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Common fixed point theorem for compatible mappings of type (A-1) in complete fuzzy metric space

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ABSTRACT

The purpose of this paper is to obtain common fixed point theorem for compatible maps of type (A-1) on complete fuzzy metric space .Our result improves the result of Khan M.S. [7].

Keywords: Common fixed point, Compatible Mappings , Compatible Mappings of type (A-1), Fuzzy Metric Space, Complete Fuzzy Metric Space **2010 AMS Subject Classification :** 47H10, 54H25.

INTRODUCTION

The concept of fuzzy sets was first given by Zadeh [14] in 1965. Then Kramosil and Michalek[8] introduced the concept of fuzzy metric space and George and Veeramani[4] modified the notion of fuzzy metric with help of continuous t-norms.

The improving commutativity in fixed point theorems by using weakly commuting maps in metric space was initiated by Sessa [10] .Later on , this method was enlarged to compatible maps by Jungck[5] . Cho[2,3] introduced the concept of compatible maps of type (α) and compatible maps of type (β) in fuzzy metric space. Singhet.al.[12] proved fixed pointtheorems in a fuzzy metric space. Recently in 2012 Jain et.al.[6] proved various fixed point theorems using the concept of semi compatible mapping

The concept of type A-compatible and S-compatible was given by Pathak and Khan [7]. Pathak et.al. [9] renamed A-compatible and S- compatible as compatible mappings of type (A-1) and compatible mappings of type (A-2) respectively.

B.Singh et.al.[12] proved fixed point theorems in fuzzy metric space and Menger space using the concept of semi-compatibility ,weak compatibility and compatibility of type (β) respectively. The idea of fuzzy 2- metric space and fuzzy 3- metric space were used by

Sushil Sharma [11] and obtained some fruitful results.

Preliminaries
Definition 1.1 [13] Let X be any set . A Fuzzy set A in X is a function with domain X and Values in [0,1].

Definition 1.2[4] A Binary operation * : [0,1] ×[0,1] →[0,1] is called a continuous t-norms if an topological monoid with unit 1 such that a*b≤ c*d whenever a≤c and b≤d , for all a,b,c,d in [0,1].

Definition 1.3[4] The triplet (X,M, *) is said to be a Fuzzy metric space if , X is an arbitrary set , * is a continuous t- norm and M is a fuzzy set on $X^2\times(0,\infty)$ satisfying the following conditions; for all x,y,z in X and s,t > 0,
(i) M(x,y,0) = 0 , M(x,y,t)>0,
(ii) M(x,y,t) = 1 ,for all t > 0 if and only if x=y,
(iii) M(x,y,t) = M(y,x,t),
(iv) M(x,y,t) * M(y,z,s) ≤ M(x,z, t+s),
(v) M(x,y,t) : [0,∞) →[0,1] is left continuous.

Definition 1.4 [4] A sequence $\{x_n\}$ in a fuzzy metric space (X,M,*) is called a Cauchy Sequence if , $\lim_{n\to\infty} M(X_{n+p}, X_n, t) = 1$ for every t.>0 and for each p>0.
A fuzzy metric space(X, M,*) is Complete if ,every Cauchy sequence in X converges in X.

Definition 1.5 [4] A sequence $\{X_n\}$ in a fuzzy metric space (X,M,*) is said to be Convergent to x in X if , $\lim_{n\to\infty} M(X_n, X, t) = 1$, for each t>0.

Definition 1.6 [4] Two self mappings P and Q of a fuzzy metric space (X,M,*) are said to be Compatible , if $\lim_{n\to\infty} M(PQx_n, QPx_n, t) = 1$ whenever $\{x_n\}$ is a sequence such that $\lim_{n\to\infty} Px_n = \lim_{n\to\infty} Qx_n = z$, for some z in X .

Definition 1.7 [1] Self mappings P and Q of a fuzzy metric space (X, M, *) are said to be Compatible of type (A) if $\lim_{n\to\infty} M(PQx_n, QQx, t) = \lim_{n\to\infty} (QPx_n, PPx_n, t) = 1$ for all t>0, whenever $\{x_n\}$ is a sequence such that $\lim_{n\to\infty} Px_n = \lim_{n\to\infty} Qx_n = z$, for some z in X.

Definition 1.8 [5] Self mappings P and Q of a fuzzy metric space are said to be compatible of type (A-1) , if $\lim_{n\to\infty} (QPx_n, PPx_n, t) = 1$ for all t>0, whenever $\{x_n\}$ is a sequence such that $\lim_{n\to\infty} Px_n = \lim_{n\to\infty} Qx_n = z$, for some z in X.

Lemma 1.9[12] Let $\{y_n\}$ is a sequence in an FM- space . If there exists a positive number k<1 such that $M(y_{n+2}, y_{n+1}, kt) \geq M(y_{n+1}, y_n, t)$, t>0 , n ϵ N, then $\{y_n\}$ is a Cauchy sequence in X .

Lemma 1.10 [2] If for two points x, y in X and a positive number k < 1 M(x,y,kt) ≥ M(x,y,t) , then x = y. Next we give some properties of compatible mappings of type (A-1) which will be used in our main theorem.

