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Common fixed point theorem in M-Fuzzy metric space satisfying integral type contractive conditions

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ABSTRACT

Bhatia et.al [10] proved a theorem using common EA property which assures the existence of a fixed point for four weak compatible mappings in the framework of fuzzy metric space. Motivated by this, we prove a common fixed point theorem using common E.A. property for three pairs of weakly compatible mapping satisfying integral type contractive conditions in M-fuzzy metric space. An example is also given in the support of the theorem.

AMS Mathematics Subject Classification: 47H10, 54H25. **Keywords**: Fixed point, weakly compatible mappings, E. A. property,M-fuzzy metric space.

INTRODUCTION

It proved a turning point in the development of mathematics when the notion of fuzzy set was introduced by Zadeh [9] which laid the foundation of fuzzy mathematics. Since then to use this concept in topology and analysis many authors have expansively developed the theory of fuzzy sets and application. George and Veeramani [2] and Kramosil and Michalek [8] introduced the concept of fuzzy topological spaces induced by fuzzy metric which have very important applications in quantum particle physics particularly in connections with both string and E-infinity theory which were given and studied by El Naschie [4-7] and [15]. Dhage [3] introduced the notion of generalized metric or D-metric spaces and proved several fixed point theorems in it. Recently Sedghi and Shobe [13] introduced D*-metric spaces [14]. Based on D*- metric concepts, they [13] define M-fuzzy metric space and proved a common fixed point theorems in M-fuzzy metric space using weak compatible mappings. Recently Bhatia et.al [10] proved some common fixed point theorems using EA property in fuzzy metric space.

In this paper we prove some common fixed point theorem for three pairs of weak compatible mappings using common EA property and integral type contractive conditions ,which extend and generalizes the results of Bhatia et.al [10].

MATERIALS AND METHODS

Definition1.1. A binary operation $*: [0, 1] \times [0, 1] \rightarrow [0, 1]$ is a continuous t-norm if it satisfies the following conditions

- (1) * is associative and commutative,
- (2) * is continuous,
- (3) a * 1 = a for all $a \in [0, 1]$,

(4) $a * b \le c * d$ whenever $a \le c$ and $b \le d$, for each $a, b, c, d \in [0, 1]$.

Two typical examples of continuous t-norm are a * b = ab and a * b = min(a, b).

Definition1.2. A 3-tuple (X, M, *) is called a M-fuzzy metric space if X is an arbitrary (non-empty) set, * is a continuous t-norm, and M is a fuzzy set on $X^3 \times (0, \infty)$, satisfying the following conditions for each x, y, z, a $\in X$ and t, s > 0,

(1) M(x, y, z, t) > 0,

(2) M(x, y, z, t) = 1 if and only if x = y = z,

(3) $M(x, y, z, t) = M(p\{x, y, z\}, t)$, (symmetry) where p is a permutation function,

(4) $M(x, y, a, t) * M(a, z, z, s) \le M(x, y, z, t + s)$,

(5) M(x, y, z, .) : $(0, \infty) \rightarrow [0, 1]$ is continuous

Remark 1.1.Let (X, M, *) be a M-fuzzy metric space. We prove that for every t > 0,

M(x, x, y, t) = M(x, y, y, t). Because for each $\varepsilon > 0$ by triangular inequality we have

(i) $M(x, x, y, \epsilon + t) \ge M(x, x, x, \epsilon) * M(x, y, y, t) = M(x, y, y, t)$

(ii) $M(y, y, x, \epsilon + t) \ge M(y, y, y, \epsilon) * M(y, x, x, t) = M(y, x, x, t).$

By taking limits of (i) and (ii) when $\varepsilon \rightarrow 0$, we obtain M(x, x, y, t) = M(x, y, y, t).

Let (X, M, *) be a M-fuzzy metric space. For t > 0, the open ball BM(x, r, t) with center $x \in X$ and radius 0 < r < 1 is defined by BM $(x, r, t) = \{y \in X : M(x, y, y, t) > 1 - r \}$.

