Delta) f_7 d_1 = \begin{pmatrix} (1,6)(1,7)(2,6)(2,7)(3,6)(3,7)(4,6)(4,7)(5,6)(5,7) \\ (1,6)(3,7)(2,6)(4,7)(3,6)(5,7)(4,6)(1,7)(5,6)(2,7) \end{pmatrix}$$

$$(\Gamma\times\Delta)f_8d_1=\begin{pmatrix}(1,6)(1,7)(2,6)(2,7)(3,6)(3,7)(4,6)(4,7)(5,6)(5,7)\\(1,6)(4,7)(2,6)(5,7)(3,6)(1,7)(4,6)(2,7)(5,6)(3,7)\end{pmatrix}$$
$$(\Gamma\times\Delta)f_9d_1=\begin{pmatrix}(1,6)(1,7)(2,6)(2,7)(3,6)(3,7)(4,6)(4,7)(5,6)(5,7)\\(1,6)(5,7)(2,6)(1,7)(3,6)(2,7)(4,6)(3,7)(5,6)(4,7)\end{pmatrix}$$
$$(\Gamma\times\Delta)f_{10}d_1=\begin{pmatrix}(1,6)(1,7)(2,6)(2,7)(3,6)(3,7)(4,6)(4,7)(5,6)(5,7)\\(2,6)(1,7)(3,6)(2,7)(4,6)(3,7)(5,6)(4,7)(1,6)(5,7)\end{pmatrix}$$
$$(\Gamma\times\Delta)f_{11}d_1=\begin{pmatrix}(1,6)(1,7)(2,6)(2,7)(3,6)(3,7)(4,6)(4,7)(5,6)(5,7)\\(2,6)(3,7)(3,6)(4,7)(4,6)(5,7)(5,6)(1,7)(1,6)(2,7)\end{pmatrix}$$
$$(\Gamma\times\Delta)f_{12}d_1=\begin{pmatrix}(1,6)(1,7)(2,6)(2,7)(3,6)(3,7)(4,6)(4,7)(5,6)(5,7)\\(2,6)(4,7)(3,6)(5,7)(4,6)(1,7)(5,6)(2,7)(1,6)(3,7)\end{pmatrix}$$
$$(\Gamma\times\Delta)f_{13}d_1=\begin{pmatrix}(1,6)(1,7)(2,6)(2,7)(3,6)(3,7)(4,6)(4,7)(5,6)(5,7)\\(2,6)(5,7)(3,6)(1,7)(4,6)(2,7)(5,6)(3,7)(1,6)(4,7)\end{pmatrix}$$
$$(\Gamma\times\Delta)f_{14}d_1=\begin{pmatrix}(1,6)(1,7)(2,6)(2,7)(3,6)(3,7)(4,6)(4,7)(5,6)(5,7)\\(3,6)(1,7)(4,6)(2,7)(5,6)(3,7)(1,6)(4,7)(2,6)(5,7)\end{pmatrix}$$
$$(\Gamma\times\Delta)f_{15}d_1=\begin{pmatrix}(1,6)(1,7)(2,6)(2,7)(3,6)(3,7)(4,6)(4,7)(5,6)(5,7)\\(3,6)(2,7)(4,6)(3,7)(5,6)(4,7)(1,6)(5,7)(2,6)(1,7)\end{pmatrix}$$
$$(\Gamma\times\Delta)f_{16}d_1=\begin{pmatrix}(1,6)(1,7)(2,6)(2,7)(3,6)(3,7)(4,6)(4,7)(5,6)(5,7)\\(3,6)(4,7)(4,6)(5,7)(5,6)(1,7)(1,6)(2,7)(2,6)(3,7)\end{pmatrix}$$
$$(\Gamma\times\Delta)f_{17}d_1=\begin{pmatrix}(1,6)(1,7)(2,6)(2,7)(3,6)(3,7)(4,6)(4,7)(5,6)(5,7)\\(3,6)(5,7)(4,6)(1,7)(5,6)(2,7)(1,6)(3,7)(2,6)(4,7)\end{pmatrix}$$
$$(\Gamma\times\Delta)f_{18}d_1=\begin{pmatrix}(1,6)(1,7)(2,6)(2,7)(3,6)(3,7)(4,6)(4,7)(5,6)(5,7)\\(4,6)(1,7)(5,6)(2,7)(1,6)(3,7)(2,6)(4,7)(3,6)(5,7)\end{pmatrix}$$
$$(\Gamma\times\Delta)f_{19}d_1=\begin{pmatrix}(1,6)(1,7)(2,6)(2,7)(3,6)(3,7)(4,6)(4,7)(5,6)(5,7)\\(4,6)(2,7)(5,6)(3,7)(1,6)(4,7)(2,6)(5,7)(3,6)(1,7)\end{pmatrix}$$
$$(\Gamma\times\Delta)f_{20}d_1=\begin{pmatrix}(1,6)(1,7)(2,6)(2,7)(3,6)(3,7)(4,6)(4,7)(5,6)(5,7)\\(4,6)(3,7)(5,6)(4,7)(1,6)(5,7)(2,6)(1,7)(3,6)(2,7)\end{pmatrix}$$
$$(\Gamma\times\Delta)f_{21}d_1=\begin{pmatrix}(1,6)(1,7)(2,6)(2,7)(3,6)(3,7)(4,6)(4,7)(5,6)(5,7)\\(4,6)(5,7)(5,6)(1,7)(1,6)(2,7)(2,6)(3,7)(3,6)(4,7)\end{pmatrix}$$
$$(\Gamma\times\Delta)f_{22}d_1=\begin{pmatrix}(1,6)(1,7)(2,6)(2,7)(3,6)(3,7)(4,6)(4,7)(5,6)(5,7)\\(5,6)(1,7)(1,6)(2,7)(2,6)(3,7)(3,6)(4,7)(4,6)(5,7)\end{pmatrix}$$
$$(\Gamma\times\Delta)f_{23}d_1=\begin{pmatrix}(1,6)(1,7)(2,6)(2,7)(3,6)(3,7)(4,6)(4,7)(5,6)(5,7)\\(5,6)(2,7)(1,6)(3,7)(2,6)(4,7)(3,6)(5,7)(4,6)(1,7)\end{pmatrix}$$
$$(\Gamma\times\Delta)f_{24}d_1=\begin{pmatrix}(1,6)(1,7)(2,6)(2,7)(3,6)(3,7)(4,6)(4,7)(5,6)(5,7)\\(5,6)(3,7)(1,6)(4,7)(2,6)(5,7)(3,6)(1,7)(4,6)(2,7)\end{pmatrix}$$
$$(\Gamma\times\Delta)f_{25}d_1=\begin{pmatrix}(1,6)(1,7)(2,6)(2,7)(3,6)(3,7)(4,6)(4,7)(5,6)(5,7)\\(5,6)(4,7)(1,6)(5,7)(2,6)(1,7)(3,6)(2,7)(4,6)(3,7)\end{pmatrix}$$
$$(\Gamma\times\Delta)f_1d_2=\begin{pmatrix}(1,6)(1,7)(2,6)(2,7)(3,6)(3,7)(4,6)(4,7)(5,6)(5,7)\\(1,7)(1,6)(2,7)(2,6)(3,7)(3,6)(4,7)(4,6)(5,7)(5,6)\end{pmatrix}$$
$$(\Gamma\times\Delta)f_2d_2=\begin{pmatrix}(1,6)(1,7)(2,6)(2,7)(3,6)(3,7)(4,6)(4,7)(5,6)(5,7)\\(2,7)(2,6)(3,7)(3,6)(4,7)(4,6)(5,7)(5,6)(1,7)(1,6)\end{pmatrix}$$

