

Chemically Reacting on MHD Oscillatory Slip Flow in a Planer Channel with Varying Temperature and Concentration

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ABSTRACT

In this paper, unsteady MHD mixed convective oscillatory flow of an electrically conducting optically thin fluid flow through a planer channel filled with saturated porous medium. The effect of buoyancy, heat source, thermal radiation and chemical reaction are taken into account embedded with slip boundary condition, varying temperature and concentration. The closed-form analytical solutions are obtained for the momentum, energy and concentration equations. The influences of the various parameters entering into the problem in the velocity, temperature and concentration fields are discussed with the help of graphs. The effect of skin friction, the rate of heat and mass transfer coefficients at the walls are discussed qualitatively.

INTRODUCTION

Oscillatory flows has known to result in higher rates of heat and mass transfer, many studies have been done to understand its characteristics in different systems such as reciprocating engines, pulse combustors and chemical reactors. Cramer, K. R. and Pai, S. I. [1] taken transverse applied magnetic field and magnetic Reynolds number are assumed to be very small, so that the induced magnetic field is negligible Muthucumaraswamy *et al.* [2] have studied the effect of homogenous chemical reaction of first order and free convection on the oscillating infinite vertical plate with variable temperature and mass diffusion. Sharma [3] investigate the effect of periodic heat and mass transfer on the unsteady free convection flow past a vertical flat plate in slip flow regime when suction velocity oscillates in time. Chaudhary and Jha [4] studied the effects of chemical reactions on MHD micropolar fluid flow past a vertical plate in slip-flow regime. Anjalidevi *et al.* [5] have examined the effect of chemical reaction on the flow in the presence of heat transfer and magnetic field. Muthucumaraswamy *et al.* [6] have investigated the effect of thermal radiation effects on flow past an impulsively started infinite isothermal vertical plate in the presence of first order chemical reaction. Moreover, Al-Odat and Al-Azab [7] studied the influence of magnetic field on unsteady free convective heat and mass transfer flow along an impulsively started semi-infinite vertical plate taking into account a homogeneous chemical reaction of first order. The effect of radiation on the heat and fluid flow over an unsteady stretching surface has been analyzed by El-Aziz [8]. Singh *et. al.* [9] studied the heat transfer over stretching surface in porous media with transverse magnetic field. Singh *et. al.* [10] and [11] also investigated MHD oblique stagnation-point flow towards a stretching sheet with heat transfer for steady and unsteady cases. Elbashedy *et. al.* [12] investigated the effects of thermal radiation and magnetic field on unsteady boundary layer mixed convection flow and heat transfer problem from a vertical porous stretching surface. Ahmed Sahin studied influence of chemical reaction on transient MHD free Convective flow over a vertical plate.

Recently, the chemical reaction, heat and mass transfer on MHD flow over a vertical stretching surface with heat source and thermal stratification have been presented by Kandasamy *et al.* [13]. The opposing buoyancy effects on simultaneous heat and mass transfer by natural convection in a fluid saturated porous medium investigated by

Angirasa *et al.*[14]. Ahmed [15] investigates the effects of unsteady free convective MHD flow through a porous medium bounded by an infinite vertical porous plate. Ahmed Sahin [16] studied the Magneto hydrodynamic and chemical reaction effects on unsteady flow, heat and mass transfer characteristics in a viscous, incompressible and electrically conduction fluid over a semi-infinite vertical porous plate in a slip-flow regime. V. Sri Hari Babu and G. V. Ramana Reddy [17] analyzed the Mass transfer effects on MHD mixed convective flow from a vertical surface with Ohmic heating and viscous dissipation. Satya Sagar Saxena and G. K. Dubey [18] studied the effects of MHD free convection heat and mass transfer flow of visco-elastic fluid embedded in a porous medium of variable permeability with radiation effect and heat source in slip flow regime. Unsteady MHD heat and mass transfer free convection flow of polar fluids past a vertical moving porous plate in a porous medium with heat generation and thermal diffusion was analysed by Satya Sagar Saxena and G. K. Dubey [19]. Sudeerbabu *et al* [20] analyzed the radiation and chemical reaction effects on an unsteady MHD convection flow past a vertical moving porous plate embedded in a porous medium with viscous dissipation. Unsteady MHD free convection flow and mass transfer near a moving vertical plate in the presence of thermal radiation is studied by Seethamahalakshmi *et al* [21].