A subset A of X is called open set if for each $x \in A$ there exist t > 0 and 0 < r < 1 such that

 $BM(x, r, t) \subseteq A.$

A sequence $\{x_n\}$ in X converges to x if and only if $M(x, x, x_n, t) \rightarrow 1$ as $n \rightarrow \infty$, for each t > 0. It is called a Cauchy sequence if for each $0 < \epsilon < 1$ and t > 0, there exist $n_0 \in N$ such that $M(x_n, x_n, x_m, t) > 1 - \epsilon$ for each $n, m \ge n_0$. The M-fuzzy metric (X, M, *) is said to be complete if every Cauchy sequence is convergent.

Example 1.1.Let X is a nonempty set and D is the D-metric on X. Denote a * b = a.b for all $a, b \in [0,1]$. For each t $\in]0, \infty[$, define

M(x, y, z, t) = t/[t + D(x, y, z)]

for all x, y, $z \in X$. It is easy to see that (X, M, *) is a M-fuzzy metric space.

Lemma 1.1.Let (X, M, *) is a fuzzy metric space. If we define M: $X^3 \times (0, \infty) \rightarrow [0, 1]$ by

M(x, y, z, t) = M(x, y, t) * M(y, z, t) * M(z, x, t) for every x, y, z in X, then (X, M, *) is a M-fuzzy metric space

Proof.

(1) It is easy to see that for every x, y, $z \in X$, $M(x, y, z, t) > 0 \forall t > 0$.

(2) M(x, y, z, t) = 1 if and only if M(x, y, t) = M(y, z, t) = M(z, x, t) = 1 if and only if x = y = z.

(3) $M(x, y, z, t) = M(p\{x, y, z\}, t)$, where p is a permutation function.

 $\begin{array}{l} (4) \ M(x,\,y,\,z,\,t+s) = M(x,\,y,\,t+s) * M(y,\,z,\,t+s) * M(z,\,x,\,t+s) \\ \geq M(x,\,y,\,t\,) * M(y,\,a,\,t\,) * M(a,\,z,\,s) * M(z,\,a,\,s) * M(a,\,x,\,t\,) = M(x,\,y,\,a,\,t\,) * M(a,\,z,\,s) * M(z,\,a,\,s) * M(z,\,z,\,s) \\ \end{array}$

y, a , t) \ast M(a, z, z, s) for every s > 0.

Lemma 1.2.Let (X, M, *) be a M-fuzzy metric space. Then M(x, y, z, t) is non decreasing with respect to t, for all x, y, z in X

Proof. By definition 1.2 (4) for each x, y, z, $a \in X$ and t, s > 0 we have

 $M(x,\,y,\,a,\,t\;)*M(a,\,z,\,z,\,s)\leq M(x,\,y,\,z,\,t+s).$

If set a = z we get $M(x, y, z, t) * M(z, z, z, s) \le M(x, y, z, t + s)$, that is $M(x, y, z, t + s) \ge M(x, y, z, t)$.

Definition 1.3.Let (X, M, *) be a M-fuzzy metric space. M is said to be continuous function on

 $X^3 \times (0, \infty)$ if $\lim M(x_n, y_n, z_n, t_n) = M(x, y, z, t)$ as $n \to \infty$.

Whenever a sequence $\{(x_n, y_n, z_n, t_n)\}$ in $X^3 \times (0, \infty)$ converges to a point $(x, y, z, t) \in X^3 \times (0, \infty)$ i.e.

 $\lim x_n = x, \lim y_n = y, \lim z_n = z \text{ and } \lim M(x, y, z, t_n) = M(x, y, z, t_n) \text{ as } n \to \infty.$

Lemma 1.3.Let (X, M, *) be a M-fuzzy metric space. Then M is continuous function on $X^3 \times (0, \infty)$.

Definition 1.4. Let A and B be two self-mappings of a M-fuzzy metric space (X, M, *).

We say that A and B satisfy the common property (E), if there exists a sequence $\{x_n\}$ such that

 $\lim M(Axn, u, u, t) = \lim M(Bxn, u, u, t) = 1 \text{ as } n \rightarrow \infty$

for some $u \in X$ and t > 0.

Example 1.2.Let X=R and M(x, y, z, t) = t/ [t + |x - y| + |y - z| + |x - z|]

for every x, y, $z \in X$ and t > 0. Let A and B, Ax = 2x + 1, Bx = x + 2.