$$(\Gamma\times\Delta)f_3d_2=\begin{pmatrix}(1,6)(1,7)(2,6)(2,7)(3,6)(3,7)(4,6)(4,7)(5,6)(5,7)\\(3,7)(3,6)(4,7)(4,6)(5,7)(5,6)(1,7)(1,6)(2,7)(2,6)\end{pmatrix}$$
$$(\Gamma\times\Delta)f_4d_2=\begin{pmatrix}(1,6)(1,7)(2,6)(2,7)(3,6)(3,7)(4,6)(4,7)(5,6)(5,7)\\(4,7)(4,6)(5,7)(5,6)(1,7)(1,6)(2,7)(2,6)(3,7)(3,6)\end{pmatrix}$$
$$(\Gamma\times\Delta)f_5d_2=\begin{pmatrix}(1,6)(1,7)(2,6)(2,7)(3,6)(3,7)(4,6)(4,7)(5,6)(5,7)\\(5,7)(5,6)(1,7)(1,6)(2,7)(2,6)(3,7)(2,6)(4,7)(4,6)\end{pmatrix}$$
$$(\Gamma\times\Delta)f_6d_2=\begin{pmatrix}(1,6)(1,7)(2,6)(2,7)(3,6)(3,7)(4,6)(4,7)(5,6)(5,7)\\(1,6)(1,7)(2,6)(2,7)(3,6)(3,7)(4,6)(4,7)(5,6)(5,7)\end{pmatrix}$$
$$(\Gamma\times\Delta)f_7d_2=\begin{pmatrix}(1,6)(1,7)(2,6)(2,7)(3,6)(3,7)(4,6)(4,7)(5,6)(5,7)\\(1,7)(3,6)(2,7)(4,6)(3,7)(5,6)(4,7)(1,6)(5,7)(2,6)\end{pmatrix}$$
$$(\Gamma\times\Delta)f_8d_2=\begin{pmatrix}(1,6)(1,7)(2,6)(2,7)(3,6)(3,7)(4,6)(4,7)(5,6)(5,7)\\(1,7)(4,6)(2,7)(5,6)(3,7)(1,6)(4,7)(2,6)(5,7)(3,6)\end{pmatrix}$$
$$(\Gamma\times\Delta)f_9d_2=\begin{pmatrix}(1,6)(1,7)(2,6)(2,7)(3,6)(3,7)(4,6)(4,7)(5,6)(5,7)\\(1,7)(5,6)(2,7)(1,6)(3,7)(2,6)(4,7)(3,6)(5,7)(4,6)\end{pmatrix}$$
$$(\Gamma\times\Delta)f_{10}d_2=\begin{pmatrix}(1,6)(1,7)(2,6)(2,7)(3,6)(3,7)(4,6)(4,7)(5,6)(5,7)\\(2,7)(1,6)(3,7)(2,6)(4,7)(3,6)(5,7)(4,6)(1,7)(5,6)\end{pmatrix}$$
$$(\Gamma\times\Delta)f_{11}d_2=\begin{pmatrix}(1,6)(1,7)(2,6)(2,7)(3,6)(3,7)(4,6)(4,7)(5,6)(5,7)\\(2,7)(3,6)(3,7)(4,6)(4,7)(5,6)(5,7)(1,6)(1,7)(2,6)\end{pmatrix}$$
$$(\Gamma\times\Delta)f_{12}d_2=\begin{pmatrix}(1,6)(1,7)(2,6)(2,7)(3,6)(3,7)(4,6)(4,7)(5,6)(5,7)\\(2,7)(4,6)(3,7)(5,6)(4,7)(1,6)(5,7)(2,6)(1,7)(3,6)\end{pmatrix}$$
$$(\Gamma\times\Delta)f_{13}d_2=\begin{pmatrix}(1,6)(1,7)(2,6)(2,7)(3,6)(3,7)(4,6)(4,7)(5,6)(5,7)\\(2,7)(5,6)(3,7)(1,6)(4,7)(2,6)(5,7)(3,6)(1,7)(4,6)\end{pmatrix}$$
$$(\Gamma\times\Delta)f_{14}d_2=\begin{pmatrix}(1,6)(1,7)(2,6)(2,7)(3,6)(3,7)(4,6)(4,7)(5,6)(5,7)\\(3,7)(1,6)(4,7)(2,6)(5,7)(3,6)(1,7)(4,6)(2,7)(5,6)\end{pmatrix}$$
$$(\Gamma\times\Delta)f_{15}d_2=\begin{pmatrix}(1,6)(1,7)(2,6)(2,7)(3,6)(3,7)(4,6)(4,7)(5,6)(5,7)\\(3,7)(2,6)(4,7)(3,6)(5,7)(4,6)(1,7)(5,6)(2,7)(1,6)\end{pmatrix}$$
$$(\Gamma\times\Delta)f_{16}d_2=\begin{pmatrix}(1,6)(1,7)(2,6)(2,7)(3,6)(3,7)(4,6)(4,7)(5,6)(5,7)\\(3,7)(4,6)(4,7)(5,6)(5,7)(1,6)(1,7)(2,6)(2,7)(3,6)\end{pmatrix}$$
$$(\Gamma\times\Delta)f_{17}d_2=\begin{pmatrix}(1,6)(1,7)(2,6)(2,7)(3,6)(3,7)(4,6)(4,7)(5,6)(5,7)\\(3,7)(5,6)(4,7)(1,6)(5,7)(2,6)(1,7)(3,6)(2,7)(4,6)\end{pmatrix}$$
$$(\Gamma\times\Delta)f_{18}d_2=\begin{pmatrix}(1,6)(1,7)(2,6)(2,7)(3,6)(3,7)(4,6)(4,7)(5,6)(5,7)\\(4,7)(1,6)(5,7)(2,6)(1,7)(3,6)(2,7)(4,6)(3,7)(5,6)\end{pmatrix}$$
$$(\Gamma\times\Delta)f_{19}d_2=\begin{pmatrix}(1,6)(1,7)(2,6)(2,7)(3,6)(3,7)(4,6)(4,7)(5,6)(5,7)\\(4,7)(2,6)(5,7)(3,6)(1,7)(4,6)(2,7)(5,6)(3,7)(1,6)\end{pmatrix}$$
$$(\Gamma\times\Delta)f_{20}d_2=\begin{pmatrix}(1,6)(1,7)(2,6)(2,7)(3,6)(3,7)(4,6)(4,7)(5,6)(5,7)\\(4,7)(3,6)(5,7)(4,6)(1,7)(5,6)(2,7)(1,6)(3,7)(2,6)\end{pmatrix}$$
$$(\Gamma\times\Delta)f_{21}d_2=\begin{pmatrix}(1,6)(1,7)(2,6)(2,7)(3,6)(3,7)(4,6)(4,7)(5,6)(5,7)\\(4,7)(5,6)(5,7)(1,6)(1,7)(2,6)(2,7)(3,6)(3,7)(4,6)\end{pmatrix}$$
$$(\Gamma\times\Delta)f_{22}d_2=\begin{pmatrix}(1,6)(1,7)(2,6)(2,7)(3,6)(3,7)(4,6)(4,7)(5,6)(5,7)\\(5,7)(1,6)(1,7)(2,6)(2,7)(3,6)(3,7)(4,6)(4,7)(5,6)\end{pmatrix}$$

$$(\Gamma\times\Delta)f_{23}d_2=\begin{pmatrix}(1,6)(1,7)(2,6)(2,7)(3,6)(3,7)(4,6)(4,7)(5,6)(5,7)\\(5,7)(2,6)(1,7)(3,6)(2,7)(4,6)(3,7)(5,6)(4,7)(1,6)\end{pmatrix}$$
$$(\Gamma\times\Delta)f_{24}d_2=\begin{pmatrix}(1,6)(1,7)(2,6)(2,7)(3,6)(3,7)(4,6)(4,7)(5,6)(5,7)\\(5,7)(3,6)(1,7)(4,6)(2,7)(5,6)(3,7)(1,6)(4,7)(2,6)\end{pmatrix}$$
$$(\Gamma\times\Delta)f_{25}d_2=\begin{pmatrix}(1,6)(1,7)(2,6)(2,7)(3,6)(3,7)(4,6)(4,7)(5,6)(5,7)\\(5,7)(4,6)(1,7)(5,6)(2,7)(1,6)(3,7)(2,6)(4,7)(3,6)\end{pmatrix}$$

Renaming the symbols as
(1,6)—›1, (1,7)—›2, (2,6)—›3, (2,7)—›4, (3,6)—›5,
(3,7)—›6, (4,6)—›7, (4,7)—›8, (5,6)—›9, (5,7)—›10
The permutations in cyclic form are
G_1 = {(1), (1 3 5 7 9), (1 5 9 3 7), (1 7 3 9 5), (1 9 7 5 3), (2 10 8 6 4), (2 4 6 8 10), (2 6 10 4 8), (2 8 4 10 6), (1 2 3 4 5 6 7 8 9 10), (1 2 5 6 9 10 3 4 7 8), (1 2 7 8 3 4 9 10 5 6), (1 2 9 10 7 8 5 6 3 4), (1 4 3 6 5 8 7 10 9 2), (1 4 5 8 9 2 3 6 7 10), (1 4 7 10 3 6 9 2 5 8), (1 4 9 2 7 10 5 8 3 6), (1 6 3 8 5 10 7 2 9 4), (1 6 5 10 9 4 3 8 7 2), (1 6 7 2 3 8 9 4 5 10), (1 6 9 4 7 2 5 10 3 8), (1 8 3 10 5 2 7 4 9 6), (1 8 5 2 9 6 3 10 7 4), (1 8 7 4 3 10 9 6 5 2), (1 8 9 6 7 4 5 2 3 10), (1 10 3 2 5 4 7 6 9 8), (1 10 5 4 9 8 3 2 7 6), (1 10 7 6 3 2 9 8 4 5 4), (1 10 9 8 7 6 5 4 3 2), (1 3 5 7 9)(2 10 8 6 4), (1 3 5 7 9)(2 4 6 8 10), (1 3 5 7 9)(2 8 4 10 6), (1 3 5 7 9)(2 6 10 4 8), (1 5 9 3 7)(2 6 10 4 8), (1 5 9 3 7)(2 8 4 10 6), (1 5 9 3 7)(2 4 6 8 10), (1 5 9 3 7)(2 10 8 6 4), (1 7 3 9 5)(2 4 6 8 10), (1 7 3 9 5)(2 6 10 4 8), (1 7 3 9 5)(2 10 8 6 4), (1 7 3 9 5)(2 8 4 10 6), (1 9 7 5 3)(2 10 7 6 4), (1 9 7 5 3)(2 4 6 8 10), (1 9 7 5 3)(2 6 10 4 8), (1 9 7 5 3)(2 8 4 10 6), (1 2)(3 4)(5 6)(7 6)(9 10),(1 5)(2 9)(3 6)(5 8)(7 10), (1 6)(2 7)(3 8)(4 9)(5 10),(1 8)(2 5)(3 10)(4 7)(6 9),(1 10)(2 3)(4 5)(6 7)(8 9)}
Now, $|G_1|$ = 50 = 2×5^2
4.1.1 G_1 has a unique Sylow 5-subgroup H_1 of order 25 given by
H_1 = {(1), (1 3 5 7 9), (1 5 9 3 7), (1 7 3 9 5), (1 9 7 5 3), (2 10 8 6 4), (2 4 6 8 10), (2 6 10 4 8), (2 8 4 10 6), (1 3 5 7 9)(2 10 8 6 4), (1 3 5 7 9)(2 4 6 8 10), (1 3 5 7 9)(2 8 4 10 6), (1 3 5 7 9)(2 6 10 4 8), (1 5 9 3 7)(2 6 10 4 8), (1 5 9 3 7)(2 8 4 10 6), (1 5 9 3 7)(2 4 6 8 10), (1 5 9 3 7)(2 10 8 6 4), (1 7 3 9 5)(2 4 6 8 10), (1 7 3 9 5)(2 6 10 4 8), (1 7 3 9 5)(2 10 8 6 4), (1 7 3 9 5)(2 8 4 10 6), (1 9 7 5 3)(2 10 7 6 4), (1 9 7 5 3)(2 4 6 8 10), (1 9 7 5 3)(2 6 10 4 8), (1 9 7 5 3)(2 8 4 10 6)}
Thus $|H_1|$= 5^2 which is a p-group