Proposition 1.11[7] Let S and T be self maps of an FM- space X . If the pair (S,T) are Compatible of type (A-1) and Sz = Tz for some z in X then STz = TTz.

Proposition 1.12[7] Let S and T be self maps of an FM –space X with t*t >t for all t in [0,1] . If the pair (S,T) are compatible of type (A-1) and $Sx_n, Tx_n \to z$ for some z in Xand a sequence $\{x_n\}$ in X then $TTx_n \to Sz$ if S is continuous at z.

Proposition 1.13 [7] Let S and T be self maps of an FM- space X . If the pair (S,T) are Compatible of type (A-1) and Sz = Tz for some z in X then TSz = SSz .

Main Result
We prove the following theorem.

Theorem 2.1 : Let A ,B, S and T be self maps on a complete fuzzy metric space (X, M, *) where * is continuous t-norm defined by a*b = min {a,b} satisfying the following conditions
(i). A(X) ⊆ T(X) and B(X) ⊆ S(X)
(ii). S and T are continuous.
(iii). For each x,y ϵX and t > 0 ,

$$M(Ax, By, t) \geq \phi[\min\{M(Sx, Ty, t) . M(By, Sx, t)\}, \tfrac{1}{2}\{M(Ax, Ty, t) + M(By, Ay, t)\}]$$

Where ϕ : [0,1]→[0,1] is a continuous function such that $\phi(1) = 1$,$\phi(0) =0$

and $\phi(a) > a$, for each $0<a<1$.

If (A,S) and (B,T) are compatible mappings of type (A-1) , then A,B ,S and T have a unique common fixed point in X.

Proof : Let $x_0 \epsilon\ X$ *be any* arbitary point. Then there exist x_1 and $x_2 \in X$ such that $Ax_0 = Tx_1$ and $Bx_1 = Sx_2$

Thus , we can construct a sequence a sequence $\{y_n\}$ and $\{x_n\}$ in X such that

$y_{2n+1}= Ax_{2n} = Tx_{2n+1}$,

$y_{2n+2} = Bx_{2n+1} = Sx_{2n+2}$, for n = 0,1,2 ,…

Then, by (iii) , put $x = x_{2n}$ and $y = x_{2n+1}$ we get

$M(Ax_{2n} , Bx_{2n+!}, t) \geq \emptyset$ [min { M (Sx_{2n}, Tx_{2n+1}, t) . M (Bx_{2n}, Sx_{2n+1},t) },

½ { $M(Ax_{2n}, Tx_{2n+1},t)$ + $M(Bx_{2n+1} , Ax_{2n+1}, t)$ }]

$M(y_{2n+1},y_{2n+2} , t) \geq \emptyset$ [min {M ($y_{2n}, y_{2n+1},t)$. $M(y_{2n+2} , y_{2n},t)$ },

½ { $M(y_{2n+1}, y_{2n+1},t) + M(y_{2n+2} ,y_{2n+2},t)$}]

$M(y_{2n+1},y_{2n+2} , t) \geq \emptyset$ [min { M ($y_{2n}, y_{2n+1},t)$. $M(y_{2n+2} , y_{2n},t)$ }, 1 }]

Hence , by the definition of $\emptyset$, we get

$M(y_{2n+1},y_{2n+2} , t) \geq M (y_{2n}, y_{2n+1},t)$

Similarly , we have

$M(y_{2n+2} , y_{2n+3} ,t) \geq M(y_{2n+1},y_{2n+2},t)$,

In general

$M(y_{n+1},y_n,t) \geq M(y_n, y_{n-1},t)$

Therefore, { $M(y_{n+1,}y_{n,}t)$ } is an increasing sequence of positive real numbers ,
in [0,1] and tends to limit $L \leq 1$. If $L <1$ then

$M(y_{n+1},y_n,t) \geq M(y_n, y_{n-1},t)$. On letting $n\to\infty$ we get

$\lim_{n\to\infty} M(y_{n+1} ,y_n,t) \geq \emptyset (\lim_{n\to\infty} M (y_n, y_{n-1},t)$

$L\geq \emptyset (L) = L$ (Since $\emptyset(a) > a$) , a contradiction .

Now for any positive integer m ,

$M(y_n,y_{n+m}, ,t) \geq M(y_n,y_{n+1},t/m) * M(y_{n+1}, y_{n+2} , t/m) * \ldots *\ldots* M(y_{n+m-1},y_{n+m}, t/m)$

Letting $n\to\infty$,we have

$\lim_{n\to\infty} M(y_n, y_{n+m}, t) = 1*1*1*\ldots*1 =1$.

Thus ,

$\lim_{n\to\infty} M(y_n, y_{n+m}, t) = 1$

This shows that $\{y_n\}$ is a Cauchy sequence in X , which is complete .

Therefore , $\{y_n\}$ converges to a point z_1 in X . Hence the subsequence $\{Ax_{2n}\}$,$\{Sx_{2n}\}$,

$\{Tx_{2n+!}\}$ and $\{Bx_{2n+1}\}$ also converges to z_1.