Consider the sequence $x_n=1/n+1$, n=1,2,3... Thus we have

 $\lim M(Axn, 3, 3, t) = \lim M(Bxn, 3, 3, t) = 1 \text{ as } n \rightarrow \infty$

for every t > 0. Then A and B satisfying the property (E).

Definition 1.5. Let A and S be mappings from a M-fuzzy metric space (X, M, *) into itself. Then the mappings are said to be weak compatible if they commute at their coincidence point, that is, Ax = Sx implies that ASx = SAx.

Definition 1.6. Let A and S be mappings from a M-fuzzy metric space (X, M, *) into itself. Then the mappings are said to be compatible if

$$\begin{split} & \text{Lim } M(ASx_n, SAx_n, SAx_n, t \) = 1, \ \forall \ t > 0 \\ & n \to \infty \end{split}$$
 whenever $\{x_n\}$ is a sequence in X such that $\lim Ax_n = \lim Sx_n = x \in X \text{ for } n \to \infty . \end{split}$

Branciari -Integral contractive type condition [1]: For a given $\varepsilon > 0$, there exists a real number $c \in (0, 1)$ and a locally Lebesgue-integrable function $g : [0, \infty) \rightarrow [0, \infty)$ such that

 $\int_0^{d(fx,fy)} g(t) dt \le c \int_0^{d(x,y)} g(t) dt \text{ and } \int_0^c g(t) dt > 0, \text{ for all } x, y \text{ in } X \text{ and for each } \varepsilon > 0.$

Also, Branciari-Integral contractive type condition is a generalization of Banach contraction map if g(t) = 1 for all t ≥ 0 .

RESULTS AND DISCUSSION

Let Ψ (t) : R⁺ \rightarrow R is Lebesgue-integrable mapping which is summable, non-negative and

 $\int_0^t \Psi(t) dt > 0, \text{ for each } \varepsilon > 0 \text{ and if } \int_0^{M(x,y,z,g(t))} \Psi(t) dt \ge \int_0^{M(x,y,z,t)} \Psi(t) dt \text{ then } t < 0 \text{ for each } \varepsilon > 0 \text{ and if } \int_0^{M(x,y,z,g(t))} \Psi(t) dt = 0$

x = y = z, where $g : (0,\infty) \rightarrow (0,\infty)$ is such that g(t) < t for every t and X is a M- fuzzy metric space on $X^3 \times (0,\infty)$ $\forall x, y, z \in X, t > 0.$

Theorem2.1. Let A, B, I, J,R and S be self mappings of a M-fuzzy metric space (X,M, *) satisfying

(i) $A(X) \subseteq R(X)$, $B(X) \subseteq S(X)$, $I(X) \subseteq J(X)$ and J(X) or R(X) or S(X) is a closed subspace of X. (ii) Pairs (A, J), (B,R) and (I,S) are weakly compatible. (iii) Any two pairs from (A,J), (B,R) and (I,S) satisfy common property (E) and (iv) $\int_{0}^{M(Ax_{i}By_{i}Iz_{i}g(t))} \Psi(t) dt \geq \int_{0}^{m(Ax_{i}By_{i}Iz_{i}t)} \Psi(t) dt$, where

m(Ax, By, Iz, t) = min{M(Jx, Ry, Sz, t), M(Jx, Ax, By, t), M(Ry, By, Iz, t), M(Sz, Iz, Ax, t), M(Jx, Ry, Iz, t), M(Ax, Ry, Sz, t), M(Jx, By, Sz, t), M(Jx, By, Iz, t), M(Ax, Ry, Iz, t), M(Ax, By, Sz, t)} $\forall x, y, z \in X$, t >0, where g : (0, ∞) \rightarrow (0, ∞) is such that g(t) < t for every t. Then A,B, I,J, R and S have a unique common fixed point.

Proof: Since the pairs (A, J) and (B,R) satisfy common property (E) therefore there exists sequences $\{x_n\}$ and $\{y_n\}$ s.t. Lim M(Axn, v, v, t) = lim M(Jxn, v, v, t) = 1 and

 $\lim M(Byn, v, v, t) = \lim M(Ryn, v, v, t) = 1 \text{ as } n \rightarrow \infty.$

Therefore $\lim Byn = \lim Ryn = \lim Axn = \lim Jxn = v$ as $n \to \infty$ for $v \in X$.

