

Computational analysis of pulsatile biofluid in a locally expanded vessel under the action of magnetic field

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ABSTRACT

The transient impact of magnetic field on the drug delivery and heat transfer rate was investigated through an aneurysm artery. Vessel walls were considered as porous media, and blood viscosity was assumed to show non-Newtonian behaviour. The transient solution was used for solving governing equations in porous and free regions simultaneously while both magnetization and Lorentz exerted the external forces. These forces resulted in vortex formation and flow flux to the aneurysm region. The effects of the magnetic field on the velocity, temperature, heat transfer rate and shear stress distribution through the dilated vessel were further studied. The results showed that applying of a magnetic field intensity (Mnf) of 10^9 creates high shear stress and v-velocity on aneurysm region, and by doubling Mnf, these factors increase 2X and 1.38X respectively. Furthermore, by doubling the magnetic field intensity, the maximum temperature showed an increase of 5.2% on the dilation region and Nusselt number, indicating that heat transfer rate rises sharply with increasing Lorentz part of the magnetic field.

Keywords: Biomagnetic fluid, Drug delivery, Aneurysm artery, Heat transfer, Blood flow

INTRODUCTION

The inflating of vessel's wall or blood-filled dilation is an aneurysm disease which occurs due to weakening of endothelial layer [1]. The disease is classified in two types; saccular and fusiform. The former, is like a small sack which develops in small medullar vessels[2]. In the latter type, the vessel swells symmetrically around the aneurysm region[3]. Recently, drug delivery has been carried out through the intravenous injection which travels to heart as the target, and distributes in the whole body. This method may not be efficient, and could even damage intact cells. The magnetic target delivery is one of the new efficient and non-destructive drug delivery methods [4, 5]. This method could be utilized for many diseases like stenosis, thrombosis, and aneurysm for focusing the drug in a specific region [6]. During the last decade, numerous investigations have been done in this area. A theoretical study on the magnetic drug delivery was reported by Grief and Richardson who focused on the two dimensional model for ferrofluid motion in blood vessel[7]. A high gradient magnetic separation was also proposed to study magnetic drug delivery [8][9]. This method can employing for many disease as stenosis[10], thrombosis [11] and aneurysm[12] for focusing the drug in the specific regions in inner wall of blood vessel. Wang et al. developed a hydrodynamic modeling of biofluid in magnetic drug delivery [13]. A mathematical model for natural biomagnetic fluid base on the Ferro hydro dynamic (FHD) principles was further suggested[14]. Since electrical conductivity of blood flow is high, to obtain accurate results, it is essential to consider Lorentz force which appears due to Magneto-hydrodynamic (MHD) [15][16].The current work reports for the first time, the transient solving of the blood flow equations in free and porous media simultaneously considering the aneurysm geometry and non-Newtonian behavior of biofluid. The extended Navier-Stokes, Brinkman and energy equations under the influence of external magnetic field governing in the free and porous regions of an aneurysm vessel are studied too. The work also presents the dimensionless equations, boundary conditions and simulation.

MATHEMATICAL FORMULATION

A viscous, non-Newtonian, laminar, incompressible, transient and two dimensional biofluid flow in an aneurysms geometry was assumed. The geometry is shown in

Fig 1[13]. In this model, the aneurysm region was assumed as a part of circle. The vessel walls were considered as porous media which made the model closing to the actual vessel. The thickness and the other diminution of vessel are shown in

Fig 1. The D is the artery height; each porous wall had 0.15D thickness. In this study, $L_T=10 \times D$, $R=0.4 \times D$ which is related to r as: $R=r+0.15 \times D$