4.2 Consider the permutation groups C_2 and D_2
C_2 = {(1),(1234567),(1357246),(1473625),(1526374),(1642753),(1765432)}
D_2 ={(1),(89)} acting on the sets S_2 ={1,2,3,4,5} and Δ ={8,9} respectively.
Let $P=C_2^{\Delta}$={$f:\Delta\longrightarrow C_2$}. Then |P| = $|C_2|^{|\Delta|}$=7^2=49
After following the same procedure as in 4.1, we obtained permutations in cyclic form as
G_2 = {(1), (1 3 5 7 9 11 13), (2 4 6 8 10 12 14), (1 5 9 13 3 7 11), (2 6 10 14 4 8 12), (1 7 13 5 11 3 9), (1 8 14 6 12 4 10), (1 9 3 11 5 13 7), (2 10 4 12 6 14 8), (1 11 7 3 13 9 5), (2 12 8 4 14 10 6), (1 13 11 9 7 5 3), (2 14 12 10 8 6 4), (2 4 6 8 10 12 14), (2 6 10 14 4 8 12), (2 8 14 6 12 4 10), (2 12 8 4 14 10 6), (2 10 4 12 6 14 8), (2 14 12 10 8 6 4), (1 3 5 7 9 11 13), (1 3 5 7 9 11 13)(2 6 10 14 4 8 12), (1 3 5 7 9 11 13)(2 8 14 6 2 4 10), (1 3 5 7 9 11 13), (2 10 4 12 6 14 8), (1 3 5 7 9 11 13), (2 12 8 4 14 10 6), (1 3 5 7 9 11 13)(2 14 12 10 8 6 4), (1 5 9 13 3 7 11), (1 5 9 13 3 7 11)(2 4 6 8 10 12 14), (1 5 9 13 3 7 11)(2 8 14 6 12 4 10), (1 5 9 13 3 7 11)(2 10 4 12 6 14 8), (1 5 9 13 3 7 11)(2 12 8 4 14 10 6), (1 5 9 13 3 7 11)(2 14 12 10 8 6 4), (1 7 13 5 11 3 9), (1 7 13 5

11 3 9)(2 4 6 8 10 12 14), (1 7 13 5 11 3 9)(2 6 10 14 4 8 12), (1 7 13 5 11 3 9)(2 10 4 12 6 14 8), (1 7 13 5 11 3 9)(2 12 8 4 14 10 6), (1 7 13 5 11 3 9)(2 14 12 10 8 6 4), (1 9 3 11 5 13 7), (1 9 3 11 5 13 7)(2 4 6 8 10 12 14), (1 9 3 11 5 13 7)(2 6 10 14 4 8 12), (1 9 3 11 5 13 7)(2 8 14 6 12 4 10), (1 9 3 11 5 13 7)(2 12 8 4 14 10 6), (1 9 3 11 5 13 7)(2 14 12 10 8 6 4), (1 11 7 3 13 9 5), (1 11 7 3 13 9 5)(2 4 6 8 10 12 14), (1 11 7 3 13 9 5)(2 6 10 14 4 8 12), (1 11 7 3 1 3 9 5)(2 8 14 6 12 4 10), (1 11 7 3 13 9 5),(2 10 4 12 6 14 8), (1 11 7 3 13 9 5)(2 14 12 10 8 6 4), (1 13 11 9 7 5 3), (1 13 11 9 7 5 3)(2 4 6 8 10 12 14), (1 13 11 9 7 5 3)(2 6 10 14 4 8 12), (1 13 11 9 7 5 3)(2 8 14 6 12 4 10), (1 13 11 9 7 5 3)(2 10 4 12 6 1 4 8), (1 13 11 9 7 5 3)(2 12 8 4 14 10 6), (1 2)(3 4)(5 6)(7 8)(9 10)(11 12)(1 3 14), (1 4 5 8 9 12 13 2 3 6 7 10 11 14), (1 6 9 14 3 8 11 2 5 10 13 4 7 12), (1 8 13 6 11 4 9 2 7 14 5 12 3 10), (1 10 3 12 5 4 7 2 9 4 11 6 13 8), (1 12 7 4 13 10 5 2 11 8 3 14 9 6), (1 14 11 10 7 6 3 2 13 12 9 8 5 4), (1 2 3 4 5 6 7 8 9 10 11 12 13 14), (1 2 5 6 9 10 13 14 3 4 7 8 11 12), (1 2 7 8 13 14 5 6 11 12 3 4 9 10), (1 2 9 10 3 4 11 1 2 5 6 13 14 7 8), (1 2 11 12 7 8 3 4 13 14 9 10 5 6), (1 2 13 14 11 12 9 10 7 8 5 6 3 4), (1 4 3 6 5 8 7 10 9 12 11 14 13 2), (1 4 7 9 13 2 5 8 11 14 3 6 9 12), (1 4 9 12 3 6 11 14 5 8 13 2 7 10), (1 4 11 14 7 10 3 6 13 2 9 12 5 8), (1 4 13 2 11 14 9 12 7 10 5 8 3 6), (1 4)(2 13)(3 6)(5 8)(7 10)(9 12)(1 1 14), (1 6 5 10 9 14 13 4 3 8 7 12 11 2), (1 6 7 12 13 4 5 10 11 2 3 8 9 14), (1 6 11 2 7 12 3 8 13 4 9 14 5 10), (1 6 13 4 11 2 9 14 7 12 5 10 3 8), (1 6)(2 11)(3 8)(4 13)(5 10)(7 12)(9 14), (1 6 3 8 5 10 7 12 9 14 11 2 13 4), (1 8 7 14 13 6 5 12 11 4 3 10 9 2), (1 8 9 2 3 10 11 4 5 12 13 6 8 1 4), (1 8 11 4 7 14 3 10 13 6 9 2 5 12), (1 8)(2 9)(3 10)(4 11)(5 12)(6 13)(7 14), (1 8 13 10 5 12 7 14 9 2 11 4 13 6), (1 8 5 12 9 2 13 6 3 10 7 14 11 4), (1 10 9 4 3 12 11 6 5 14 13 8 7 2), (1 10 11 6 7 2 3 12 13 8 9 4 5 14), 1 10 13 8 11 6 9 4 7 2 5 14 3 12), (1 10)(2 7)(3 12)(4 9)(5 14)(6 11)(8 13), (1 10 5 14 9 4 13 8 3 12 7 2 11 6), (1 10 7 2 13 8 5 14 11 6 3 12 9 4), (1 12 11 8 7 4 3 14 13 10 9 6 5 2), (1 12 13 14 5 2 7 4 9 6 7 4 5 2 3 14), (1 12)(2 5)(3 14)(4 7)(6 9)(8 11)(10 13), (1 12 3 14 5 2 7 4 9 6 11 8 13 10), (1 12 5 2 9 6 13 10 3 14 7 4 11 8), (1 12 9 6 3 14 11 8 5 2 13 10 7 4), (1 14 13 12 11 10 9 8 7 6 5 4 3 2), (1 14)(2 3)(4 5)(6 7)(8 9)(10 11)(12 13), (1 14 3 2 5 4 7 6 9 8 11 10 13 12), (1 14 5 4 9 8 13 12 3 2 7 6 11 10), (1 14 7 6 13 12 5 4 11 10 3 2 9 8), (1 14 9 8 3 2 11 10 5 4 13 12 7 6)}