Since , (A,S) and (B,T) are Compatible mappings of type (A-1) , and

From proposition 1.12 , we have

$AAx_{2n} \rightarrow Sz_1$. (1)

$BBx_{2n+1} \rightarrow Tz_1$ (2)

Now, by (iii),

$M(AAx_{2n}, BBx_{2n+1}, t) \geq \emptyset$ [min $\{M(SAx_{2n}, TBx_{2n+!}, t) . M(BBx_{2n},SAx_{2n},t) \}$, ½ $\{ M(AAx_{2n}, TBx_{2n},t) + M(BBx_{2n+1}, ABx_{2n+!},t)\}$]

Now, letting $n\to\infty$ and using (1) ,(2) and proposition 1.11, we get

$M (Sz_1, Tz_1,t) \geq \emptyset$ [min$\{M(Sz_1 ,Tz_1,t). M(Tz_1, Sz_1,t) \}$, ½ $\{M(Sz_1,Tz_1,t) + M(Tz_1,Sz_1,t) \}$]

$M (Sz_1, Tz_1,t) \geq \emptyset$ [min $\{ M(Sz_1 ,Tz_1,t) , 1 \}$]

$M(Sz_1 ,Tz_1,t) \geq M(Sz_1 ,Tz_1,t)$

It follows that $Sz_1= Tz_1$ (3)

Now by (iii) putting $x = z_1$ and $y = Bx_{2n+1}$, we get

$M(Az_1, BBx_{2n+1},t) \geq \emptyset$ [min $\{ M(Sz_1, TBx_{2n+1},t) . M(BBx_{2n+1}, Sz_1,t) \}$, ½ $\{ M(Az_1,TBx_{2n+!},t) + M(BBx_{2n+1},ABX_{2n+1},t) \}$]

Again taking limit as $n\to\infty$, using (1) and (2) , we have

$M(Az_1,Tz_1,t) \geq \emptyset$ [min$\{M(Sz_1,Tz_1,t) .M(Tz_1,Sz_1,t)\}$, ½ $\{M(Az_1,Tz_1,t) + M(Tz_1,Az_1,t)\}$]

$M(Az_1, Tz_1,t) \geq M(Sz_1,Tz_1,t)$

It follows that $Az_1= Sz_1$ (4)

Now by (iii) putting $x = z_1$and $y= z_1$, we get

$M(Az_1, Bz_1,t) \geq \emptyset$[min$\{M(Sz_1,Tz_1,t). M(Bz_1,Sz_1,t) \}$, ½ $\{M(Az_1,Tz_1,t) +M(Bz_1,Az_1,t)\}$]

$M(Az_1,Bz_1,t) \geq \emptyset$ [min$\{M(Sz_1,Sz_1,t) . M(Bz_1 Az_1,t)\}$, ½ $\{M(Az_1,Tz_1,t) + M(Tz_1, Az_1,t)\}$]

$M(Az_1,Bz_1,t) \geq \emptyset$ [min $\{1.M(Bz_1,Az_1,t)\},1\}$]

$M(Az_1,Bz_1,t) \geq M(Bz_1,Az_1,t)$

It follows that $Az_1 = Bz_1$ (5)

Hence , from (3), (4) and (5) , we get

$Az_1 = Bz_1 = Sz_1 = Tz_1$ (6)

Now , we have to show that $Bz_1 = z_1$

From (iii) putting $x=x_{2n}$ and $y = z_1$,we get

$M(Ax_{2n},Bz_1,t) \geq \emptyset\, [\min\{ M(Sx_{2n},Tz_1,t).M(Bz_1,Sx_{2n},t)\},½\{M(Ax_{2n},Tz_1,t)+M(Bz_1,Az_1,t)\}]$

Taking limit $n\rightarrow\infty$, and using (6), we get

$M(z_1,Bz_1,t) \geq \emptyset\, [\min\{ M(z_1,Bz_1,t).M(Bz_1,z_1,t)\}, ½ \{M(z_1,Bz_1,t)+M(Bz_1,Bz_1,t)\}]$

$M(z_1,Bz,t) \geq M(z_1,Bz,t)$

And , hence we get $z_1 = Bz_1$

Thus we have $z_1 = Az_1 = Bz_1 = Sz_1 = Tz_1$.

Hence z_1 is a common fixed point of A,B,S and T.

Uniqueness – Let z_2 be another fixed point of A,B,S and T. Then

$M(z_1,z_2,t) = M(Az_1,Bz_2,t)$

$\geq \emptyset[\min\{ M(Sz_1,Tz_2,t) . M(Bz_2,Sz_1,t)\}, ½ \{ M(Az_1,Tz_2,t)+M(Bz_2,Az_2,t)\}]$

$\geq \emptyset\, [\min \{M(z_1,z_2,t) .M(z_2,z_1,t) \}, ½ \{M(z_1,z_2,t)+ M(z_1,z_2,t)\}]$

$\geq M(z_1,z_2,t)$

Therefore by lemma 1.10 ,we get $z_1 = z_2$.

Hence z_1 is the unique common fixed point of A,B ,S and T.

CONCLUSION

In this paper we introduce the concept of compatible mapping of type **(**A-1) in fuzzy metric space .

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