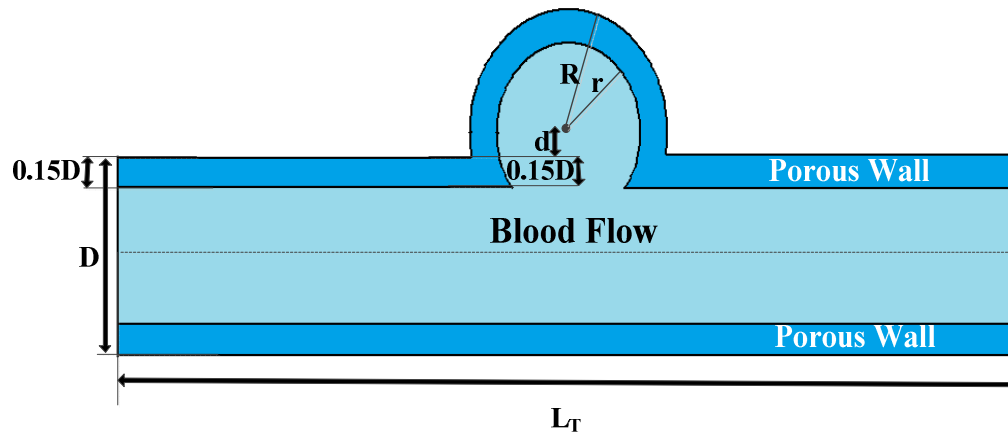


Fig 1. An aneurysm blood model[13]

The governing equations on the biomagnetic flow in the free and porous regions were described by extended Navier-Stokes and Brinkman equations below.

2.1 Heat transfer and fluid flow equations

Continuity equation:

$$\nabla \cdot \vec{V}^* = 0 \tag{1}$$

Momentum equation [17]:

$$\rho \frac{D\vec{V}^*}{Dt^*} = -\nabla p^* + \eta \nabla^2 \vec{V}^* + \vec{J}^* \times \vec{B}^* + \mu_0 M^* \nabla H^* \tag{2}$$

Energy equation [17]:

$$\rho C_p \frac{DT^*}{Dt^*} + \mu_0 T^* \frac{\partial M^*}{\partial T^*} \frac{\partial H^*}{\partial t^*} - \frac{\vec{J}^* \cdot \vec{J}^*}{\sigma^*} = k \nabla^2 T^* + \eta \left(2 \left(\frac{\partial u^*}{\partial x^*} \right)^2 + 2 \left(\frac{\partial v^*}{\partial y^*} \right)^2 + \left(\frac{\partial v^*}{\partial x^*} + \frac{\partial u^*}{\partial y^*} \right)^2 \right) \tag{3}$$

where $\vec{V}^* = (u^*, v^*)$ is the two dimensional velocity field; ρ is the fluid density; P is the pressure; η is the dynamic viscosity; μ_0 is the magnetic permeability of vacuum, M^* is the magnetization, H is the magnetic field intensity, B^* is the magnetic induction where $B^* = \mu_0(M^* + H^*)$, σ^* is the electrical conductivity of the fluid, J is the density of the electrical current, T^* is the temperature, k is the thermal conductivity, and C_p is the specific heat at constant pressure. The terms “ $\mu_0 M^* \nabla H^*$ ” and “ $\vec{J}^* \times \vec{B}^*$ ” are entered into the momentum equations due to the

magnetization and Lorentz forces. In the energy equation the additional term $\mu_0 T^* \frac{\partial M^*}{\partial T^*} \frac{\partial H^*}{\partial t^*}$ is appeared because

of Magneto Hydro Dynamic (MHD) effect while $\frac{\vec{J}^* \cdot \vec{J}^*}{\sigma^*}$ is the symbol of Ferro Hydro Dynamic (FHD) and Joule

heating impact. In this study, the power-law model was considered for the blood viscosity which is defined by the equation below [18]:

$$\eta = \eta_0 |\dot{\gamma}|^n \quad (4)$$

η_0 and n are constant parameters, and $|\dot{\gamma}|$ is shear rate magnitude.

The Brinkman equations for the porous regions are expressed as:

Continuity equation:

$$\nabla \cdot \vec{V}_p^* = 0 \quad (5)$$

Momentum equation:

$$\frac{\rho}{\varepsilon_p} \frac{D\vec{V}_p^*}{Dt^*} = -\nabla p^* + \frac{\eta}{\varepsilon_p} \nabla^2 \vec{V}_p^* - \frac{\eta}{k_{br}} \vec{V}_p^* + J \times B + \mu_0 M^* \nabla H^* \quad (6)$$

ε_p and k_{br} are the porosity and permeability of vessel walls and the $\vec{V}_p^* = (u_p^*, v_p^*)$ describes the velocity field inside the porous regions. The magnetization and Lorentz forces are exerted to the porous regions as well, and the temperature distribution is calculated by equations (3) similar to free flow.