To the best of the author's knowledge, studies pertaining to oscillatory flow investigations in a planer channel with variable temperature and concentration have not received much attention. Therefore, the main goal here is to study the chemical reaction effects on unsteady MHD oscillatory slip flow in an optically thin fluid through a planer channel in the presence of a temperature-dependent heat source. The closed form solutions for velocity, temperature, skin friction, concentration, Nusselt number, and Sherwood number are presented. The effects of pertinent parameters on fluid flow of heat and mass transfer characteristics are studied in detail and presented graphically and discussed qualitatively.

2. FORMATION OF THE PROBLEM

We consider the unsteady mixed convection, two dimensional slip flow of an electrically conducting, heat generating, optically thin and chemically reacting oscillatory fluid flow in a planer channel filled with porous medium in the presence of thermal radiation with temperature and concentration variation. Take a Cartesian coordinate system (x', y') where x' - axis is taken along the flow and y' - axis is taken normal to the flow direction. A uniform transverse magnetic field of magnitude B_0 is applied in the presence of thermal and Solutal buoyancy effects in the direction of y' - axis. Then, assuming a Boussinesq incompressible fluid model, the equations governing the motion are given as:

$$\frac{\partial v'}{\partial y'} = 0 \quad (1)$$

$$\frac{\partial u'}{\partial t'} = -\frac{1}{\rho} \frac{\partial P'}{\partial x'} + \nu \frac{\partial^2 u'}{\partial y'^2} + g\beta(T' - T_1') + g\beta^*(C' - C_1') - \frac{\nu}{K'} u' - \frac{\sigma B_0^2}{\rho} u' \quad (2)$$

$$\frac{\partial T'}{\partial t'} = \frac{k}{\rho C_p} \frac{\partial^2 T'}{\partial y'^2} - \frac{1}{\rho} \frac{\partial q_r}{\partial y'} + \frac{Q(T' - T_1')}{\rho C_p} \quad (3)$$

$$\frac{\partial C'}{\partial t'} = D \frac{\partial^2 C'}{\partial y'^2} - K'_r (C' - C_1') \quad (4)$$

The appropriate boundary conditions of the problem are

$$u' = L_1 \frac{\partial u'}{\partial y'}, \quad T' = T_1' + \delta_r^* \frac{\partial T'}{\partial y'}, \quad C' = C_1' + \delta_c^* \frac{\partial C'}{\partial y'} \quad \text{at } y' = 0 \quad (5)$$

$$u' = 0, \quad T' = T_2' + \delta_t^* \frac{\partial T'}{\partial y'}, \quad C' = C_2' + \delta_c^* \frac{\partial C'}{\partial y'} \quad \text{at } y' = d \quad (6)$$

where u', v' - the velocity components in the x', y' directions respectively, ν - the kinematics viscosity, k - the thermal conductivity, β - the coefficient of volume expansion due to temperature, β^* - the coefficient of volume expansion due to concentration, ρ - the density, σ - the electrical conductivity of the fluid, g - the acceleration due to gravity, T' - the temperature, T_1' - wall temperature of the fluid, q_r - the radiation heat flux, C' - the concentration,

C'_1 - wall concentration of the fluid and K'_r - chemical reaction parameter, L_1 - mean free path, C_p - specific heat at a constant pressure and D - mass diffusivity.

The radiative heat flux (Cogley *et al.* [22]) is given by

$$\frac{\partial q'_r}{\partial y'} = 4(T'_1 - T)I', \text{ where } I' = \int_0^\infty K_{\lambda w} \frac{\partial e_{b\lambda}}{\partial T} d\lambda \quad (7)$$

where $K_{\lambda w}$ - radiation absorption coefficient at the wall, $e_{b\lambda}$ - Planck's function.

