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Chemical reaction interaction on unsteady MHD free convective radiative flow past an oscillating plate embedded in porous media with thermal diffusion

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ABSTRACT

The present paper deals analytically the influence of chemical reaction on unsteady MHD free convective mass transfer flow of a viscous incompressible fluid past an infinite oscillating plate in presence of thermal radiation and thermo - diffusion (Soret) effects. Cogly-Vincentine-Gilles equilibrium approximate model is incorporated in the energy equation and the effect of first- order homogeneous chemical reaction has also been considered in the species concentration equation. The governing system of partial differential equations subject to favorable boundary conditions is solved in closed form by using one-sided Laplace transform technique. The influence of various pertinent parameters on the velocity, concentration, skin-friction, Nusselt and Sherwood numbers of thermally assisting, chemically reactive flow has been investigated through graphs and tables. It is observed that, the first order chemical reaction reduces the fluid velocity, plate friction and fluid concentration but enhances the mass transfer rate. It is also observed that, the fluid velocity, skin-friction and Nusselt number increase with rise in parametric values of Soret number.

Key words: MHD Free convection, porous media, thermal diffusion, thermal radiation, chemical reaction.

Nomenclature

B_0 Strength of the applied magnetic field
$\bar{C}$ Species concentration
C_p Specific heat at constant pressure
$\bar{C}_{w^*}$ Species concentration at the plate
$\bar{C}_\infty$ Species concentration in the free stream
D_M Co-efficient of mass diffusion
D_T Co-efficient of thermal diffusion
erf Error function
erfc Complementary error function
F Chemical reaction parameter (Non-dimensional)
Gm Grashof number for mass transfer
Gr Grashof number for heat transfer
k Thermal conductivity
K Permeability parameter (Non- dimensional)
$\bar{K}$ Permeability parameter (Dimensional)
K_l Chemical reaction parameter (Dimensional)

M Magnetic parameter (square of the Hartmann number)
Nu Nusselt number
Pr Prandtl number
R Radiation parameter
s Laplace Transform parameter
Sc Schmidt number
Sh Sherwood number
Sr Soret number
t Time variable(Non-Dimensional)
$\bar{t}$ Time variable (Dimensional)
$\bar{T}$ Fluid temperature
$\bar{T}_{w^*}$ Temperature at the plate
$\bar{T}_\infty$ Temperature in the free stream
u First component of fluid velocity (Non-Dimensional)
$\bar{u}$ First component of fluid velocity (Dimensional)
$\tilde{u}$ Laplace Transform of u, $\tilde{u}$ = L (u)
u_0 Mean plate velocity / Scale of plate velocity
y y – Co-ordinate (Dimensional)
$\bar{y}$ y – Co-ordinate (Non-Dimensional)

Greek Symbols
β Coefficient of thermal volume expansion
β^* Coefficient of solutal volume expansion
ρ Fluid density
ν Kinematic coefficient of viscosity
σ Electrical conductivity
θ Non-dimensional temperature
$\tilde{\theta}$ Laplace Transform of θ, $\tilde{\theta}$ = L(θ)
ϕ Non-dimensional species concentration
$\tilde{\phi}$ Laplace Transform of ϕ , $\tilde{\phi}$ = L (ϕ)
τ Skin - friction (Non-dimensional)
$\bar{\omega}$ Frequency of Oscillation (Dimensional)
$\bar{\omega}\bar{t}$ Phase angle (Dimensional)
ωt Phase angle (Non-dimensional)

Subscripts
w^* Condition on the walls
∞ Free stream conditions

INTRODUCTION

Many problems in nature are time dependent as such unsteady free convective flow past infinite or semi-infinite plate has got an important place in research due to its significant technological applications. Stokes [1] initially studied unsteady flow by considering internal friction of fluids on motion of pendulum. Soundalgekar [2] investigated free convection effects on Stokes problem for a vertical plate. Perdikis and Takhar [3] considered the free convection effects on flow past a moving vertical infinite porous plate. In recent times, free convective mass transfer flow has become more significant, as the effects of heat transfer along with mass transfer are the dominant features in many engineering applications such as rocket nozzles, cooling of nuclear reactors, high sinks in turbine blades, high speed aircrafts and their atmospheric re-entry, chemical devices, process equipment etc. Hossain and Mandal [4] studied the effects of mass transfer and free convection on unsteady MHD flow past a vertical porous plate with variable suction. Gokhale and Samman [5] considered the effects of mass transfer on the transient free convective flow of a dissipative fluid along a semi-infinite vertical plate with constant heat flux. On the other hand, flows through porous media are very much prevalent in nature and therefore have become of principal interest in many scientific and engineering applications. Due to many applications in engineering, geophysical and astrophysical problems which combine heat and mass transfer by free convection in porous media, various researchers were attracted to this field for several decades. A comprehensive review of the study of convective heat transfer mechanisms through porous media in relation to the applications to the above areas had been made by Nield and Bejan [6]. Anghel et al. [7] studied free convection boundary layer flow over a vertical surface embedded in a porous medium. The case of MHD free convective heat and mass transfer flow past a vertical flat plate embedded in porous media was considered by Alam and Rahman [8].