Since $B(X) \subseteq S(X)$, there exists sequences $\{z_n\}$ in X s.t. lim Byn = lim Szn $\forall n$.

Therefore $\lim Szn = v \text{ as } n \to \infty$.

Let $\lim Izn = p$ as $n \to \infty$, therefore from (iv)

$$\int_{0}^{M(Axn,Byn,Izn,g(t))} \Psi(t) dt \geq \int_{0}^{m(Axn,Byn,Izn,t)} \Psi(t) dt, \text{ where }$$

 $m(Axn, Byn, Izn, t) = \min\{M(Jx_n, Ry_n, Sz_n, t), M(Jx_n, Ax_n, By_n, t), M(Ry_n, By_n, Iz_n, t), M(Sz_n, Iz_n, Ax_n, t), M(Jx_n, Ry_n, Iz_n, t), M(Ax_n, Ry_n, Sz_n, t), M(Jx_n, By_n, Sz_n, t), M(Jx_n, By_n, Iz_n, t), M(Ax_n, Ry_n, Iz_n, t), M(Ax_n, By_n, Sz_n, t)\}, which implies$

$$\int_{0}^{M(v,v,p,g(t))} \Psi(t) dt \geq \int_{0}^{M(v,v,p,t)} \Psi(t) dt, \text{ as as } n \to \infty \text{ and}$$

$$\begin{split} m(v, v, p, t) &= \min\{M(v, v, v, t), M(v, v, v, t), M(v, v, p, t), M(v, p, v, t), M(v, v, p, t), M(v, v, v, v, v, v, t), M(v$$

Therefore $M(v, v, p, g(t)) \ge M(v, v, p, t)$ which implies p = v.

Thus lim Izn $= v as n \rightarrow \infty$.

Now let us suppose J(X) is a closed subspace of X. Then v = Ju for some $u \in X$.

$$\int_{0}^{M(Au,Byn,Izn,g(t))} \Psi(t) dt \geq \int_{0}^{M(Au,v,v,t)} \Psi(t) dt, \text{ as}$$

$$\begin{split} M(Au, v, v, t) &= \min\{M(Ju, Ry_n, Sz_n, t), M(Ju, Au, By_n, t), M(Ry_n, By_n, Iz_n, t), M(Sz_n, Iz_n, Au, t), M(Ju, Ry_n, Iz_n, t), \\ M(Au, Ry_n, Sz_n, t), M(Ju, By_n, Sz_n, t), M(Ju, By_n, Iz_n, t), M(Au, Ry_n, Iz_n, t), M(Au, By_n, Sz_n, t)\}, \\ \end{split}$$

$$\int_0^{M(Au,v,p,g(t))} \Psi(t) dt \ge \int_0^{M(Au,v,v,t)} \Psi(t) dt, \text{ as } n \to \infty \text{ and}$$

 $m(Au, v, v, t) = min\{M(v, v, v, t), M(v, Au, v, t), M(v, v, v, t), M(v, v, Au, t), M(v, v, v, t), M(Au, v, v, t), M(v, v, v, t), M(v, v, v, t), M(Au, v, v, t), M(Au, v, v, t)\} = M(Au, v, v, v, t)$

Therefore $M(Au, v, v, g(t)) \ge M(Au, v, v, t)$ which implies Au = v.

Since the pair (A,J) is weakly compatible therefore AJu = JAu and Au = Ju = v. Thus AAu = AJu = JAu = JJu which implies Av = Jv.

Since $A(X) \subseteq R(X)$ there exist $w \in X$ s.t. v = Au = Rw.

