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Common fixed point theorem for four random operators in Hilbert space

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ABSTRACT

The object of this paper is to obtain a common fixed point theorem for four continuous random operators satisfying certain contractive conditions in Separable Hilbert space, which generalizes and extend the results of Beg and Shahzad [5, 8].

Keywords: Separable hilbert space, random operators, random fixed point
AMS Subject Classification: 47H10, 54H25

INTRODUCTION

Prague school of probilists started the study of random fixed point theory in the 1950s [1, 9].Random fixed point theorems are stochastic generalization of classical fixed point theorems. The survey article by Bharucha-Reid [4] attracted the attention of several mathematicians and gave wings to this theory. The study of this theory has attracted much attention in recent years. Now this theory has become the full fledged research area and various ideas associated with random fixed point theory are used to obtain the solution of non linear random system[2,6].Beg[3,7] and Beg and Shahzad [5,8] studied the structure of common random fixed points and random coincidence points of a pair of compatible random operator in Polish Space. The aim of this paper is to prove a common fixed point theorem for four random operators using certain contractive conditions in separable Hilbert Space.

MATERIALS AND METHODS

Let (Ω, Σ) be a measurable space (Σ-Sigma algebra) and C a nonempty subset of a Separable Hilbert space H.

Definition2.1: A mapping $f: \Omega \to C$ is measurable if $f-1(B \cap C) \in \Sigma$ for each Borel subset B of H.

Definition 2.2: A mapping $T: \Omega\times C \to C$ is a random map if and only if for each $x \in C$, the mapping $T(\cdot, x): \Omega \to C$ is measurable.

Definition 2.3: A random operator is said to be continuous if for each $\omega \in \Omega$, the mapping $T(\omega, \cdot): C \to X$ is continuous.

Definition 2.4: A measurable mapping $f: \Omega \to X$ is a random fixed point of a random map $T: \Omega \times C \to C$ if $T(\omega, f(\omega)) = f(\omega)$ for each $\omega \in \Omega$.

Condition –A: Let C be a non –empty closed subset of a separable Hilbert space H. Let A, B,S ,T be four continuous random operator defined on C such that for $t \in \Omega$ $A(t,.),B(t,.),S(t,.)$ and $T(t,.): C\rightarrow C$ satisfy the condition

$\|Ax-By\|^2 < k\{\|Sx-Ty\|^2+\|Ax-Tx\|^2+\|Ax-By\|^2+\|Sx-By\|^2\}$ where $1/6 \le k < 1/3$.

RESULTS AND DISUSSION

Theorem: Let C be a non –empty closed subset of a separable Hilbert space H. Let A, B,S ,T be four continuous random operator defined on C such that for $t \in \Omega$ $A(t,.),B(t,.),S(t,.)$ and $T(t,.):C\rightarrow C$ satisfy the condition

(1) $\|Ax-By\|^2 < k\{\|Sx-Ty\|^2+\|Ax-Tx\|^2+\|Ax-By\|^2+\|Sx-By\|^2\}$ where $1/6\le k<1/3$.

(2) (A, S) and (A,T) are weakly compatible.
then A,B,S,T have a common random fixed point in C.

Proof: We construct a sequence of functions $\{g_n\}$ as $g_o: \Omega\rightarrow C$ is arbitrary measurable function for $t \in \Omega$ and n=0,1,2--------.

Define sequences $\{g'_n\},\{g''_n\}$as such that $g''_{2n}(t) = A(t, g'_{2n}) = T(t, g'_{2n+1})$,
$g''_{2n+1}(t)= B(t, g'_{2n+1})=S(t, g'_{2n+2})$

Using (1) we prove here $\{g''_n\}$is a Cauchy sequence.

If $g''_{2n}(t) = g'_{2n}(t) = g'_{2n+1}(t)$ and $g''_{2n+1}(t)= g'_{2n+1}(t)= g'_{2n+2}(t) = g''_{2n}(t)$.

Then $g''_{2n\,(t)} =A(t, g''_{2n}(t)) =T(t, g''_{2n}(t))= B(t, g''_{2n}(t)) = S(t, g''_{2n}(t))$.

Then $g''_{2n}(t)$ is a random fixed point of A,B,S&T. Therefore we suppose that no two consecutive terms of $\{g''_n\}$ are equal to two consecutive terms of $\{g'_n\}$ at same order.