Introducing the following non-dimensional quantities

$$\begin{aligned} x &= \frac{x'}{d}, \quad y = \frac{y'}{d}, \quad P = \frac{dP}{\mu u_0}, \quad u = \frac{u'}{u_0}, \quad \theta = \frac{T' - T'_1}{T_2 - T_1}, \quad \phi = \frac{C' - C_1}{C_2 - C_1}, \\ t &= \frac{u_0 t'}{d}, \quad \text{Re} = \frac{u_0 d}{\nu}, \quad \gamma = \frac{K'}{d^2}, \quad M = \frac{\sigma B_0^2 d^2}{\mu}, \quad \text{Gr} = \frac{g\beta(T_2 - T_1)d^2}{\nu u_0}, \\ R &= \frac{4I'd^2}{k}, \quad \text{Gc} = \frac{g\beta^*(C_2 - C_1)d^2}{\nu u_0}, \quad \text{Pe} = \frac{\rho C_p u_0 d}{k}, \quad \alpha = \frac{Qd^2}{k}, \\ \text{Sc} &= \frac{D}{u_0 d}, \quad K_r = \frac{K'_r d}{u_0}, \quad d_2 = \frac{\delta_T^*}{d}, \quad d_1 = \frac{\delta_C^*}{d}, \end{aligned} \quad (8)$$

In view of the above dimensionless variables, the basic field equations (2) to (4) can be expressed in non-dimensional form as

$$\text{Re} \frac{\partial u}{\partial t} = -\frac{\partial P}{\partial t} + \text{Gr}\theta + \text{Gm}C + \frac{\partial^2 u}{\partial y^2} - \left(M + \frac{1}{K}\right)u \quad (9)$$

$$\text{Pe} \frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial y^2} - (R + \alpha)\theta \quad (10)$$

$$\frac{\partial \phi}{\partial t} = \text{Sc} \frac{\partial^2 \phi}{\partial y^2} - K_r \phi \quad (11)$$

The corresponding boundary conditions for $t > 0$ are transformed to:

$$\begin{aligned} u &= \gamma \frac{\partial u}{\partial y}, \quad \theta = d_2 \frac{\partial \theta}{\partial y}, \quad \phi = d_1 \frac{\partial \phi}{\partial y} \quad \text{at } y = 0 \\ u &= 0, \quad \theta = 1 + d_2 \frac{\partial \theta}{\partial y}, \quad \phi = 1 + d_1 \frac{\partial \phi}{\partial y} \quad \text{at } y = 1 \end{aligned} \quad (12)$$

where $P, \text{Re}, M, K, \text{Pe}, R, \text{Sc}, \lambda, d_1, d_2, \text{Gr}, \text{Gc}$ and γ are pressure, Reynolds number, magnetic parameter, permeability parameter, Peclet number, thermal radiation parameter, Schmidt number, real constant, volumetric concentration expansion, volumetric thermal expansion, thermal Grashof number, Solutal Grashof number and slip parameter respectively.

3. SOLUTION OF THE PROBLEM

In order to solve equations (9) - (11) with respect to the boundary conditions (12) for purely oscillatory flow, let us take

$$u(y, t) = u_0(y) e^{i\alpha t} \quad (13)$$

$$\theta(y, t) = \theta_0(y) e^{i\omega t} \quad (14)$$

$$\phi(y, t) = \phi_0(y) e^{i\omega t} \quad (15)$$

$$-\frac{\partial P}{\partial x} = \lambda e^{i\omega t} \quad (16)$$

Substituting the Equations (13) - (16) in Equations (9) - (12), we obtain:

$$u_0'' - N_1^2 u_0 = -[\lambda + Gr\theta_0 + Gc\phi_0] \quad (17)$$

$$\theta_0'' - N_3^2 \theta_0 = 0 \quad (18)$$

$$\phi_0'' - N_3^2 \phi_0 = 0 \quad (19)$$

where prime denotes ordinary differentiation with respect to y .