Thus from (iv) we have

$$\int_{0}^{M(Au,Bw,Jzn,g(t))} \Psi(t) dt \geq \int_{0}^{M(v,Bw,v,t)} \Psi(t) dt, \text{ as}$$

$$\begin{split} M(v, Bw, v, t) &= \min\{M(Ju, Rw, Sz_n, t), M(Ju, Au, Bw, t), M(Rw, Bw, Iz_n, t), M(Sz_n, Iz_n, Au, t), M(Ju, Rw, Iz_n, t), M(Au, Rw, Sz_n, t), M(Ju, Bw, Sz_n, t), M(Ju, Bw, Iz_n, t), M(Au, Rw, Iz_n, t), M(Au, Bw, Sz_n, t)\}, which implies \end{split}$$

$\int_0^{M(\upsilon,Bw,\upsilon,g(t))} \Psi(t) dt \geq \int_0^{M(\upsilon,Bw,\upsilon,t)} \Psi(t) dt, \text{ as}$

$$\begin{split} m(v, Bw, v, t) &= \min\{M(v, v, v, t), M(v, v, Bw, t), M(v, Bw, v, t), M(v, v, v, t), M(v, v, v, t), M(v, v, v, t), M(v, Bw, v, t), M(v, Bw, v, t), M(v, Bw, v, t), M(v, Bw, v, t)\} \\ &= M(v, Bw, v, t), M(v, v, v, t), M(v, Bw, v, t)\} \\ = M(v, Bw, v, t), M(v, v, v, t), M(v, Bw, v, t)\} \\ = M(v, Bw, v, t), M(v, v, v, t), M(v, Bw, v, t)\} \\ = M(v, Bw, v, t), M(v, v, v, t), M(v, Bw, v, t)\} \\ = M(v, Bw, v, t), M(v, v, v, t), M(v, bw, v, t)\} \\ = M(v, Bw, v, t), M(v, v, v, t), M(v, bw, v, t)\} \\ = M(v, Bw, v, t), M(v, v, v, t), M(v, bw, v, t)\} \\ = M(v, Bw, v, t), M(v, v, v, t), M(v, bw, v, t)\} \\ = M(v, Bw, v, t), M(v, v, v, t), M(v, bw, v, t)\} \\ = M(v, Bw, v, t), M(v, v, v, t), M(v, bw, v, t) \\ = M(v, Bw, v, t), M(v, bw, v, t), M(v, bw, v, t) \\ = M(v, Bw, v, t), M(v, bw, v, t) \\ = M(v, Bw, v, t), M(v, bw, v, t) \\ = M(v, Bw, v, t), M(v, bw, v, t) \\ = M(v, Bw, v, t), M(v, bw, v, t) \\ = M(v, Bw, v, t), M(v, bw, v, t) \\ = M(v, bw, v, t), M(v, bw, v, t) \\ = M(v, bw, v, t), M(v, bw, v, t) \\ = M(v, bw, v, t), M(v, bw, v, t) \\ = M(v, bw, v, t), M(v, bw, v, t) \\ = M(v, bw, v, t), M(v, bw, v, t) \\ = M(v, bw, v, t), M(v, bw, v, t) \\ = M(v, bw, v, t), M(v, bw, v, t) \\ = M(v, bw, v, t), M(v, bw, v, t) \\ = M(v, bw, v, t), M(v, bw, v, t) \\ = M(v, bw, v, t), M(v, bw, v, t) \\ = M(v, bw, v, t), M(v, bw, v, t) \\ = M(v, bw, v, t), M(v, bw, v, t) \\ = M(v, bw, v, t), M(v, bw, v, t) \\ = M(v, bw, v, t), M(v, bw, v, t) \\ = M(v, bw, v, t), M(v, bw, v, t), M(v, bw, v, t) \\ = M(v, bw, v, t), M(v, bw, v, t), M(v, bw, v, t) \\ = M(v, bw, v, t), M(v,$$

Therefore $M(v, Bw, v, g(t)) \ge M(v, Bw, v, t)$ which implies Bw = v.

Since the pair (B, R) is weakly compatible therefore BRw = RBw and Bw = Rw = v. Thus BBw = BRw = RBw = RRw which implies Bv = Rv.

Since $B(X) \subseteq S(X)$ there exist $r \in X$ s.t. v = Bw = Sr.