2.2 Magnetization equation

The magnetization property (M) determines the effect of magnetic field on the ferrofluid. Among several formulas used, the linear equation which depends on the magnetic field intensity and temperature was used in this study[19].

$$M^* = \chi_m H^* \quad (7)$$

χ_m is the magnetic susceptibility, and varies with the temperature:

$$\chi_m = \frac{\chi_0}{1 + \beta(T^* - T_0)} \quad (8)$$

χ_0 , β and T_0 are constant parameters, and electrical current through a wire generates the magnetic field. The wire is plum to the x-y plane and its magnetic field intensity is express by:

$$\begin{cases} H_x = H_0 \frac{(x-a)}{((x-a)^2 + (y-b)^2)} \\ H_y = -H_0 \frac{(y-b)}{((x-a)^2 + (y-b)^2)} \end{cases} \quad (9)$$

H_0 is the magnetic field strength which depends on the applied magnetics induction ($B = \mu_0 (H + M)$) and a,b are the location of wire.

3. Transformation of equations:

For convenient analysis, the following non-dimensional variables were defined.

$$t = \frac{\alpha}{D^2} t^* \quad x = \frac{x^*}{D} \quad y = \frac{y^*}{D} \quad u = \frac{u^*}{u_r} \quad v = \frac{v^*}{u_r} \quad p = \frac{p^*}{\rho u_r^2} \quad H = \frac{H^*}{H_0} \quad T = \frac{T^*}{\delta T} \quad (10)$$

$\alpha = \frac{k}{\rho c_p}$ and $u_r = \frac{\alpha}{D}$ are the thermal diffusivity and characteristic velocity of the fluid. By substituting these non-dimensional variables into above equations we have:

Continuity:

$$\nabla \vec{V} = 0 \quad (11)$$

x-momentum:

$$\frac{Du}{Dt} = -\frac{\partial p}{\partial x} + \text{Pr} \times |\dot{\gamma}| (\nabla^2 u) + \text{Mnf} \chi_m H \frac{\partial H}{\partial x} + \frac{Mnm}{\text{Re}} (\chi_m + 1)^2 (v H_x H_y - u H_y^2) \quad (12)$$

y-momentum:

$$\frac{Dv}{Dt} = -\frac{\partial p}{\partial y} + \text{Pr} \times |\dot{\gamma}| (\nabla^2 v) + \text{Mnf} \chi_m H \frac{\partial H}{\partial y} + \frac{Mnm}{\text{Re}} (u H_x H_y - v H_y^2) \quad (13)$$

Also Energy Equation is rewritten as below equation:

$$\begin{aligned} \frac{DT}{Dt} = & (\nabla^2 T) + Ec \times \text{Pr} \times |\dot{\gamma}| \left(2 \left(\frac{\partial u}{\partial x} \right)^2 + 2 \left(\frac{\partial v}{\partial y} \right)^2 + \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right)^2 \right) - \text{Mnf} \times Ec \times \frac{\partial \chi}{\partial T} T \times \\ & H \left(u \frac{\partial H}{\partial x} + v \frac{\partial H}{\partial y} \right) + \frac{Mnm}{\text{Re}} \times Ec \times (\chi_m + 1)^2 \times (u H_y - v H_x)^2 \end{aligned} \quad (14)$$

For the porous region the momentum equations are:

x momentum:

$$\frac{Du_p}{Dt} = -\frac{\partial p}{\partial x} + \text{Pr} \times |\dot{\gamma}| (\nabla^2 u_p) + \text{Pr} \times Da \times u_p + \text{Mnf} \times \chi_m \times H \frac{\partial H}{\partial x} + \frac{Mnm}{\text{Re}} (\chi_m + 1)^2 (v_p H_x H_y - u_p H_y^2) \quad (15)$$

y momentum:

$$\frac{Dv_p}{Dt} = -\frac{\partial p}{\partial y} + \text{Pr} \times |\dot{\gamma}| (\nabla^2 v_p) + \text{Pr} \times Da \times v_p + \text{Mnf} \chi_m H \frac{\partial H}{\partial y} + \frac{Mnm}{\text{Re}} (u H_x H_y - v H_y^2) \quad (16)$$

