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Comparative study of certain numerical algorithms for solving non linear equations

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ABSTRACT

In this paper, we introduce the comparative study of the numerical methods for solving non linear equation. Here, in particular Two step iterative method is compared with Approximation techniques by circles which includes External touch technique of circles and Orthogonal intersection technique of circles. Numerical Illustrations are given.

Key words: Newton's method, External touch technique of circles, Orthogonal intersection of circles, Two step iteration technique.

INTRODUCTION

Solving non linear equation is one of the most important problems in numerical analysis. Since the beginning of the 1980s, the Adomian decomposition method has been applied to a wide class of functional equations [2]. Adomian gives the solution as an infinite series usually converging to an accurate solution. Abbaoui et al [3] applied the standard Adomian decomposition on simple iteration method to solve the equation $f(x) = 0$, where $f(x)$ is a nonlinear function, and proved the convergence of the series solution. Babolian et al. [4] modified the standard Adomian method which proposed in [3].In recent years, the application of the homotopy perturbation method in nonlinear problems has been devoted by scientists and engineers. Abbasbandy [5] introduces a new improvement of Newton Raphson method by modified homotopy perturbation method.

One of the oldest and most basic problems in mathematics is that of solving nonlinear equations $f(x) =0$. To solve these equations, we can use iterative methods such as Newton's method and its variants. Newton's method is one of the most powerful and well-known iterative methods known to converge quadratic ally. Recently, there has been some progress on iterative methods with higher order of convergence that do require the computation of as lower-order derivatives as possible. In that direction, there has been another approach based on the Adomian decomposition method on developing iterative methods to solve the equation f(x) = 0 [3,4,6]. The Adomian decomposition method is the method which considers the solution as an infinite series usually converging to an accurate solution, and has been successfully applied to a wide class of functional equations over the last 20 years [1]. The convergence of the decomposition scrics have been investigated by several authors. Abbaoui and Cherruault [3] applied the method to solve the equation f(x) = 0 and proved the convergence of the series solution. The adomian method has been modified also so as to construct numerical schemes [3,4]. In the paper[10], Chun constructed a sequence of higher-order iterative methods based on the Adomian decomposition method.

Vinay Kanwar et. al.[13] introduced new numerical techniques for solving non linear equations includes Approximation techniques by circles. It includes mainly two algorithms which are External touch techniques of circles and Orthogonal intersection technique of circles. Among these two, in most of the cases rate of convergence External Touch technique of circles is fast than Newton's algorithm. But the major drawback of these Circles techniques is that in each iteration, we have to evaluate three functional values viz. $f(x), f'(x)$ *and* $f''(x)$, unlike Newton's method. Another major advantage of the above Circle techniques over Newton's method is that they do not fail if the gradient of the function is zero or nearly zero during the iterative cycle. If the root is required to four or five decimal degree of accuracy, then these Circle techniques required the same number of iterations as Newton and Secant technique. When the root is desired to several decimal places, circle techniques required more number of iterations than Newton's method. Finally if $|f'(x) \geq 2|$ then Circle techniques are superior to Bisection, Regula- Falsi, Modified Regula- Falsi, and are as good as Newton's and Secant techniques.

Chun[10] and Basto[9] have proposed and studied several methods for non linear equations with higher order convergence by using the decomposition technique of Adomian[8]. They have used the higher order differential derivatives which is a drawback. Jishe Feng[11] introduced a new two step method for solving non linear equations for solving non linear equations which does not been involved the higher order derivative of the function.

Here in this paper we compare the rate of convergence of two step iterative method with the External Touch technique of circles and Orthogonal intersection of circles. Numerical experiments shows that the rate of convergence of two step iterative procedure is faster than the above Circle techniques.

2. Algorithms for comparison

2.1 Approximation Techniques by circles

Consider the equation f(x) = 0--(1) whose one or more roots to be found. Let y = f(x)--(2) represents the graph of the function f(x) and assume that an initial estimate x_0 is known for the desired root of the equation (1). A circle C_1 of radius $f(x_0+h)$ is drawn with centre at any point $(x_0 + h, f(x_0 + h))$ on the curve of the function (2) where h is small positive or negative quantity. Another circle C_2 with radius $f(x_0 - h)$ and centre at $(x_0 - h, f(x_0 - h))$ is drawn on the curve of function f(x) such that it touches (or intersects) the circle C_1 externally (or orthogonally)

Let $x_1 = x_0 + h$, $|h| << 1$ be the first approximation to the required root of equation (1).