Now consider $t \in \Omega$
$\| g''_{2n+1}(t)- g''_{2n+2}(t)\|^2=\| B(t, g'_{2n+1}(t))- A(t, g'_{2n+2}(t))\|^2=\| A (t, g'_{2n+2}(t)) -B(t,g'_{2n+1}(t))\|^2$
$\le k \{\| S (t, g'_{2n+2}(t)) -T(t, g'_{2n+1}(t))\|^2+\| A(t, g'_{2n+2}(t))- T(t, g'_{2n+2}(t))\|^2 +$
$\| A (t, g'_{2n+2}(t)) -B(t,g'_{2n+1}(t))\|^2+\| S (t, g'_{2n+2}(t))- B(t, g'_{2n+1}(t))\|^2\}$.implies
$\| g''_{2n+1}(t)- g''_{2n+2}(t)\|^2\le k \{\| g''_{2n+1}(t)- g''_{2n}(t)\|^2+\| g''_{2n+2}(t)- g''_{2n+1}(t)\|^2 +$
$\| g''_{2n+1}(t)- g''_{2n+2}(t)\|^2 +\| g''_{2n+1}(t)- g''_{2n+1}(t)\|^2\}$,implies
$(1-2k) \| g''_{2n+1}(t)- g''_{2n+2}(t)\|^2\le k \{\| g''_{2n+1}(t)- g''_{2n}(t)\|^2\}$,implies
$\| g''_{2n+1}(t)- g''_{2n+2}(t)\|^2\le (k/1-2k) \| g''_{2n+1}(t)- g''_{2n}(t)\|^2$,

This implies $\| g''_{2n+1}(t) - g''_{2n+2}(t)\|\le \lambda \| g''_{2n}(t)- g''_{2n+1}(t)\|$ where $\lambda=\sqrt{(2k/1-k)} < 1$

In general $\| g''_n(t)- g''_{n+1}(t)\|\le \lambda^n\| g''_o(t)- g''_1(t)\|$ for all $t \in \Omega$.

As $n\rightarrow\infty$, $\| g''_n(t)- g''_{n+1}(t)\|\rightarrow 0$.

Therefore for all $t \in \Omega$, $\{g''_n(t)\}$ is a Cauchy sequence and hence convergent in Hilbert space H.

For $t \in \Omega$, $\{g''_n(t)\}\rightarrow g''(t)$ as $n\rightarrow\infty$.Since C is closed , $g''(t) : C\rightarrow C$.

Now, we shall prove that $g''(t)$ is a fixed point of A,B,S,T. For $t \in \Omega$.
$\| g''(t)- A(t, g''(t))\|^2=\| g''(t)- g''_{2n+1}(t)+ g''_{2n+1}(t)- A(t, g''(t))\|^2$
$\le 2\| g''(t)- g''_{2n+1}(t)\|^2+2 \|g''_{2n+1}(t)- A(t, g''(t))\|^2$
$= 2\| g''(t)- g''_{2n}(t) \|^2+2 \|B(t,g'_{2n+1}(t)) - A(t, g''(t))\|^2$
$\le 2\| g''(t)- g''_{2n}(t) \|^2+2[k\{\| S(t, g'' (t))- T(t, g'_{2n+1}(t))\|^2+$
$\| A(t, g'' (t))- T(t, g''(t))\|^2 +\| A(t, g''(t))- B(t,g'_{2n+1}(t)) \|^2 +$

$\| S(t, g''(t)) - B(t, g'_{2n+1}(t)) \|^2\}]$
$\leq 2\| g''(t) - g''_{2n}(t) \|^2 + 2[k\{\| S(t, g''(t)) - g''_{2n}(t)) \|^2 + \| A(t, g''(t)) - T(t, g''(t)) \|^2 + \| A(t, g''(t)) - g''(t)) \|^2 + \| S(t, g''(t)) - g''(t) \|^2\}]$.