The non-dimensional parameters which appear in the 11-15 equations are:

$$\text{Re} = \frac{h \rho u_r}{\eta_0 \times \left(\frac{\alpha}{D^2} \right)^{-0.4}} = \frac{\rho \alpha}{\eta_0 \times \left(\frac{\alpha}{D^2} \right)^{-0.4}} \quad (\text{Reynolds number}) \quad (17)$$

$$Ec = \frac{u_r^2}{C_p \delta T} = \frac{\alpha^2}{C_p \delta T D^2} \quad (\text{Eckert number}) \quad (18)$$

$$\text{Pr} = \frac{\eta_0 \times \left(\frac{\alpha}{D^2} \right)^{-0.4}}{\rho \alpha} \quad (\text{Prandtl number}) \quad (19)$$

$$Da = \frac{D^2}{k_{br}} \quad (\text{Darcy number}) \quad (20)$$

$$Mnf = \frac{\mu_0 H_0^2}{\rho u_r^2} = \frac{\mu_0 H_0^2 D^2}{\rho \alpha^2} \text{ (Magnetic number of FHD)} \tag{21}$$

$$Mnm = \frac{\mu_0^2 H_0^2 D^2 \sigma}{\eta} \text{ (Magnetic number of MHD)} \tag{22}$$

3.1. Boundary conditions

The implemented non-dimensional boundary conditions applied for solving equations 11-16 are listed below:

- The upper and lower plates are at constant temperature (T_1) and according to no slip condition; the velocity is zero on them.
- The pressure is zero at the out let and the temperature profile reaches to the fully developed condition.
- The uniform non-dimensional axial velocity ($u=1$) at constant temperature (T_1) is entered to the domain.
- For obtaining a continuous velocity field between the free and porous interface, the velocity is equal.

The constant values used in the current study, are listed in Table1.

Table 1. Parameters and numerical values

Parameters	Numerical value	Parameters	Numerical value
ρ	1050 $kg.m^{-3}$	Ec	8.7×10^{-6}
η_0	$35 \times 10^{-3} kg.m^{-1}.s^{-1}$	α	$1.2210^{-7} m^2.s^{-1}$
Re	0.23	k	$1.832 \times 10^{-3} Joule.^0 K^{-1} m^{-1} s^{-1}$
T_1	300 $^0 K$	Mn_F	$[10^5:5 \times 10^5]$
δT	30 $^0 K$	Mn_M	150
Pr	4.386	Da	100-400-800
C_p	$14.65 J.(Kg.K)^{-1}$	χ_0	0.06
n	0.6	β	$5.6 \times 10^{-3} K^{-1}$

The magnetic field was used for drug delivery process since it was able to transfer the blood flow to aneurysm, and create vortex inside the sac which results to an increase in the contact time of drug around the disease area. The streamline and temperature contour without action of magnetic field are depicted in

Fig 2As shown, the blood does not flow into the aneurysm region. Therefore, the drug particles injected to the blood, are not adsorbed by the target region. In fluid mechanics, the temperature directly relates to the velocity, and the faster the fluid flows, the higher temperature it has. Due to cross-sectional expansion in aneurysm disease, the flow rate decreases in aneurysm region, and consequently its temperature declines.