Thus from (iv) we have

$$\int_{0}^{M(Au,Bw,Ir,g(t))} \Psi(t) dt \geq \int_{0}^{M(v,v,Ir,t)} \Psi(t) dt, \text{ as}$$

$$\begin{split} M(v, v, Ir, t) &= \min\{M(Ju, Rw, Sr, t), M(Ju, Au, Bw, t), M(Rw, Bw, Ir, t), M(Sr, Ir, Au, t), M(Ju, Rw, Ir, t), M(Au, Rw, Sr, t), M(Ju, Bw, Sr, t), M(Ju, Bw, Ir, t), M(Au, Rw, Ir, t), M(Au, Bw, Sr, t)\}, which implies \end{split}$$

$$\int_{0}^{M(v,v,Ir,g(t))} \Psi(t) dt \geq \int_{0}^{M(v,v,Ir,t)} \Psi(t) dt, \text{ as}$$

$$\begin{split} m(v, v, Ir, t) &= \min\{M(v, v, v, t), M(v, v, v, t), M(v, v, Ir, t), M(v, Ir, v, t), M(v, v, Ir, t), M(v, v, v, v, v, v, v), M(v, v, v, v), M(v, v, v, v, v), M(v, v, v, v), M(v, v, v, v),$$

Now

$$\int_{0}^{M(Av,v,v,g(t))} \Psi(t) dt \geq \int_{0}^{M(Av,Bw,Ir,g(t))} \Psi(t) dt \geq \int_{0}^{M(Av,v,v,t)} \Psi(t) dt \text{ where }$$

 $M(Av, v, v, t) = min\{M(Jv, Rw, Sr, t), M(Jv, Av, Bw, t), M(Rw, Bw, Ir, t), M(Sr, Ir, Av, t), M(Jv, Rw, Ir, t), M(Av, Rw, Sr, t), M(Jv, Bw, Sr, t), M(Jv, Bw, Ir, t), M(Av, Rw, Ir, t), M(Av, Bw, Sr, t)\}$, which implies

$$\int_0^{M(Av,v,v,g(t))} \Psi(t) dt \geq \int_0^{M(Av,v,v,t)} \Psi(t) dt, \text{ as}$$

 $\begin{array}{l} m(Av, v, v, t) = \min\{M(Av, v, v, t), M(Av, Av, v, t), M(v, v, v, t), M(v, v, Av, t), M(v, v, v, t), M(Av, v, v, t)\} = M(Av, v, v, v, t) \\ Therefore M(Av, v, v, g(t)) \ge M(Av, v, v, t) \text{ which implies } Av = v. \text{ Thus } Av = Jv = v. \end{array}$

Similarly Bv = v and Iv = v.

Hence Av = Bv = Iv = Jv = Rv = Sv = v which implies v is common fixed point of A, B, I, J, R and S.

Uniqueness : Let us suppose 'q' be a fixed point of A, B, I, J, R and S other than v. Then Aq = Bq = Iq = Jq = Rq = Sq = q. For uniqueness

$$\int_0^{M(q,v,v,g(t))} \Psi(t) dt \geq \int_0^{M(Aq,Bv,Jv,g(t))} \Psi(t) dt \geq \int_0^{M(q,v,v,t)} \Psi(t) dt$$

where

$$\begin{split} M(q, v, v, t) &= \min\{M(Jq, Rv, Sv, t), M(Jq, Aq, Bv, t), M(Rv, Bv, Iv, t), M(Sv, Iv, As, t), M(Jq, Rv, Iv, t), M(Aq, Rv, Sv, t), M(Jq, Bv, Sv, t), M(Jq, Bv, Iv, t), M(Aq, Rv, Iv, t), M(Aq, Bv, Sv, t)\}, \\ \end{split}$$

$$\int_{0}^{M(q,v,v,g(t))} \Psi(t) dt \geq \int_{0}^{M(q,v,v,t)} \Psi(t) dt, \text{ as }$$