As $\{g''_{2n+1}(t)\}$,$\{g''_{2n}(t)\}$ are subsequences of $\{g''_n(t)\}$ therefore as $n \to \infty$, $\{g''_{2n+1}(t)\} \to g''(t)$, $\{g''_{2n}(t)\} \to g''(t)$,which implies $\| g''(t) - A(t, g''(t)) \|^2 \leq 2\| g''(t) - g''(t) \|^2 + 2[k\{\| S(t, g''(t)) - g''(t)) \|^2 + \| A(t, g''(t)) - T(t, g''(t)) \|^2 + \| A(t, g''(t)) - g''(t)) \|^2 +$
$\| S(t, g''(t)) - g''(t)) \|^2\}]$.
i.e. $\| g''(t) - A(t, g''(t)) \|^2 (1-2k) \leq 2[k\{\| S(t, g''(t)) - g''(t)) \|^2 + \| A(t, g''(t)) - T(t, g''(t)) \|^2 + \| S(t, g''(t)) - g''(t)) \|^2\}]$

As (A,S) and (A,T) are weakly compatible so
$\| g''(t) - A(t, g''(t)) \|^2 (1-2k) \leq 2[2k\| A(t, g''(t)) - g''(t)) \|^2] \leq 4k\| A(t, g''(t)) - g''(t)) \|^2$.
i.e. $0 \leq (6k-1) \| A(t, g''(t)) - g''(t)) \|$
i.e. $(6k-1) \| A(t, g''(t)) - g''(t)) \| \geq 0$.
implies $A(t, g''(t)) = g''(t)$ for all $t \in \Omega$ (As $k \geq 1/6$)
Similarly for all $t \in \Omega$,$B(t, g''(t)) = g''(t)$;$S(t, g''(t)) = g''(t)$;$T(t, g''(t)) = g''(t)$.

Again if $V: \Omega \times C \to C$ is a continuous random operator on non empty subset C of a separable Hilbert space H then for any measurable function $f: \Omega \to C$ the function $h(t)=V(t, f(t))$ is also measurable.

It follows that $\{g''_n(t)\}$ is a sequence of measurable functions .It follows that $g''(t)$ is a measurable function. Hence $g''(t): \Omega \to C$ is a common random fixed point of A,B,S,T.

Uniqueness
Let $h''(t)$ be another fixed point common to A,B,S,T.
i .e for $t \in \Omega$, $A(t, h''(t)) = h''(t)$; $B(t, h''(t)) = h''(t)$; $S(t, h''(t)) = h''(t)$;$T(t, h''(t)) = h''(t)$.
Then for $t \in \Omega$,$\| g''(t) - h''(t)) \|^2 = \| A(t, g''(t)) - B(t, h''(t)) \|^2$
$< k\{\| S(t, g''(t)) - T(t, h''(t)) \|^2 + \| A(t, g''(t)) - T(t, g'{}'(t)) \|^2 + \| A(t, g''(t)) - B(t, h'{}'(t)) \|^2 +$
$\| S(t, g''(t)) - B(t, h'{}'(t)) \|^2\}$
$< 2k\{\| g''(t) - h''(t) \|^2 + \| g''(t) - g''(t) \|^2 + \| g''(t) - h''(t) \|^2 + \| g''(t) - h''(t) \|^2\}$.
i.e $\| g''(t) - h''(t)) \|^2 \leq 3.2k\| g''(t) - h''(t) \|^2 \leq 6k\| g''(t) - h''(t) \|^2$.
i.e $(6k-1)\| g''(t) - h''(t) \| \geq 0$.
i.e $\| g''(t) - h''(t) \| \geq 0$.Thus $g''(t) = h''(t)$ for all $t \in \Omega$. Hence proved.

Application
Example: Let $Ax = x$, $Bx = 1+x$,$Sx = 2+x$ and $Tx = 2+x$ and $\{x_n\}$and $\{y_n\}$ are defined as
$x_{n = 1+1/n}$, $y_{n = 1+1/n}$ .Clearly (A,S) and (A,T) are weakly compatible as
$ASx = A(2+x) = (2+x)$ and $SAx = Sx = (2+x)$; $ATx = A(2+x) = (2+x)$ and
$Tax = Tx = (2+x)$.

Also, as $n \to \infty$, $\| A x_n - B y_n \|^2 < k\{\| S x_n - T y_n \|^2 + \| A x_n - T x_n \|^2 + \| A x_n - B y_n \|^2 + \| S x_n - B y_n \|^2\}$
implies $\| 1 - 2 \|^2 < k\{\| 3 - 3 \|^2 + \| 1 - 3 \|^2 + \| 1 - 2 \|^2 + \| 3 - 2 \|^2\}$
implies $1 \leq 6k$ implies $k \geq 1/6$.

Therefore, all the conditions of above theorem are satisfied.

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