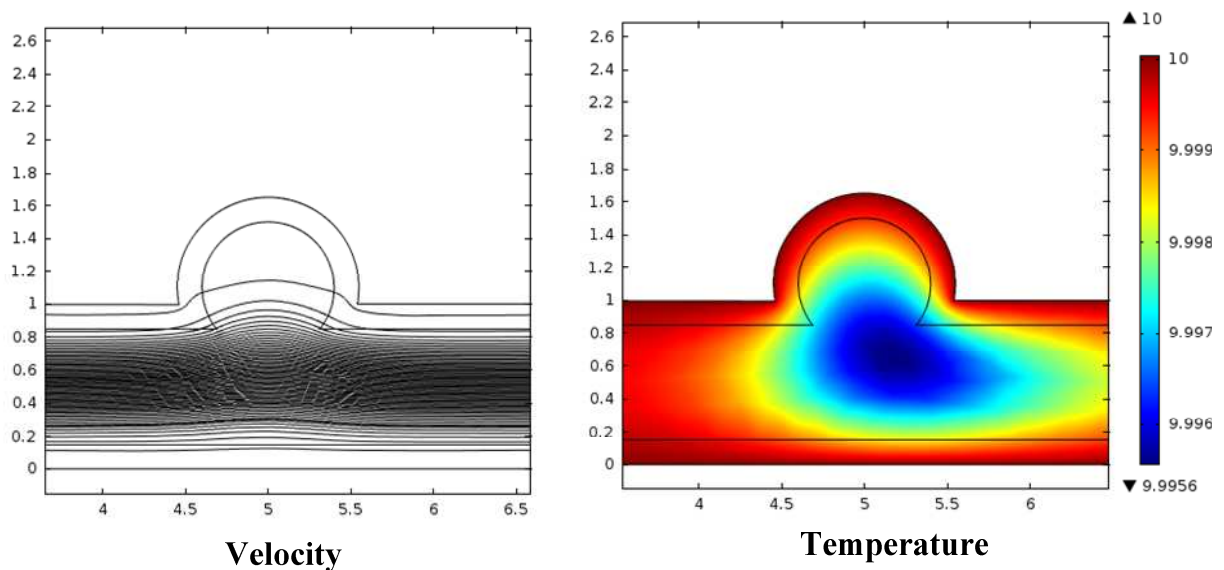


Fig 2. Streamline and temperature contour in aneurysm artery

4. SIMULATION RESULT

shows the effect of magnetic field on the streamline and temperature contours at three different times. As observed, the magnetic field creates vortex inside the bulge which is time-dependent because of increasing the viscosity and inertia forces with the passage of the time. At $t=0.2$, the inertia and viscosity forces overcome the magnetic force. Therefore, only weak vortexes are formed. With the passage of the time, the magnetic force affects the biofluid severely, and creates bigger and stronger vortex on the aneurysm region. In addition, by exerting the magnetic field, the aneurysm temperature is increased, and the maximum temperature is focused on the dilation region. This would enhance the drug activation.

The magnetic field intensity has a direct impact on the penetration velocity, temperature, and shear stress. The effect of this parameter on the v-velocity and temperature of aneurysm region is shown in Fig 4 and Fig 5, respectively. As presented in these figures, with increasing of the magnetic field, the maximum shear stress, velocity, and temperature increase significantly. By doubling and tripling the magnetic field intensity from 10^9 , the maximum velocity (temperature) rises about 2X and 3X (1.05X and 1.14X), respectively.