The corresponding boundary conditions can be written as

$$\begin{aligned} u_0 = \gamma u_0', \theta_0 = d_2 \theta_0', \phi_0 = d_1 \phi_0' \quad \text{at } y = 0 \\ u_0 = 0, \theta_0 = 1 + d_2 \theta_0', \phi_0 = d_1 \phi_0' \quad \text{at } y = 1 \end{aligned} \quad (20)$$

Solving equations (17) - (19) under the boundary conditions (20), we obtain the velocity, temperature and concentration distribution in the boundary layer as:

$$u(y, t) = \left[\begin{aligned} &N_7 + N_8 \sinh(N_1 y) + N_9 \cosh(N_3 y) + N_{10} \sin(N_2 y) \\ &+ N_{11} \sinh(N_1 y) + N_{12} \cosh(N_1 y) \end{aligned} \right] e^{i\omega t}$$

$$\theta(y, t) = N_6 \sin(N_2 y) e^{i\omega t}$$

$$\phi(y, t) = [N_4 \sinh(N_3 y) + N_5 \cosh(N_3 y)] e^{i\omega t}$$

where

$$N_1 = \sqrt{i\omega Re + M^2 + 1/K}; N_2 = \sqrt{R + \alpha - i\omega Pe}; N_3 = \sqrt{\frac{K_r + i\omega}{Sc}}$$

$$N_4 = \frac{1}{(1 - d_2 N_3^2) \sinh(N_3)}; N_5 = d_2 N_3 N_4; N_6 = \frac{1}{\sin(N_2) - d_1 N_2 \cos(N_2)};$$

$$N_7 = \frac{\lambda}{N_1^2}; N_8 = \frac{N_4 Gc}{N_1^2 - N_3^2}; N_9 = \frac{N_4 Gc}{N_1^2 - N_3^2}; N_{10} = \frac{N_6 Gr}{N_1^2 + N_2^2};$$

$$N_{11} = \frac{\left[\begin{aligned} &N_7 (\cosh(N_1) - 1) - N_8 (N_3 \gamma \cosh(N_1) + \sinh(N_3)) \\ &+ N_9 (\cosh(N_1) - \cos(N_3)) - N_{10} (N_2 \gamma \cosh(N_1) + \sinh(N_2)) \end{aligned} \right]}{N_1 \gamma \cosh(N_1) + \sinh(N_1)};$$

$$N_{12} = (N_1 N_{11} + N_2 N_{10} + N_3 N_8) \gamma - N_7 - N_9;$$

The shear stress, the coefficient of the rate of heat transfer and the rate of mass transfer at any point in the fluid can be characterized by

$$\tau^* = -\mu \frac{\partial u}{\partial y}, Nu^* = -k \frac{\partial T}{\partial y}, Sh^* = -D \frac{\partial C}{\partial y}$$

In dimensionless form

$$\tau = \frac{\tau^* d}{\mu u_0} = -\frac{\partial u}{\partial y}, Nu = -\frac{Nu^* d}{x(T_1 - T_0)} = -\frac{\partial \theta}{\partial y}, Sh = -\frac{Sh^* d}{C_1 - C_0} = -\frac{\partial \phi}{\partial y}$$

The skin-friction (τ), the Nusselt (Nu) and the Sherwood number (Sh) at the walls $y = 0$ and $y = 1$ are given by

$$\tau_0 = -\frac{\partial u}{\partial y}\bigg|_{y=0}, \quad \tau_1 = -\frac{\partial u}{\partial y}\bigg|_{y=1}$$

$$Nu_0 = -\frac{\partial \theta}{\partial y}\bigg|_{y=0}, \quad Nu_1 = -\frac{\partial \theta}{\partial y}\bigg|_{y=1},$$

$$Sh_0 = -\frac{\partial \phi}{\partial y}\bigg|_{y=0}, \quad Sh_1 = -\frac{\partial \phi}{\partial y}\bigg|_{y=1}$$