Now, $|G_2|$ = 98 = 2×7^2

4.2.1 G_2 has a unique Sylow 7-subgroup H_2 of order 49 given by

H_2 = {(1), (1 3 5 7 9 11 13)(2 4 6 8 10 12 14), (1 5 9 13 3 7 11)(2 6 10 14 4 8 12), (1 7 13 5 11 3 9), (1 8 14 6 12 4 10), (1 9 3 11 5 13 7)(2 10 4 12 6 14 8), (1 11 7 3 13 9 5)(2 12 8 4 14 10 6), (1 13 11 9 7 5 3)(2 14 12 10 8 6 4), (2 4 6 8 10 12 14), (2 6 10 14 4 8 12), (2 8 14 6 12 4 10), (2 12 8 4 14 10 6), (2 10 4 12 6 14 8), (2 14 12 10 8 6 4), (1 3 5 7 9 11 13), (1 3 5 7 9 11 13)(2 6 10 14 4 8 12), (1 3 5 7 9 11 13)(2 8 14 6 2 4 10), (1 3 5 7 9 11 13)(2 10 4 12 6 14 8), (1 3 5 7 9 11 13)(2 12 8 4 14 10 6), (1 3 5 7 9 11 13)(2 14 12 10 8 6 4), (1 5 9 13 3 7 11), (1 5 9 13 3 7 11)(2 4 6 8 10 12 14), (1 5 9 13 3 7 11)(2 8 14 6 12 4 10), (1 5 9 13 3 7 11)(2 10 4 12 6 14 8), (1 5 9 13 3 7 11)(2 12 8 4 14 10 6), (1 5 9 13 3 7 11)(2 14 12 10 8 6 4), (1 7 13 5 11 3 9), (1 7 13 5 11 3 9)(2 4 6 8 10 12 14), (1 7 13 5 11 3 9)(2 6 10 14 4 8 12), (1 7 13 5 11 3 9)(2 10 4 12 6 14 8), (1 7 13 5 11 3 9)(2 12 8 4 14 10 6), (1 7 13 5 11 3 9)(2 14 12 10 8 6 4), (1 9 3 11 5 13 7), (1 9 3 11 5 13 7)(2 4 6 8 10 12 14), (1 9 3 11 5 13 7)(2 6 10 14 4 8 12), (1 9 3 11 5 13 7)(2 8 14 6 12 4 10), (1 9 3 11 5 13 7)(2 12 8 4 14 10 6), (1 9 3 11 5 13 7)(2 14 12 10 8 6 4), (1 11 7 3 13 9 5), (1 11 7 3 13 9 5)(2 4 6 8 10 12 14), (1 11 7 3 13 9 5)(2 6 10 14 4 8 12), (1 11 7 3 1 3 9 5)(2 8 14 6 12 4 10), (1 11 7 3 13 9 5)(2 10 4 12 6 14 8), (1 11 7 3 13 9 5)(2 14 12 10 8 6 4), (1 13 11 9 7 5 3), (1 13 11 9 7 5 3)(2 4 6 8 10 12 14), (1 13 11 9 7 5 3)(2 6 10 14 4 8 12), (1 13 11 9 7 5 3)(2 8 14 6 12 4 10), (1 13 11 9 7 5 3)(2 10 4 12 6 1 4 8), (1 13 11 9 7 5 3)(2 12 8 4 14 10 6),

$|H_2| = 7^2 = 49$ is a p-group.

4.3 Consider the permutation groups C_3 and D_3

C_3 = {(1),(123),(132)}

D_3 ={(1) (456),(465)} acting on the sets S_3 ={1,2,3} and Δ ={4,5,6} respectively.

Let $P = C_3^{\Delta}=\{f: \Delta \longrightarrow C_3\}$. Then $|P| = |C_3|^{|\Delta|}=3^3=27$

After applying the same procedure as in 4.1, we obtained a permutation group in cyclic form as

4.3.1 G_3 = {(1), (1 4 7), (1 7 4), (2 5 8), (2 8 5), (3 6 9), (3 9 6), (1 2 3 4 5 6 7 8 9), (1 2 6 4 5 9 7 8 3), (1 2 6 7 8 3 4 5 9), (1 2 9 7 8 6 4 5 3), (1 2 9 4 5 3 7 8 6), (1 2 3 7 8 9 4 5 6), (1 3 2 4 6 5 7 9 8), (1 3 2 7 9 8 4 6 5), (1 3 5 4 6 8 7 9 2), (1 3 5 7 9 2 4 6 8), (1 3 8 4 6 2 7 9 5), (1 3 8 7 9 5 4 6 2), (1 5 3 4 8 6 7 2 9), (1 5 3 7 2 9 4 8 6), (1 5 6 7 2 3 4 8 9), (1 5 9 4 8 3 7 2 6), (1 5 9 7 2 6 4 8 3), (1 5 6 4 8 9 7 2 3), (1 6 2 4 9 5 7 3 8), (1 6 2 7 3 8 4 9 5), (1 6 8 4 9 2 7 3 5), (1 6 8 7 3 5 4 9 2), (1 6 5 8 3 2 4 9 8), (1 6 5 4 9 8 7 3 2), (1 8 3 4 2 6 7 5 9), (1 8 3 7 5 9 4 2 6), (1 8 6 7 5 3 4 2 9), (1 8 6 4 2 9 7 5 3), (1 8 9 4 2 3 7 5 6), (1 8 9 7 5 6 4 2 3), (1 9 2 7 6 8 4 3 5), (1 9 2 4 3 5 7 6 8), (1 9 5 4 3 8 7 6 2), (1 9 5 7 6 2 4 3 8), (1 9 8 4 3 2 7 6 5), (1 9 8 7 6 5 4 3 2), (1 4 7)(3 9 8), (1 4 7)(2 8 5), (1 4 7)(3 6 9), (1 4 7)(2 5 8), (1 7 4)(2 8 5), (1 7 4)(3 6 9), (1 7 4)(3 9 6), (1 7 4)(2 5 8), (2 8 5)(3 6 9), (2 8 5)(3 9 6), (2 5 8)(3 6 9), (2 5 8)(3 9 6), (1 2 6)(3 7 8)(4 5 9), (1 2 9)(3 4 5)(6 7 8), (1 2 3)(4 5 6)(7 8 9), (1 4 7)(2 8 5)(3 9 6), (1 4 7)(2 8 5)(3 6 9), (1 4 7)(2 5 8)(3 9 6), (1 4 7)(2 5 8)(3 6 9), (1 5 9)(2 6 7)(3 4 8), (1 5 6)(2 3 7)(4 8 9), (1 5 3)(2 9 7)(4 8 6), (1 6 8)(2 4 9)(3 5 7), (1 6 2)(3 8 7)(5 4 9), (1 6 5)(2 7 3)(4 9 8), (1 7 4)(2 8 5)(3 9 6), (1 7 4)(2 8 5)(3 6 9), (1 7 4)(2 5 8)(3 9 6), (1 7 4)(2 5 8)(3 6 9), (1 8 6)(2 9 4)(3 7 5), (1 8 3)(2 6 4)(2 5 7), (1 8 9)(2 3 4)(5 6 7), (1 9 5)(2 7 6)(3 8 4), (1 9 8)(2 4 3)(5 7 6), (1 9 2)(3 5 4)(6 8 7), (1 3 2)(4 6 5)(7 9 8), (1 3 5)(2 7 9)(4 6 8), (1 3 8)(2 4 6)(5 7 9)}

G_3 is a p-group of order 81 given by $|G_3| = 3^4$

4.4 Consider the permutation groups C_4 and D_4

C_4 = {(1),(12345),(13524),(14253),(15432)}

D_4 ={(1),(678),(687)} acting on the sets S_4 ={1,2,3,4,5} and Δ={6,7,8} respectively.

Let $P = C_4^{\Delta}=\{f: \Delta \longrightarrow C_4\}$. Then $|P| = |C_4|^{|\Delta|} =5^3=125$

Applying the same procedure as in 4.1, we obtained a permutation group in cyclic form as

G_4 = {(1), (1 4 7 10 13)(2 5 8 11 14)(3 6 9 12 15), (1 7 13 4 10)(2 8 14 5 11)(3 9 15 6 12), (1 10 4 13 7)(2 11 5 14 8)(3 12 6 15 9), (1 13 10 7 4)(2 14 11 8 5)(3 15 12 9 6), (2 5 8 11 14)(3 9 15 6 12), (2 5 8 11 14)(3 12 6 15 9), (2 5 8 11 14)(3 15 12 9 6), (2 8 14 5 11)(3 6 9 12 15), (2 8 14 5 11)(3 12 6 15 9), (2 8 15 5 11)(3 15 12 9 6), (2 11 5 14 8)(3 6 9 12 15), (2 11 5 14 8)(3 6 9 12 15), (2 11 5 14 8)(3 9 15 6 12), (2 11 5 14 8)(3 15 12 9 6), (2 14 11 8 5)(3 6 9 12 15), (2 14 11 8 5)(3 9 15 6 12), (2 14 11 8 5)(3 12 6 15 9), (1 4 7 10 13)(3 9 15 6 12), (1 4 7 10 13)(3 12 6 15 9), (1 4 7 10 13)(3 15 12 9 6), (1 4 7 10 13)(2 8 14 4 11), (1 4 7 10 13)(2 8 14 5 11)(3 12 6 15 9), (1 4 7 10 13)(2 8 14 5 11)(3 15 12 9 6), (1 4 7 10 13)(2 11 5 14 8), (1 4 7 10 13)(2 11 5 14 8)(3 9 15 6 12), (1 4 7 10 13)(2 11 5 14 8)(3 15 12 9 6),(1 4 7 10 13)(2 14 11 8 5), (1 4 7 10 13)(2 14 11 8 5)(3 9 15 6 12), (1 4 7 4 13)(2 14 11 8 5)(3 12 6 15 9), (1 7 13 4 10)(3 6 9 12 15), (1 7 13 4 10)(3 12 6 15 9), (1 7 13 4 10)(3 15 12 9 6), (1 7 13 4 10)(2 5 8 11 14), (1 7 13 4 10)(3 15 12 9 6), (1 7 13 4 10)(2 5 8 11 14), (1 7 13 4 10)(2 5 8 11 14)(3 12 6 15 9), (1 7 13 4 10)(2 5 8 11 14)(3 15 12 9 6), (1 7 13 4 10)(2 11 5 14 8), (1 7 13 4 10)(2 11 5 14 8)(3 6 9 12 15), (1 7 13 4 10)(2 11 5 14 8)(3 15 12 9 6), (1 7 13 4 10)(2 14 11 8 5), (1 7 13 4 10)(2 14 11 8 5)(3 6 9 12 15), (1 7 13 4 10)(2 14 11 8 5)(3 12 6 15 9), (1 10 4 13 7)(3 6 9 12 15), (1 10 4 13 7)(3 9 15 6 12), (1 10 4 13