$$\begin{split} m(q,\,v,\,v,\,t) &= \min\{M(q,\,v,\,v,\,t),\,M(q,\,q,\,v,\,t),\,M(v,\,v,\,v,\,t),\,M(v,\,v,\,q,\,t),\,M(q,\,v,\,v,\,v,\,t),\,M(q,\,v,\,v,\,v,\,t),\,M(q,\,v,\,v,\,v,\,t),\,M(q,\,v,\,v,\,v,\,t),\,M(q,\,v,\,v,\,v,\,t),\,M(q,\,v,\,v,\,v,\,t),\,M(q,\,v,\,v,\,v,\,t),\,M(q,\,v,\,v,\,v,\,t),\,M(q,\,v,\,v,\,v,\,t),\,M(q,\,v,\,v,\,v,\,t),\,M(q,\,v,\,v,\,v,\,t),\,M(q,\,v,\,v,\,v,\,t),\,M(q,\,v,\,v,\,v,\,t),\,M(q,\,v,\,v,\,v,\,t),\,M(q,\,v,\,v,\,v,\,t),\,M(q,\,v,\,v,\,v,\,t),\,M(q,\,v,\,v,\,v,\,t),\,M(q,\,v,\,v,\,v,\,t),\,M(q,\,v,\,v,\,v,\,t),\,M(q,\,v,\,v,\,v,\,v),\,M(q,\,v,\,v,\,v,\,v),\,M(q,\,v,\,v,\,v),\,M(q,\,v,\,v,\,v),\,M(q,\,v,\,v,\,v),\,M(q,\,v,\,v,\,v),\,M(q,\,v,\,v,\,v),\,M(q,\,v,\,v,\,v),\,M(q,\,v,\,v,\,v),\,M(q,\,v,\,v,\,v),\,M(q,\,v,\,v,\,v),\,M(q,\,v,\,v,\,v),\,M(q,\,v,\,v,\,v),\,M(q,\,v,\,v),\,M(q,\,v,\,v),\,M(q,\,v,\,v),\,M(q,\,v,\,v),\,M(q,\,v,\,v),\,M(q,\,v,\,v),\,M(q,\,v,\,v),\,M(q,\,v,\,v),\,M(q,\,v,\,v),\,M(q,\,v$$

Therefore $M(q, v, v, g(t)) \ge M(q, v, v, t)$ which implies q = v.

Hence v is unique common fixed point of A, B, I, J,R and S.

Example 1.3. Let $X = [1, \infty]$ and M(x, y, z, t) = 2 t/[2t + |x - y| + |y - z| + |x - z|] for all t > 0 and $x, y, z \in X$. Let A, B, I, J, R and S are self mappings on X defined by

Ax = Bx = Ix = 2 and

 $J(x) = 0 \text{ if } x \in (-\infty, 1)$ 2 if $x \in [1, \infty)$,

 $R(x) = 1/4 \text{ if } x \in (-\infty, 1)$ 2 if $x \in [1, \infty)$,

$$\begin{split} S(x) &= 1/5 \text{if } x \in (-\infty, 1) \\ 2 \text{ if } x \in [1, \infty), \end{split}$$

We define g which satisfies the conditions in above theorem. Also, consider the sequences

 $\begin{array}{l} \{x_n\} = \{1+2/n\}, \{y_n\} = \{1+3/n\} \text{ and } \{z_n\} = \{1+4/n\}. \end{array} \\ \begin{array}{l} \text{Then}(A,J) \text{ and } (B,R) \text{ satisfy the common property } (E). \\ \text{Clearly} \\ A(X) \subseteq R \ (X), \ B(X) \ \subseteq S(X), \ I(X) \subseteq J(X) \text{ and } J(X) \text{ or } R(X) \text{ or } S(X) \text{ is a closed subspace of } X \end{array} . \end{array}$

Now, $Ax = 2 = Jx \quad \forall x \in X$.therefore AJx = A2 = 2 and JAx = J2 = 2.Thus AJx = JAx, which implies the pair (A,J) is weakly compatible. Similarly, pairs (B,R) and (I,S) are weakly compatible.

Also, M(Axn, 2, 2, t) = M(A(1+2/n), 2, 2, t) = M (2, 2, 2, t) = 2 t/[2t + |2-2| + |2-2| + |2-2| = 2t/2t=1, as $n \rightarrow \infty$. Similarly M(Jxn, 2, 2, t) = 1, M(Byn, 2, 2, t) = 1 and M(Ryn, 2, 2, t) = 1 as $n \rightarrow \infty$. Thus pairs (A,J) and (B,R) satisfy common property E.A. Also contractive condition (iv) is also satisfied by mappings A,B,I,J,R and S.Hence,2 is fixed point of A,B,I,J,R and S.

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