RESULTS AND DISCUSSION

Numerical evaluation for the analytical solution of this problem is performed and the results are illustrated graphically in Figs. 1–12 to show the interesting features of significant parameters on velocity, temperature, concentration, skin friction, Nusselt number and Sherwood number distributions in the planer channel. Throughout the computations we employ

$$t = 1, M = 2, K = 2, Re = 1, Gr = 2, Gc = 1, R = 2, \alpha = 3, Pe = 4, K_r = 2, Sc = 1, \\ g = 0.1, d_1 = 0.002 \text{ and } d_2 = 0.002 \text{ unless otherwise stated.}$$

The effect of magnetic intensity of the velocity profiles is shown in Fig. 1. While all other participating parameters in the velocity field are held constant and magnetic intensity is increased, a drop in the velocity field is noticed. When the magnetic intensity is relatively small the velocity field increases initially and thereafter it decreases substantially. The effect of magnetic field is found to have zero effect on the velocity field as we move far away from the plate. The effect of the frequency of excitation over the velocity field is illustrated in Fig. 2. It is noticed that as the frequency of excitation increases the velocity field decreases. Further, it is observed that as we move far away from the plate the velocity decreases rapidly initially and thereafter the decrease is found to be slow. Fig. 3 illustrates the effect of thermal Grashof number on velocity field. While all other parameters in the velocity field are held constant, increase in the thermal Grashof number contributes to the raise in the velocity field. However, the effect is found to be absolutely zero as we move away from the bounding surface. The effect of the Solutal Grashof number on the velocity profiles is illustrated in Fig. 4. The increase in the Solutal Grashof number contributes to the raise in the velocity field. Moreover, it is observed that the effect is almost zero as we move far away from the plate. Fig.5 displays that the increases in the permeability coefficient of porous medium act against the porosity of the porous medium which increase the fluid velocity. Fig.6. represents that the increase in the slip parameter has the tendency to reduce the friction forces which increases the fluid velocity. Fig. 7 illustrates that increase in the radiation parameter increases the temperature distribution because large values of radiation parameter oppose the conduction over radiation, thereby which increases the buoyancy force and increases the thickness of the thermal boundary layer. Fig. 8 represents that the increase in the heat source parameter significantly increase the thermal buoyancy effects which raise fluid temperature. Increase in temperature variation parameter coincides with the decrease of heat transfer and the curves could be seen in Fig. 9. Figs. 10 & 11 illustrate that Schmidt number and concentration variation parameter are used to increase the mass transfer. Fig. 12 shows that we obtain a destructive type chemical reaction because the concentration decreases for increasing the chemical reaction parameter which indicates that the diffusion rates can be tremendously changed by the chemical reaction.

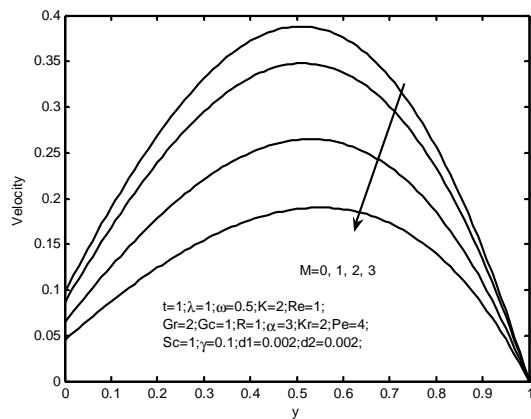


Fig. 1. Velocity profiles for different values of M

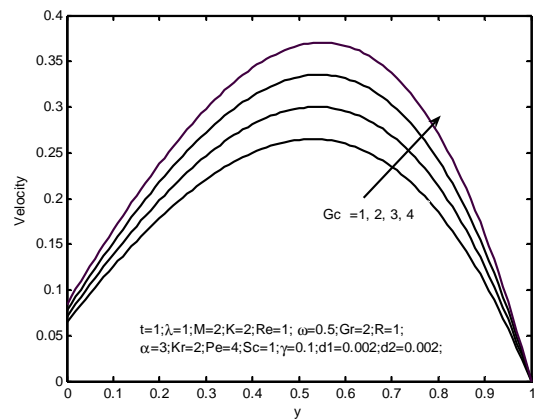


Fig. 4. Velocity profiles for different values of Gc.