7)(3 15 12 9 6), (1 10 4 13 7)(2 5 8 11 14), (1 10 4 13 7)(2 5 8 11 14)(3 9 15 6 12), (1 10 4 13 7)(2 5 8 11 14)(3 15 12 9 6), (1 10 4 13 7)(2 8 14 5 11), (1 10 4 13 7)(2 8 14 5 11)(3 6 9 12 15), (1 10 4 13 7)(2 8 14 5 11)(3 15 12 9 6), (1 10 4 13 7)(2 14 11 8 5), (1 10 4 13 7)(2 14 11 8 5)(3 6 9 12 15), (1 10 4 13 7)(2 14 11 8 5)(3 9 15 6 12), (1 13 10 7 4)(3 6 9 12 15), (1 13 10 7 14)(3 9 15 6 12),(1 13 10 7 14)(3 12 6 15 9), (1 13 10 7 4)(2 5 8 11 14), (1 13 10 7 4)(2 5 8 11 14)(3 9 15 6 12), (1 13 10 7 4)(2 5 8 11 14)(3 12 6 15 9), (1 13 10 7 4)(2 8 14 5 11), (1 13 10 7 4)(2 8 14 5 11)(3 6 9 12 15), (1 13 10 7 4)(2 8 14 5 11)(3 12 6 15 9), (1 13 10 7 4)(2 11 5 14 8), (1 13 10 7 4)(2 11 5 14 8)(3 6 9 12 15), (1 13 10 7 4)(2 11 5 14 8)(3 9 15 6 12)(3 6 9 12 15), (3 9 15 6 12), (3 12 6 15 9), (3 15 12 9 6), (2 5 8 11 14), (2 8 14 5 11), (2 11 5 14 8), (2 14 11 8 5), (1 4 7 10 13), (1 7 13 4 10), (1 10 4 13 7), (1 13 10 7 4), (1 4 7 10 13)(2 5 8 11 14), (1 4 7 10 13)(2 5 8 11 14)(3 9 15 6 12), (1 4 7 10 13)(2 5 8 11 14)(3 12 6 15 9),(1 4 7 10 13)(2 5 8 11 14)(3 15 12 9 6), (1 4 7 10 13)(3 6 9 12 15), (1 4 7 10 13)(2 8 14 5 11)(3 6 9 12 15), (1 4 7 10 13)(2 11 5 14 8)(3 6 9 12 15), (1 4 7 10 13)(2 14 11 8 5)(3 6 9 12 15), (2 5 8 11 14)(3 6 9 12 15), (1 17 13 4 10)(2 5 8 11 14)(3 6 9 12 15), (1 10 4 13 7)(2 5 8 11 14)(3 6 9 12 15), (1 13 10 7 4)(2 5 8 11 14)(3 6 9 12 15), (1 7 13 4 10)(2 8 14 5 11)(3 6 9 12 15), (1 7 13 4 10)(2 8 14 15 11)(3 12 6 15 9), (1 7 13 4 10)(2 8 14 15 11)(3 15 12 9 6), (1 7 13 4 10)(3 9 15 6 12), (1 7 13 4 10)(2 5 8 11 14)(3 9 15 6 12), (1 7 13 4 10)(2 11 5 14 8)(3 9 15 6 12), (1 7 13 4 10)(2 14 11 8 5)(3 9 15 6 12), (2 8 14 5 11)(3 9 15 6 12), (1 4 7 10 13)(2 8 14 5 11)(3 9 15 6 12), (1 10 4 13 7)(2 8 11 14 5 11)(3 9 15 6 12), (1 13 10 7 4)(2 8 14 5 11)(3 9 15 6 12), (1 10 4 13 7)(2 11 5 14 8), (1 10 4 13 7)(2 11 5 14 8)(3 6 9 12 15), (1 10 4 13 7)(2 11 5 14 8)(3 9 15 6 12), (1 10 4 13 7)(2 11 5 14 8)(3 15 12 9 6), (1 10 4 13 7)(3 12 6 15 9), (1 10 4 13 7)(2 5 8 11 14)(3 12 6 15 9), (1 10 4 13 7)(2 8 14 5 11)(3 12 6 15 9), (1 10 4 13 7)(2 14 11 8 5)(3 12 6 15 9),(2 11 5 14 8)(3 12 6 15 9), (1 4 7 10 13)(2 11 5 14 8)(3 12 6 15 9), (1 7 13 4 10)(2 11 5 14 8)(3 12 6 15 9), (1 13 10 7 4)(2 11 5 14 8)(3 12 6 15 9), (1 13 10 7 4)(2 14 11 8 5), (1 13 10 7 4)(2 14 11 8 5)(3 6 9 12 15), (1 13 10 7 4)(2 14 11 8 5)(3 9 15 6 12), (1 13 10 7 4)(2 14 11 8 5)(3 12 6 15 9), (1 13 10 7 4)(3 15 12 9 6), (1 13 10 7 4)(2 5 8 11 14)(3 15 12 9 6), (1 13 10 7 4)(2 8 14 5 11)(3 15 12 9 6), (1 13 10 7 4)(2 11 5 14 8)(3 15 12 9 6), (2 14 11 8 5)(3 15 12 9 6), (1 4 7 10 13)(2 14 11 8 5)(3 15 12 9 6), (1 7 13 4 10)(2 14 11 8 5)(3 15 12 9 6), (1 10 4 13 7)(2 14 11 8 5)(3 15 12 9 6), (1 2 15 4 5 3 7 8 6 10 11 9 13 14 12), (1 2 15 7 8 6 13 14 12 4 5 3 10 11 9), (1 5 6 10 14 15 4 8 9 13 2 3 7 11 12), (1 5 6 13 2 3 10 14 15 7 11 12 4 8 9), (1 5 6)(2 3 13)(4 8 9)(7 11 12)(10 14 15), (1 5 12 10 14 6 4 8 15 13 2 9 7 11 3), (1 5 12 4 8 15 7 11 3 10 14 6 13 2 9), (1 5 12 7 11 3 13 2 9 4 8 15 10 14 6), (1 5 15 13 2 12 10 14 9 7 11 6 4 8 3), (1 5 15 4 8 3 7 11 6 10 14 9 13 2 12),(1 5 15 10 14 9 4 8 3 13 2 12 7 11 6), (1 5 3)(2 15 13)(4 8 6)(7 11 9)(10 14 12), (1 5 3 7 11 9 13 2 15 4 8 6 10 14 12), (1 5 3 10 14 12 4 8 6 13 2 15 7 11 9), (1 8 9 10 2 3 4 11 12 13 5 6 7 14 15), (1 2 3)(4 5 6)(7 8 9)(10 11 12)(13 14 15), (1 5 9 10 14 3 4 8 12 13 2 6 7 11 15), (1 8 15 4 11 3 7 14 6 10 2 9 13 5 12), (1 11 6 13 8 3 10 5 15 7 2 12 4 14 9), (1 14 12 7 5 3 13 11 9 4 2 15 10 8 6), (1 2 6 10 11 15 4 5 9 13 14 3 7 8 12), (1 2 6 13 14 3 10 11 15 7 8 12 4 5 9), (1 2 6)(3 13 14)(4 5 9)(7 8 12)(10 11 15), (1 2 9 10 11 3 4 5 12 13 14 6 7 8 15), (1 2 9)(3 10 11)(4 5 12)(6 13 14)(7 8 15), (1 2 9 4 5 12 7 8 15 10 11 3 13 14 6), (1 2 12 13 14 9 10 11 6 7 8 3 4 5 15), (1 2 13)(3 7 8)(4 5 15)(6 10 11)(9 13 14), (1 2 12 7 8 3 13 14 9 4 5 15 10 11 6), (1 2 15)(3 4 5)(6 7 8)(9 10 11)(12 13 14), (1 8 9)(2 3 10)(4 11 12)(5 6 13)(7 14 15),(1 8 9 4 11 12 7 14 15 10 2 3 13 5 6), (1 8 12 10 2 6 4 11 15 13 5 9 7 14 3), (1 8 12 4 11 15 7 14 3 10 2 6 13 5 9), (1 8 12 7 14 3 13 5 9 4 11 15 10 2 6), (1 8 3)(2 12 10)(4 11 6)(5 15 13)(7 14 9), (1 8 3 4 11 6 7 14 9 10 2 12 13 5 15), (1 8 3 13 5 15 10 12 7 14 9 4 11 6), (1 8 6 4 11 9 7 14 12 10 2 15 13 5 3), (1 8 6 7 14 12 13 5 3 4 11 9 10 2 15), (1 8 6 13 5 3 10 2 15 7 14 12 4 11 9), (1 11 12 13 8 9 10 5 6 7 2 3 4 14 15), (1 11 12)(2 3 7)(4 14 15)(5 6 10)(8 9 13), (1 11 12 7 2 3 13 8 9 4 14 15 10 5 6), (1 11 15 13 8 12 10 5 9 7 2 6 4 14 3), (1 11 15 4 14 3 7 2 6 10 5 9 13