Now consider the following two cases

(i) External touch technique of circles

(ii) Orthogonal intersection technique of circles

2.1.1: External Touch technique of circles

A circle C_1 of radius $f(x_0+h)$ is drawn with centre at any point $(x_0+h, f(x_0+h))$ on the curve of the function $y=g(x)$ where h is small positive or negative quantity. Another circle C_2 with radius $f(x_0-h)$ and centre at $(x_0-h, f(x_0-h))$ is drawn on the curve of the function f(x) such that it touches the circle C_1 externally.

Let $x_1 = x_0 + h$, $|h|<1$. Since the circles C_2 and C_1 touches externally, we have

$$h^2 = f(x_0+h)f(x_0-h).$$

Expanding $f(x_0+h)$ and $f(x_0-h)$ by Taylor's series (omitting fourth and higher powers of h) and simplifying, we can conclude that

$$h = \pm \frac{f(x_o)}{\sqrt{1+f'^2(x_o)-f(x_o)f''(x_o)}} \qquad \text{---- (3)}$$

where h can be taken positive or negative according as x_0 lies in the left or right of true root or slope of the curve at $(x_0, f(x_0))$ is positive or negative. If x_0 lies in the left of true root, then h is taken as positive otherwise negative. Therefore, we get the first approximation to the root as

$$x_1 = x_0 \pm h$$

That is,

$$x_1 = x_0 \pm \frac{f(x_o)}{\sqrt{1+f'^2(x_o)-f(x_o)f''(x_o)}}$$

Since $g(x) = f'(x)$, it follows that

The general iteration formula for successive approximate minimizing point of the non-linear function f is

$$x_{n+1} = x_n \pm \frac{f(x_n)}{\sqrt{1+f'^2(x_n)-f(x_n)f''(x_n)}} \qquad \text{----(4)}$$

The sufficient condition for convergence in the interval containing the root is given by $f(x_n)f''(x_n) < 1+f'^2(x_n)$

2.1.2: Orthogonal intersection technique of circles

A circle C_1 of radius $f(x_0+h)$ is drawn with centre at any point $(x_0+h, f(x_0+h))$ on the curve of the function $y = f(x)$ where h is small positive or negative quantity. Another circle C_2

with radius $f(x_0 - h)$ and centre at $(x_0 - h, f(x_0 - h))$ is drawn on the curve of the function f(x) such that C_2 intersects C_1 orthogonally.

Let $x_1 = x_0 + h$, $|h| < 1$. Since the circles C_2 and C_1 intersects orthogonally, we have

$$f^2(x_0 + h) + f^2(x_0 - h) = (x_0 + h - x_0 + h)^2 + (f(x_0 + h) - f(x_0 - h))^2$$
$$4h^2 = 2\ f(x_0 + h) f(x_0 - h)$$

Expanding $f(x_0 + h)$ and $f(x_0 - h)$ by Taylor's series (omitting fourth and higher powers of h) and simplifying, we can conclude that

$$h = \pm \frac{f(x_o)}{\sqrt{2 + f'^2(x_o) - f(x_o) f''(x_o)}} \qquad \text{---- (5)}$$

where h can be taken positive or negative according as x_0 lies in the left or right of true root or slope of the curve at $(x_0, f(x_0))$ is positive or negative. If x_0 lies in the left of true root, then h is taken as positive otherwise negative. Therefore, we get the first approximation to the root as $x_1 = x_0 \pm h$

That is, $$x_1 = x_0 \pm \frac{f(x_o)}{\sqrt{2 + f'^2(x_o) - f(x_o) f''(x_o)}}$$

The general iteration formula for successive approximate minimizing point of the non-linear function f is

$$x_{n+1} = x_n \pm \frac{f(x_n)}{\sqrt{2 + f'^2(x_n) - f(x_n) f''(x_n)}} \qquad \text{----(6)}$$

The sufficient condition for convergence in the interval containing the root is given by $f(x_n) f''(x_n) < 2 + f'^2(x_n)$

The rate of convergence of the above two methods is quadratic convergence [13].