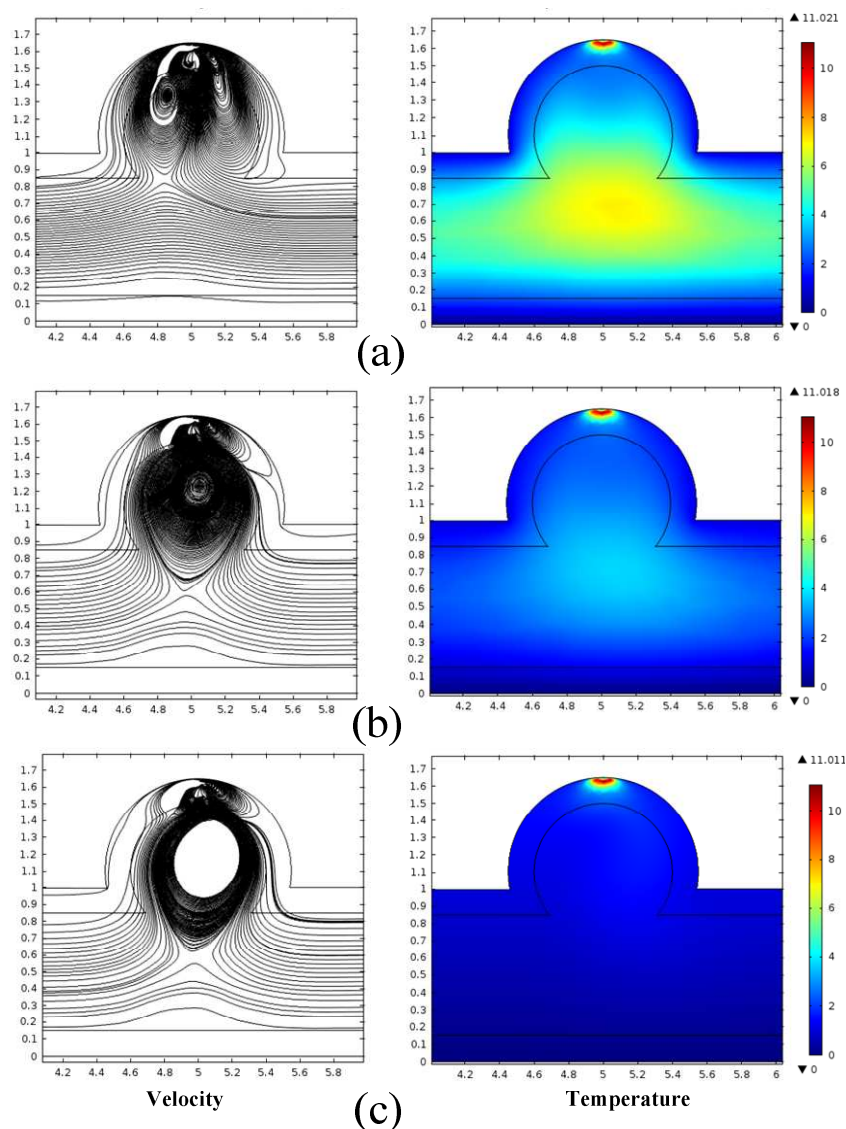


Figure 3. Stream functions and temperature contours with $Mn=10^9$ and $Mn=10^2$ in different time steps, (a) $t=0.2s$, (b) $t=0.5s$ and (c) $t=1s$

By strengthening the magnetic field, the bigger vortexes are created influencing wider area. As such, the velocity gradient increases, and higher shear stress is applied to the critical region.

Fig 6 shows the shear stress in the dilation area. As can be seen, by applying magnetic field with two (three) times stronger than 10^9 , the absolute maximum shear stress enhances 1.38X (1.85X) in the aneurysm region. In the cycle, the biofluid temperature is initially lower than the walls, and heat transfers from the wall to the biofluid. At longer, the biofluid becomes warmer and the heat transfer direction reverses. The magnetic field may intensify the heat transfer rate in biofluid. The Darcy number (Da) indicates the porosity of walls, and significantly influences the Nusselt number. **Error! Reference source not found.** shows the effect of Lorentz force and porosity factor (Da) on the heat transferring rate at $t=1s$. Increasing the magnetic field intensity (Mnm) results in bigger vortex formation. This may warm up the biofluid quickly, and the value of heat transfer is thus increased. The heat transfer rate also depends on the porosity of walls. In artery with enhanced porous characteristics (higher Da), the value of Nusselt number increases due to vicinity to the rigid walls and easier heat transfer. As shown in Table 2, $Da=400$, by two-folds increase in magnetic field intensity from 10^7 , the value of Nusselt number increases by 4.6%.

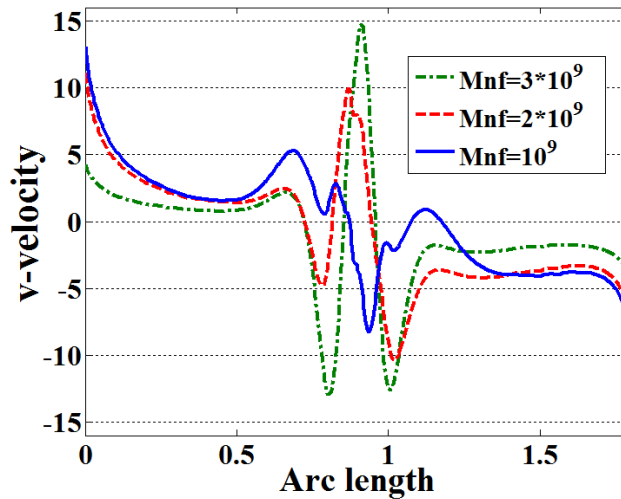


Fig 4. v-velocity in aneurysm region with different values of magnetic field intensity and $Mn_M=10^2$ at $t=1s$