8 12), (1 11 15 10 5 9 4 14 3 13 8 12 7 2 6), (1 11 3)(2 9 7)(4 14 6)(5 12 10)(8 15 13), (1 11 3 4 14 6 7 2 9 10 5 12 13 8 15), (1 11 3 13 8 15 10 5 12 7 2 9 4 14 6), (1 11 9 7 2 15 13 6 8 4 14 12 10 5 3), (1 11 9 10 5 3 4 14 12 13 8 6 7 2 15), (1 11 9 13 8 6 10 5 3 7 2 15 4 14 12), (1 14 15) (2 3 4)(5 6 7)(8 9 10)(11 12 13), (1 14 15 4 2 3 7 5 6 10 8 9 13 11 12), (1 14 15 7 5 6 13 11 12 4 2 3 10 8 9), (1 14 3)(2 6 4)(5 9 7)(8 12 10)(11 15 13), (1 14 3 7 5 9 13 11 15 4 2 6 10 8 12), (1 14 3 10 8 10 2 4 2 6 13 11 15 7 5 9), (1 14 6 4 2 9 7 5 12 10 8 15 13 11 3), (1 14 6 7 5 12 13 11 3 4 2 9 10 8 15), (1 14 6 13 11 3 10 8 15 7 5 12 4 2 9), (1 14 9 7 5 15 13 11 6 4 2 12 10 8 3), (1 14 9 10 8 3 4 2 12 13 11 6 7 5 15), (1 14 9 13 11 6 10 8 3 7 5 15 4 2 12), (1 2 3 4 5 6 7 8 9 10 11 12 13 14 15), (1 2 3 7 8 9 13 14 15 4 5 6 10 11 12)(1 2 3 10 11 12 4 5 6 13 14 15 7 8 9), (1 2 3 13 14 15 10 11 12 7 8 9 4 5 6), (1 2 6 4 5 9 7 8 12 10 11 15 13 14 2), (1 2 9 8 15 13 14 6 4 5 12 10 11 3), (1 2 12 10 11 6 4 5 15 13 14 9 7 8 3), (1 2 15 13 14 12 10 11 9 7 8 6 4 5 3), (1 5 6 4 8 9 7 11 12 10 14 15 13 2 3), (1 8 9 7 14 15 13 5 6 4 11 12 10 2 3), (1 11 12 10 5 6 4 14 15 13 8 9 7 2 3), (1 14 15 13 11 12 10 8 9 7 5 6 4 2 3), (1 5 9 7 11 15 13 2 6 4 8 12 10 14 3), (1 5 9 13 2 6 10 14 3 7 11 15 4 8 12), (1 5 9)(2 6 13)(3 10 14)(4 8 12)(7 11 15)(1 5 9 4 8 12 7 11 15 10 14 3 13 2 6), (1 5 6 7 11 12 13 2 3 4 8 9 10 14 15), (1 5 12 13 2 9 10 14 6 7 11 3 4 8 15), (1 5 15)(2 12 13)(3 4 8)(6 7 11) (9 10 14), (1 5 3 8 6 7 11 9 10 14 12 13 2 15),(1 2 6 7 8 12 13 14 3 4 5 9 10 11 15),(1 8 12 13 5 9 10 2 6 7 14 3 4 11 15), (1 11 15)(2 6 7)(3 4 14)(5 9 10)(8 12 13), (1 14 3 4 2 6 7 5 9 10 8 12 13 11 15), (1 8 15 13 5 12 10 2 9 7 14 6 4 11 3), (1 8 15)(2 9 10)(3 4 11)(5 12 13)(6 7 14), (1 8 15 7 14 6 13 5 12 4 11 3 10 2 9), (1 8 15 10 2 9 4 11 3 13 5 12 7 14 6), (1 8 9 13 5 6 10 2 3 7 14 15 4 11 12), (1 8 12)(2 6 10)(3 7 14)(4 11 15)(5 9 13), (1 8 3 7 14 9 13 5 15 4 11 6 10 2 12), (1 8 6 10 2 15 4 11 9 13 5 3 7 14 12), (1 2 9 13 14 6 10 11 3 7 8 15 4 5 12), (1 5 12)(2 9 13)(3 7 11)(4 8 15)(6 10 13), (1 11 3 7 2 9 13 8 15 4 14 6 10 5 12), (1 14 6 10 8 15 4 2 9 13 11 3 7 5 12), (1 11 6 4 14 9 7 2 12 10 5 15 13 8 3), (1 11 6 7 2 12 13 8 3 4 14 9 10 5 15), (1 11 6 10 5 15 4 14 9 13 8 3 7 2 12), (1 11 6)(2 12 2 7)(3 13 8)(4 14 9)(5 15 10), (1 11 12 4 14 15 7 2 3 10 5 6 13 8 9), (1 11 15 7 2 6 13 8 12 4 14 3 10 5 9), (1 11 3 10 5 12 4 14 6 13 8 15 7 2 9), (1 11 9)(2 15 7)(3 10 5)(4 14 12)(6 13 8), (1 2 12 4 5 7 8 3 10 11 6 13 14 9), (1 5 15 7 11 6 13 2 12 4 8 3 10 14 9),(1 8 3 10 2 12 4 11 6 13 5 15 7 14 9), (1 14 9)(2 12 4)(3 10 8)(5 15 7)(6 13 11), (1 14 12 10 8 6 4 2 15 13 11 9 7 5 3), (1 14 12 13 11 9 10 8 6 7 5 3 4 2 15), (1 14 12)(2 15 4)(3 7 5)(6 10 8)(9 13 11), (1 14 12 4 2 15 7 5 3 10 8 6 13 11 9), (1 14 15 10 8 9 4 2 3 13 11 12 7 5 6),(1 14 3 13 11 15 10 8 12 7 5 9 4 2 6), (1 14 6)(2 9 4)(3 13 11)(5 12 7)(8 15 10), (1 14 9 4 2 12 7 5 15 10 8 3 13 11 6), (1 2 15 10 11 9 4 5 3 13 14 12 7 8 6), (1 5 3 13 2 15 10 14 12 7 11 9 4 8 6), (1 8 6)(2 15 10)(3 13 5)(4 11 9)(7 14 12), (1 11 9 4 14 12 7 2 15 10 5 3 13 8 6), (1 2 3)(4 6 5)(7 9 8)(10 12 11)(13 15 14), (1 6 8 10 15 2 4 9 11 13 3 5 7 12 14), (1 9 14 4 12 2 7 15 5 10 3 8 13 6 11), (1 12 5 13 9 2 10 6 14 7 3 11 4 15 8), (1 15 11 7 6 2 13 12 8 4 3 14 10 9 5), (1 3 8 10 12 2 4 6 11 13 15 5 7 9 14), (1 3 11 13 15 8 10 12 5 7 9 2 4 6 14), (1 3 14)(2 4 6)(5 7 9)(8 10 12)(11 13 15), (1 3 5 10 12 14 4 6 8 13 15 2 7 9 11), (1 3 11)(2 7 9)(4 6 14)(5 10 12)(8 13 15), (1 3 14 4 6 2 7 9 5 10 12 8 13 15 11), (1 3 5 13 15 2 10 12 14 7 9 11 4 6 8), (1 3 8)(2 10 12)(4 6 11)(5 13 15)(7 9 14), (1 3 14 7 9 5 13 15 11 4 6 2 10 12 8), (1 3 5)(2 13 15)(4 6 8)(7 9 11)(10 12 14), (1 3 8 4 6 11 7 9 14 10 12 2 13 15 5),(1 3 11 7 9 2 13 15 8 4 6 14 10 12 5), (1 6 11 10 15 5 4 9 14 13 3 8 7 12 2), (1 6 14 13 3 11 10 15 8 7 12 5 4 9 2), (1 6 2)(3 14 13)(4 9 5)(7 12 8)(10 15 11), (1 6 5 10 15 14 4 9 8 13 3 2 7 12 11), (1 6 14 4 9 2 7 12 5 10 15 8 13 3 11), (1 6 2 7 12 8 13 3 14 4 9 5 10 15 11), (1 6 5 13 3 2 10 15 14 7 12 11 4 9 8), (1 6 11 4 9 14 7 12 2 10 15 5 13 3 8), (1 6 2 10 15 11 4 9 5 13 3 14 7 12 8), (1 6 5)(2 13 3)(4 9 8)(7 12 11)(10 15 14), (1 6 11 7 12 2 13 3 8 4 9 14 10 15 5), (1 6 14 10 15 8 4 9 2 13 3 11 7 12 5), (1 9 11 10 3 5 4 12 14 13 6 8 7 15 2), (1 9 2)(3 11 10)(4 12 5)(6 14 13)(7 15 8), (1 9 5 7 15 11 13 6 2 4 12 8 10 3 14), (1 9 8 10 3 2 4 12 11 13 6 5 7 15 14), (1 9 2 4 12 5 7 15 8 10 3 11 13 6 14), (1 9 5 7 15 11 13 6 2 4 12 8 10 3 14), (1 9 8)(2 10 3)(4 12