2.2 Two step Iterative Procedure

Consider the nonlinear equation f(x) = 0 and writing f(x + h) in Taylor's series expansion about x

$$f(x + h) = f(x) + h f'(x) + g(h)$$
$$g(h) = f(x+h) - f(x) - h\ f'(x).$$

Suppose that $f'(x) \neq 0$, one searches for a value of h such that f(x + h) = 0

$$f(x) + h\ f'(x) + g(h) = 0$$

$$h = \frac{-f(x)}{f'(x)} - \frac{g(h)}{f'(x)} \quad \text{-------} \quad (7)$$

Equation (1) can be rewritten as $h = c + N(h)$ ----- (8)

where $c = \dfrac{-f(x)}{f'(x)}$ and $N(h) = \dfrac{-g(h)}{f'(x)}$

$$N(h) = \frac{-[\,f(x+h) - f(x) - h\ f'(x)\,]}{f'(x)}$$

Here c is a constant and N(h) is a non linear function. When applying Adomian's method to (8), we use the technique of Basto and obtain

$$S = \frac{-N'(c+S^*)S + N(c+S)}{1 - N'(c+S^*)}$$

When x is sufficiently close to the real solution of f(x) = 0, $S^* \approx 0$. Thus the above equation converts to

$$S = \frac{-N'(c)S + N(c+S)}{1 - N'(c)}$$

Apply the Adomian method[4] to (8), we have

$$A_0 = N(h_0) = N(c) = \frac{N(c)}{1 - N'(c)} = -\frac{f(x+c)}{2f'(x) - f'(x+c)}$$

Now we construct the iterative method,

For $h \approx h_0 = \dfrac{-f(x)}{f'(x)}$ obtains $h + x \approx x - \dfrac{f(x)}{f'(x)}$

which yields the Newton's method $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$

For k = 1, one obtains $h \approx h_0 + N(h_0)$

which suggest the following two step iterative method[6],

$$y_n = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_{n+1} = y_n - \frac{f(y_n)}{2f'(x_n) - f'(y_n)}$$

The two step iterative method has quadratic convergence [11].

3. Numerical Illustrations

We present some examples to illustrate the efficiency of the two step iterative proposed in this paper. We compare the two step iterative method with Newton's method , External touch technique of circles, Orthogonal intersection of circles, Adomian method, Abbasbandy method, Basto method and Babolian method.

Example 3.1: Consider the equation $f_1(x) = x^3 + 4x^2 + 8x + 8 = 0, \;\; x_0 = -1$

Table 1: Number of iterations and solution obtained for the different methods

Sl. No	Method	Number of iterations	Obtained solution
1	Newton's method	1	-2.0000000
2	Two step iterative method	1	-2.0000000
3	External Touch Technique of Circles	5	-2.0000000
4	Orthogonal intersection technique of circles	6	-2.0000000
5.	Abbasbandy	2	-2.00398774
6	Basto	3	-2.00010090
7	Adomian	Slow convergence	
8	Babolian	Divergence	

Example 3.2: Consider the equation $f_2(x) = x - 2 - e^x = 0, \;\; x_0 = 2$

Table 2: Number of iterations and solution obtained for the different methods

Sl. No	Method	Number of iterations	Obtained solution
1	Newton's method	3	2.120028
2	Two step iterative method	2	2.120028
3	External Touch Technique of Circles	9	2.120028
4	Orthogonal intersection technique of circles	13	2.120028
5.	Abbasbandy	2	2.120028
6	Basto	2	2.120028
7	Adomian	6	2.120003
8	Babolian	4	2.120016

Example 3.3: Consider the equation $f_3(x) = x^2 - (1-x)^5 = 0, \;\; x_0 = 0.2$

Table 3: Number of iterations and solution obtained for the different methods

Sl. No	Method	Number of iterations	Obtained solution
1	Newton's method	3	0.345954
2	Two step iterative method	3	0.345955
3	External Touch Technique of Circles	6	0.345955
4	Orthogonal intersection technique of circles	10	0.345955
5.	Abbasbandy	2	0.345955
6	Basto	2	0.345952
7	Adomian	10	0.340622
8	Babolian	5	0.346021

Example 3.4: Consider the equation $f_4(x) = e^x - 3x^2 = 0, \ x_0 = 0$

Table 4: Number of iterations and solution obtained for the different methods

Sl. No	Method	Number of iterations	Obtained solution
1	Newton's method	5	-0.458962
2	Two step iterative method	5	-0.458962
3	External Touch Technique of Circles	5	-0.458962
4	Orthogonal intersection technique of circles	6	-0.458962
5.	Abbasbandy	5	-0.458964
6	Basto	2	-0.458992

CONCLUSION

In this paper, we have compared the two step iterative method with External Touch technique of Circles, Orthogonal intersection of Circles, Newton's Algorithm and others. It is clear from the numerical experiments that the rate of convergence of two stage iteration method is faster than External Touch Technique of Circles, Orthogonal Intersection of Circles.

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