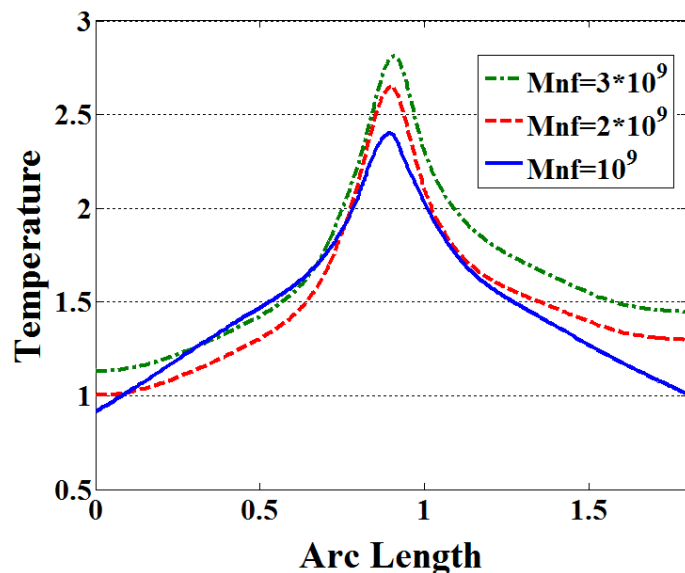


Fig 5. Increased temperature in aneurysm region with different value of magnetic field intensity and $Mn_M=10^2$ at $t=1s$

Table 2. The Nusselt number value at $t=1s$ at different magnetic fields and porosity factors with $Mnf=10^9$

Mnm	Da			
	100	400	1000	4000
0	0.00044	0.00045	0.00046	0.00048
$5 \cdot 10^6$	0.3398	0.34185	0.34289	0.35133
10^7	0.3416	0.34252	0.3437	0.35203
$5 \cdot 10^7$	0.35765	0.35836	0.36123	0.3645

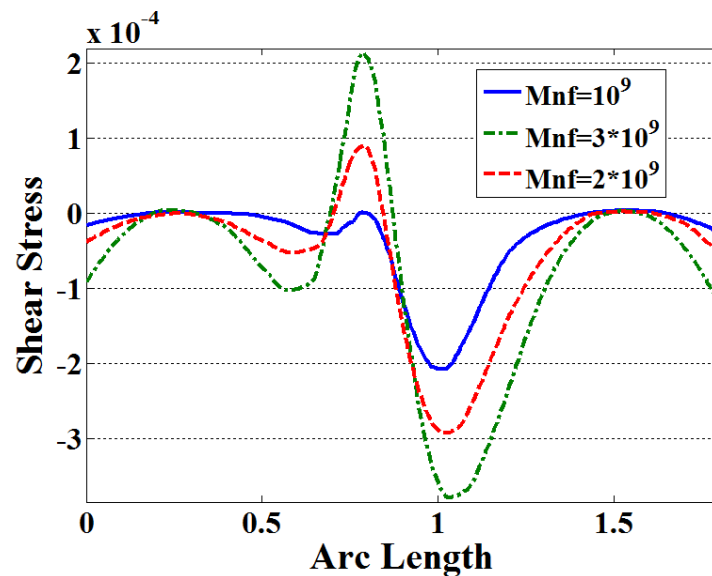


Fig 6. Shear stress in aneurysm region with different values of magnetic field intensity and $Mn_M=10^2$ at $t=1s$

CONCLUSION

We reported for the first time a transient study of magnetic field effect on an aneurysm blood vessel with porous media on the walls. The biofluid was assumed to be a non-Newtonian fluid. A current wire was fixed near the aneurysm and produced the main vortex in the dilation which was proportional to the magnetic intensity. The effect of magnetic field intensity on shear stress, v-velocity and temperature and the roll of walls porosity in heat transfer rate between the blood flow and swollen area are study in this paper. This vortex can act as a drug deliverer by carrying the drug to the target sites.

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