11)(5 13 6)(7 15 14), (1 9 11 4 12 14 7 15 2 10 3 5 13 6 8), (1 9 5 13 6 2 10 3 1 4 7 15 11 4 12 8), (1 9 8 4 12 11 7 15 14 10 3 2 13 6 5), (1 9 11 7 15 2 13 6 8 4 12 14 10 3 5), (1 9 2 13 6 14 10 3 11 7 15 8 4 12 5), (1 12 14 13 9 11 10 6 8 7 3 5 4 15 2), (1 12 2)(3 8 7)(4 15 5)(6 11 10)(9 14 13), (1 12 8 7 3 14 13 9 5 4 11 10 6 2), (1 12 11 13 9 8 10 6 5 7 3 2 4 15 14), (1 12 2 4 15 5 7 3 8 10 6 11 13 9 14), (1 12 8 10 6 2 4 15 11 13 9 5 7 3 14), (1 12 11)(2 7 3)(4 15 14)(5 10 6)(8 13 9), (1 12 14 4 15 2 7 3 5 10 6 8 13 9 11), (1 12 8 13 9 5 10 6 2 7 3 14 4 15 11), (1 12 11 7 3 2 13 9 8 4 15 14 10 6 5), (1 12 14 10 6 8 4 15 2 13 9 11 7 3 5), (1 12 2 13 9 14 10 6 11 7 3 8 4 15 5), (1 15 2)(3 5 4)(6 8 7)(9 11 10)(12 14 13), (1 15 5 4 3 8 7 6 11 10 9 14 13 12 2), (1 15 8 7 6 14 13 12 5 4 3 11 10 9 2), (1 15 14)(3 2 4)(5 7 6)(8 10 9)(11 13 12), (1 15 5 7 6 11 13 12 2 4 3 8 10 9 14), (1 15 8 10 9 2 4 3 11 13 12 5 7 6 14), (1 15 14 4 3 2 7 6 5 10 9 8 13 12 11), (1 15 2 7 6 8 13 12 14 4 3 5 10 9 11), (1 15 8 13 12 5 10 9 2 7 6 14 4 3 11), (1 15 14 7 6 5 13 12 11 4 3 2 10 9 8), (1 15 2 10 9 11 4 3 5 13 12 14 7 6 8), (1 15 5 13 12 2 10 9 14 7 6 11 4 3 8), (1 3 5 4 6 8 7 9 11 10 12 14 13 15 2), (1 3 8 7 9 14 13 15 5 4 6 11 10 12 2), (1 3 11 10 12 5 4 6 14 13 15 8 7 9 2), (1 3 14 13 15 11 10 12 8 7 9 5 4 6 2), (1 3 2 4 6 5 7 9 8 10 12 11 13 15 14), (1 3 2 7 9 8 13 15 14 4 6 5 10 12 11), (1 3 2 10 12 11 4 6 5 13 15 14 7 9 8), (1 3 2 13 15 14 10 12 11 7 9 8 4 6 5), (1 6 5 4 9 8 7 12 11 10 15 14 13 3 2), (1 9 8 7 15 14 13 6 5 4 12 11 10 3 2), (1 12 11 10 6 5 4 15 14 13 9 8 7 3 2), (1 15 14 13 12 11 10 9 8 7 6 5 4 3 2), (1 6 5 7 12 11 13 3 2 4 9 8 10 15 14), (1 6 11 13 3 8 10 15 5 7 12 2 4 9 14), (1 6 14)(2 4 9)(3 11 13)(5 7 12)(8 10 15), (1 6 2 4 9 5 7 12 8 10 15 11 13 3 14), (1 6 8 7 12 14 13 3 5 4 9 11 10 15 2), (1 6 8 13 3 5 10 15 2 7 12 14 4 9 11), (1 6 8)(2 10 15)(3 5 13)(4 9 11)(7 12 14), (1 6 8 4 9 11 7 12 14 10 15 2 13 3 5), (1 3 5 7 9 11 13 15 2 4 6 8 10 12 14), (1 9 11 13 6 8 10 3 5 7 15 2 4 12 14), (1 12 14)(2 4 15)(3 5 7)(6 8 10)(9 11 13), (1 15 2 4 3 5 7 6 8 10 9 11 13 12 14), (1 9 8 13 6 5 10 3 2 7 15 14 4 12 11), (1 9 11)(2 7 15)(3 5 10)(4 12 14)(6 8 13), (1 9 2 7 15 8 13 6 14 4 12 5 10 3 11), (1 9 5 10 3 14 4 12 8 13 6 2 7 15 11), (1 9 14 13 6 11 10 3 8 7 15 5 4 12 2), (1 9 14)(2 4 12)(3 8 10)(5 7 15)(6 11 13), (1 9 14 7 15 5 13 6 11 4 12 2 10 3 8), (1 9 14 10 3 8 4 12 2 13 6 11 7 15 5), (1 3 8 13 15 5 10 12 2 7 9 14 4 6 11), (1 6 11)(2 7 12)(3 8 13)(4 9 14)(5 10 15), (1 12 2 7 3 8 13 9 14 4 15 5 10 6 11), (1 15 5 10 9 14 4 2 8 13 12 2 7 6 11),(1 12 11 4 15 14 7 3 2 10 6 5 13 9 8), (1 12 14 7 3 5 13 9 11 4 15 2 10 6 8), (1 12 2 10 6 11 4 15 5 13 9 14 7 3 8), (1 12 8)(2 10 6)(3 14 7)(4 15 11), (5 13 9), (1 12 5 4 15 8 7 3 11 10 6 14 13 9 2), (1 12 5 7 3 11 13 9 2 4 15 8 10 6 14), (1 12 5 10 6 14 4 15 8 13 9 2 7 3 11), (1 12 5)(2 13 9)(3 11 7)(4 15 8)(6 14 10), (1 3 11 4 6 14 7 9 2 10 12 5 13 15 8), (1 6 14 7 12 5 13 3 11 4 9 2 10 15 8), (1 9 2 10 3 11 4 12 5 13 6 14 7 15 8),(1 15 8)(2 10 9)(3 11 4)(5 13 12)(6 14 7), (1 15 14 10 9 8 4 3 2 13 12 11 7 6 5), (1 15 2 13 12 14 10 9 11 7 6 8 4 3 5), (1 15 5)(2 13 12)(3 8 4)(6 11 7)(9 14 10)(1 15 8 4 3 11 7 6 14 10 9 2 13 12 5), (1 15 11 10 9 5 4 3 14 12 8 7 6 2), (1 15 11 13 12 8 10 9 5 7 6 2 4 3 14), (1 15 11)(2 7 6)(3 14 4)(5 10 9)(8 13 12), (1 15 11 4 3 14 7 6 2 10 9 5 13 12 8), (1 3 14 10 12 8 4 6 2 13 15 11 7 9 5), (1 6 2 13 3 14 10 15 11 7 12 8 4 9 5), (1 9 5)(2 13 6)(3 14 10)(4 12 8)(7 15 11), (1 12 8 4 15 11 7 3 14 10 6 2 13 9 5)}