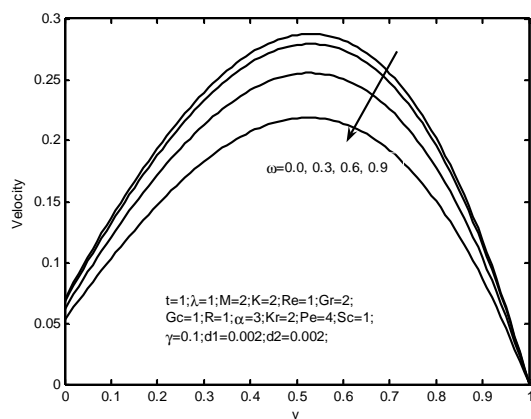


Fig. 2. Velocity profiles for different values of ω.

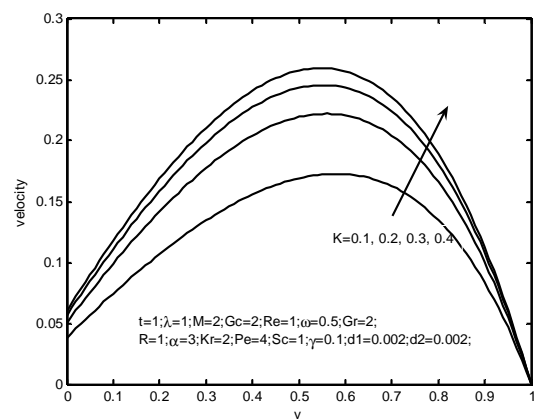


Fig. 5. Velocity profiles for different values of K.

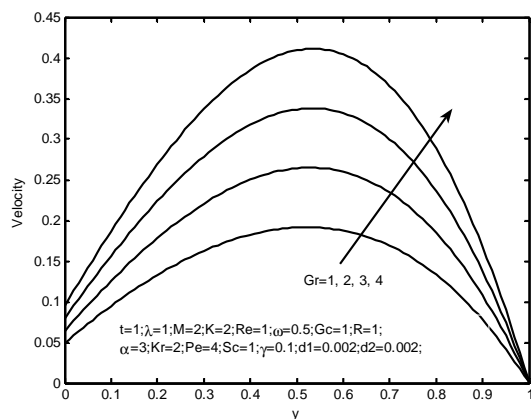


Fig. 3. Velocity profiles for different values of Gr.

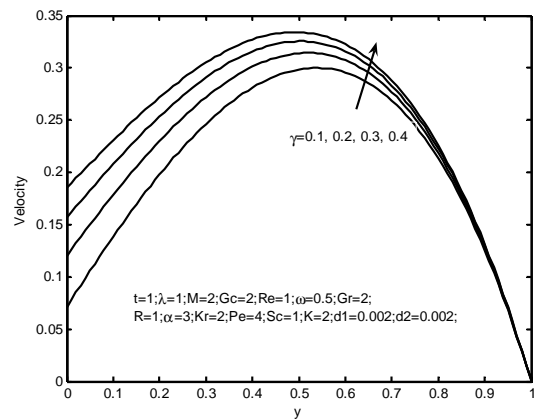


Fig. 6. Velocity profiles for different values of γ.

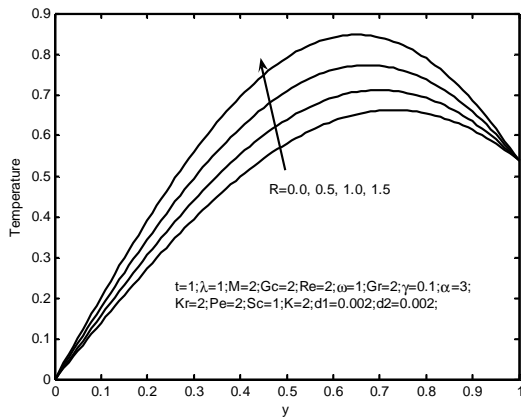


Fig. 7. Temperature profiles for different values of R.