Now, $|G_4| = 375 = 3 \times 5^3$

4.4.1 G_4 has a unique Sylow 5-subgroup H_3 of order 125 given by

H_3 = {(1), (1 4 7 10 13)(2 5 8 11 14)(3 6 9 12 15), (1 7 13 4 10)(2 8 14 5 11)(3 9 15 6 12), (1 10 4 13 7)(2 11 5 14 8)(3 12 6 15 9), (1 13 10 7 4)(2 14 11 8 5)(3 15 12 9 6), (2 5 8 11 14)(3 9 15 6 12), (2 5 8 11 14)(3 12 6 15 9), (2 5 8 11 14)(3 15 12 9 6), (2 8 14 5 11)(3 6 9 12 15), (2 8 14 5 11)(3 12 6 15 9), (2 8 15 5 11)(3 15 12 9 6), (2 11 5 14 8)(3 6 9 12 15), (2 11 5 14 8)(3 6 9 12 15), (2 11 5 14 8)(3 9 15 6 12), (2 11 5 14 8)(3 15 12 9 6), (2 14 11 8 5)(3 6 9 12 15), (2 14 11 8 5)(3 9 15 6 12), (2 14 11 8 5)(3 12 6 15 9), (1 4 7 10 13)(3 9 15 6 12), (1 4 7 10 13)(3 12 6 15 9), (1 4 7 10 13)(3 15 12 9 6), (1 4 7 10 13)(2 8 14 4 11), (1 4 7 10 13)(2 8 14 5 11)(3 12 6 15 9), (1 4 7 10 13)(2 8 14 5 11)(3 15 12 9 6), (1 4 7 10 13)(2 11 5 14 8), (1 4 7 10 13)(2 11 5 14 8)(3 9 15 6 12), (1 4 7 10 13)(2 11 5 14 8)(3 15 12 9 6), (1 4 7 10 13)(2 14 11 8 5), (1 4 7 10 13)(2 14 11 8 5)(3 9 15 6 12), (1 4 7 4 13)(2 14 11 8 5)(3 12 6 15 9), (1 7 13 4 10)(3 6 9 12 15), (1 7 13 4 10)(3 12 6 15 9), (1 7 13 4 10)(3 15 12 9 6), (1 7 13 4 10)(2 5 8 11 14), (1 7 13 4 10)(3 15 12 9 6), (1 7 13 4 10)(2 5 8 11 14), (1 7 13 4 10)(2 5 8 11 14)(3 12 6 15 9), (1 7 13 4 10)(2 5 8 11 14)(3 15 12 9 6), (1 7 13 4 10)(2 11 5 14 8), (1 7 13 4 10)(2 11 5 14 8)(3 6 9 12 15), (1 7 13 4 10)(2 11 5 14 8)(3 15 12 9 6), (1 7 13 4 10)(2 14 11 8 5), (1 7 13 4 10)(2 14 11 8 5)(3 6 9 12 15), (1 7 13 4 10)(2 14 11 8 5)(3 12 6 15 9), (1 10 4 13 7)(3 6 9 12 15), (1 10 4 13 7)(3 9 15 6 12), (1 10 4 13 7)(3 15 12 9 6), (1 10 4 13 7)(2 5 8 11 14), (1 10 4 13 7)(2 5 8 11 14)(3 9 15 6 12), (1 10 4 13 7)(2 5 8 11 14)(3 15 12 9 6), (1 10 4 13 7)(2 8 14 5 11),(1 10 4 13 7)(2 8 14 5 11)(3 6 9 12 15), (1 10 4 13 7)(2 8 14 5 11)(3 15 12 9 6), (1 10 4 13 7)(2 14 11 8 5), (1 10 4 13 7)(2 14 11 8 5)(3 6 9 12 15), (1 10 4 13 7)(2 14 11 8 5)(3 9 15 6 12), (1 13 10 7 4)(3 6 9 12 15), (1 13 10 7 14)(3 9 15 6 12), (1 13 10 7 14)(3 12 6 15 9), (1 13 10 7 4)(2 5 8 11 14), (1 13 10 7 4)(2 5 8 11 14)(3 9 15 6 12), (1 13 10 7 4)(2 5 8 11 14)(3 12 6 15 9), (1 13 10 7 4)(2 8 14 5 11), (1 13 10 7 4)(2 8 14 5 11)(3 6 9 12 15), (1 13 10 7 4)(2 8 14 5 11)(3 12 6 15 9), (1 13 10 7 4)(2 11 5 14 8), (1 13 10 7 4)(2 11 5 14 8)(3 6 9 12 15), (1 13 10 7 4)(2 11 5 14 8)(3 9 15 6 12)(3 6 9 12 15), (3 9 15 6 12), (3 12 6 15 9), (3 15 12 9 6), (2 5 8 11 14), (2 8 14 5 11), (2 11 5 14 8), (2 14 11 8 5), (1 4 7 10 13), (1 7 13 4 10), (1 10 4 13 7), (1 13 10 7 4), (1 4 7 10 13)(2 5 8 11 14), (1 4 7 10 13)(2 5 8 11 14)(3 9 15 6 12), (1 4 7 10 13)(2 5 8 11 14)(3 12 6 15 9), (1 4 7 10 13)(2 5 8 11 14)(3 15 12 9 6), (1 4 7 10 13)(3 6 9 12 15), (1 4 7 10 13)(2 8 14 5 11)(3 6 9 12 15), (1 4 7 10 13)(2 11 5 14 8)(3 6 9 12 15), (1 4 7 10 13)(2 14 11 8 5)(3 6 9 12 15), (2 5 8 11 14)(3 6 9 12 15), (1 17 13 4 10)(2 5 8 11 14)(3 6 9 12 15), (1 10 4 13 7)(2 5 8 11 14)(3 6 9 12 15), (1 13 10 7 4)(2 5 8 11 14)(3 6 9 12 15), (1 7 13 4 10)(2 8 14 5 11)(3 6 9 12 15), (1 7 13 4 10)(2 8 14 15 11)(3 12 6 15 9), (1 7 13 4 10)(2 8 14 15 11)(3 15 12 9 6), (1 7 13 4 10)(3 9 15 6 12), (1 7 13 4 10)(2 5 8 11 14)(3 9 15 6 12), (1 7 13 4 10)(2 11 5 14 8)(3 9 15 6 12),(1 7 13 4 10)(2 14 11 8 5)(3 9 15 6 12),(2 8 14 5 11)(3 9 15 6 12),(1 4 7 10 13)(2 8 14 5 11)(3 9 15 6 12),(1 10 4 13 7)(2 8 11 14 5 11)(3 9 15 6 12), (1 13 10 7 4)(2 8 14 5 11)(3 9 15 6 12), (1 10 4 13 7)(2 11 5 14 8), (1 10 4 13 7)(2 11 5 14 8)(3 6 9 12 15), (1 10 4 13 7)(2 11 5 14 8)(3 9 15 6 12), (1 10 4 13 7)(2 11 5 14 8)(3 15 12 9 6), (1 10 4 13 7)(3 12 6 15 9), (1 10 4 13 7)(2 5 8 11 14)(3 12 6 15 9), (1 10 4 13 7)(2 8 14 5 11)(3 12 6 15 9), (1 10 4 13 7)(2 14 11 8 5)(3 12 6 15 9), (2 11 5 14 8)(3 12 6 15 9), (1 4 7 10 13)(2 11 5 14 8)(3 12 6 15 9), (1 7 13 4 10)(2 11 5 14 8)(3 12 6 15 9), (1 13 10 7 4)(2 11 5 14 8)(3 12 6 15 9), (1 13 10 7 4)(2 14 11 8 5), (1 13 10 7 4)(2 14 11 8 5)(3 6 9 12 15), (1 13 10 7 4)(2 14 11 8 5)(3 9 15 6 12), (1 13 10 7 4)(2 14 11 8 5)(3 12 6 15 9), (1 13 10 7 4)(3 15 12 9 6), (1 13 10 7 4)(2 5 8 11 14)(3 15 12 9 6), (1 13 10 7 4)(2 8 14 5 11)(3 15 12 9 6), (1 13 10 7 4)(2 11 5 14 8)(3 15 12 9 6), (2 14 11 8 5)(3 15 12 9 6), (1 4 7 10 13)(2 14 11 8 5)(3 15 12 9 6), (1 7 13 4 10)(2 14 11 8 5)(3 15 12 9 6), (1 10 4 13 7)(2 14 11 8 5)(3 15 12 9 6)}

H_3 is a p-group of order 125 given by $|H_3| = 5^3$

5.0 COMPARISON OF RESSULTS

We now apply a standard program GAP to validate what we obtained in section 4.

5.1 Thus, validating our results in 4.1

GAP> C1:=GROUP((1,2,3,4,5));GROUP([(1,2,3,4,5)])

GAP> D1:=GROUP((6,7));GROUP([(6,7)])

GAP>G1:=WREATHPRODUCT(C1,D1);GROUP([(1,2,3,4,5),(6,7,8,9,10),(1,6)(2,7)(3,8)(4,9)(5,10)])
GAP> ORDER(G1);50
GAP> H1:=SYLOWSUBGROUP(G1,5);GROUP([(1,4,2,5,3)(6,9,7,10,8),(1,2,3,4,5)(6,10,9,8,7)])
GAP> ORDER(H1);25

5.2 We also validate the result in 4.2

GAP> C2:=GROUP((1,2,3,4,5,6,7));GROUP([(1,2,3,4,5,6,7)])
GAP> D2:=GROUP((8,9));GROUP([(8,9)])
GAP>G2:=WREATHPRODUCT(C2,D2);GROUP([(1,2,3,4,5,6,7),(8,9,10,11,12,13,14),(1,8)(2,9)(3,10)(4,11)(5,12)(6,13)(7,14)])
GAP> ORDER(G2);98
GAP>H2:=SYLOWSUBGROUP(G2,7);GROUP([(1,5,2,6,3,7,4)(8,12,9,13,10,14,11),(1,2,3,4,5,6,7)(8,14,13,12,11,10,9)])
GAP> ORDER(H2);49

5.3 Our result in 4.3 above is also validated as:

GAP> C3:=GROUP((1,2,3));GROUP([(1,2,3)])
GAP> D3:=GROUP((4,5,6));GROUP([(4,5,6)])
GAP> G3:=WREATHPRODUCT(C3,D3);GROUP([(1,2,3), (4,5,6), (7,8,9), (1,4,7)(2,5,8)(3,6,9)])
GAP> ORDER(G3);81

5.4 Results in 4.4 is validated as:

GAP> C4:=GROUP((1,2,3,4,5));GROUP([(1,2,3,4,5)])
GAP> D4:=GROUP((6,7,8));GROUP([(6,7,8)])
GAP>G4:=WREATHPRODUCT(C4,D4);GROUP([(1,2,3,4,5),(6,7,8,9,10),(11,12,13,14,15),(1,6,11)(2,7,12)(3,8,13)(4,9,14)(5,10,15)])
GAP> ORDER(G4);375
GAP>H3:=SYLOWSUBGROUP(G4,5);GROUP([(1,3,5,2,4)(6,8,10,7,9)(11,13,15,12,14),(1,2,3,4,5)(11,15,14,13,12), (1,5,4,3,2)(6,7,8,9,10)])
GAP> SIZE(H3);125
GAP> ORDER(H3);125
GAP> QUIT;

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