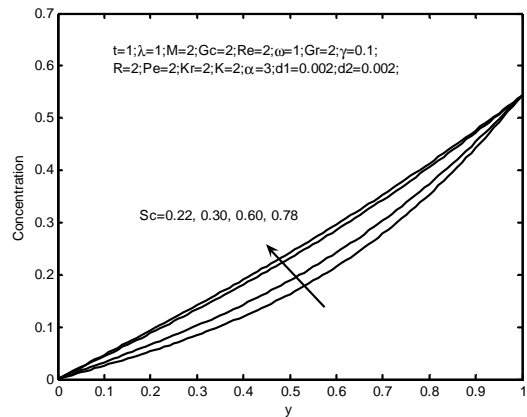


Fig. 10. Concentration profiles for different values of Sc.

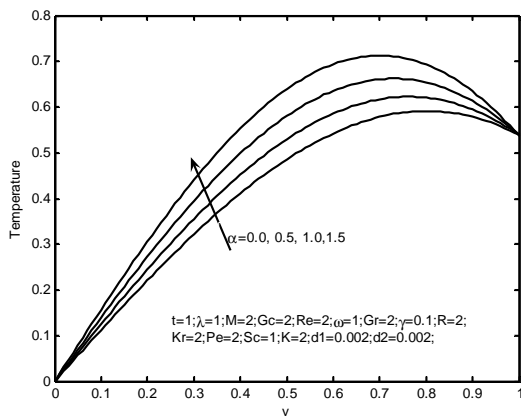


Fig. 8. Temperature profiles for different values of α .

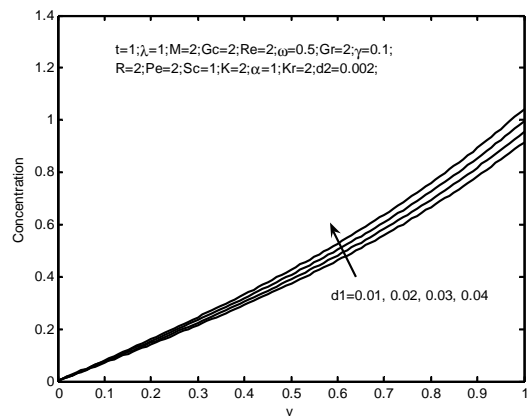


Fig. 11. Concentration profiles for different values of d_1 .

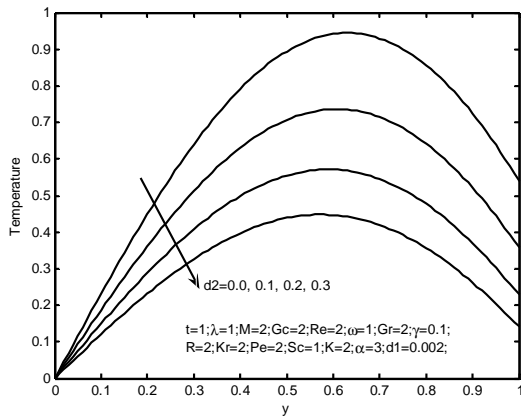


Fig. 9. Temperature profiles for different values of d_2

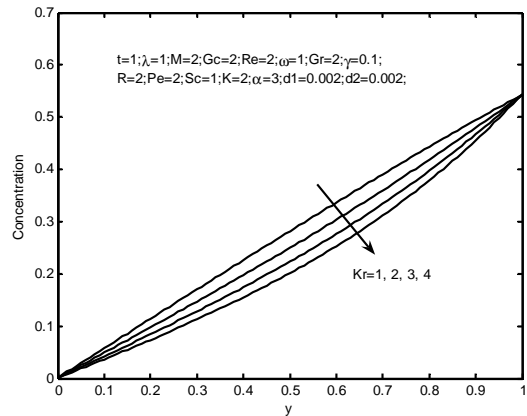


Fig. 12. Concentration profiles for different values of Kr.

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