Earlier it was considered that, the mass transfer occurs only due to concentration gradients. But after the pioneering work of Eckert and Drake [9], researchers believe that, in presence of high temperature gradient, species transportation may also take place. The process of mass transfer that occurs by the combine effects of concentration as well as temperature gradients is known as thermal diffusion or Soret effect. Significant work in Soret effect was made by Platten and Chavepeyer [10], who investigated an oscillatory motion in Benard cell. Besides the aforesaid works, some more notable contribution in this regard are made byJha and Singh [11], Dursunkaya and Worek [12], Kafoussias [13] El-Aziz [14], Afify [15] and Hayat et al. [16] etc. Recently, Ahmed et al. [17] obtained the closed form of solution for problem relating to MHD free convection heat and mass transfer flow past an oscillating plate with Soret effect.

Many processes in engineering as well as in industry occur at very high temperatures and so knowledge of radiation heat transfer becomes very significant. Cess [18] investigated radiation effects on free convective heat transfer flow. The radiation effect on a natural convective flow of an absorbing emitting liquid was considered by Novotny et al. [19]. Sattar and Kalim [20] had investigated unsteady free convection interaction with thermal radiation in a boundary layer flow past a vertical porous plate. The case of unsteady flow in presence of radiation and variable viscosity on a MHD flow past a semi-infinite flat plate with an aligned magnetic field was studied by Seddeek [21]. The free convective flow with thermal radiation and mass transfer past a moving vertical porous plate was studied by Makinde [22]. Manivannan et al. [23] investigated the effect of thermal Radiation on an isothermal vertical oscillating plate by considering variable mass Diffusion and Chemical reaction.

The study of heat and mass transfer with chemical reaction is of considerable importance in the chemical and hydrometallurgical industries. Chemical reactions can be codified as either heterogeneous or homogeneous processes, which depends on whether they occur at an interface or as a single phase volume reaction. The formation of smog is a first- order homogeneous chemical reaction. Combined heat and mass transfer with chemical reaction have been studied by various researchers like Muthucumaraswamy and Ganesan [24], Chamkha [25], Raptis and Perdikis [26], Postelnicu [27] etc. Muthucumaraswamy et al. [28] investigated first order Chemical reaction and MHD effects on flow past a vertical oscillating plate in presence of thermal radiation.

The aim of the present paper is to investigate the combined effects of thermo - diffusion (Soret), thermal radiation and first order chemical reaction on MHD free convective mass transfer flow past an infinite oscillating plate, embedded in porous media. The present study finds applications in many chemical as well as dyeing industries besides the outcome of the work can be used to develop many complex situations in the theory of heat and mass transfer of related studies.

2. Mathematical formulation of the problem:

A co-ordinate system ($\bar{x}$, $\bar{y}$) is introduced, with its $\bar{x}$ -axis along the length of the plate in the upward vertical direction and $\bar{y}$ -axis perpendicular to the plate towards the fluid region. Since the fluid is electrically conducting, a uniform magnetic field of small strength B_0 is applied normal to the plate directed to the fluid region. As the magnetic Reynolds number of the flow is considered to be less compared to the applied magnetic field, the strength of the induced magnetic field can be neglected. Using Boussinesq approximation and considering all of these effects, a fluid model is developed in terms of a system of partial differential equations combined with a set of favorable initio- boundary conditions as:

Momentum Equation

$$\frac{\partial \bar{u}}{\partial \bar{t}} = \nu \frac{\partial^2 \bar{u}}{\partial \bar{y}^2} + g\beta(\bar{T} - \bar{T}_\infty) + g\beta^*(\bar{C} - \bar{C}_\infty) - \left(\frac{\sigma B_0{}^2}{\rho} + \frac{\nu}{\bar{K}} \right) \bar{u} \tag{1}$$

Energy Equation

$$\frac{\partial \bar{T}}{\partial \bar{t}} = \frac{k}{\rho c_p} \frac{\partial^2 \bar{T}}{\partial \bar{y}^2} - \frac{1}{\rho c_p} \frac{\partial q_{\bar{y}}}{\partial \bar{y}} \tag{2}$$

Species Continuity Equation

$$\frac{\partial \bar{C}}{\partial \bar{t}} = D_M \frac{\partial^2 \bar{C}}{\partial \bar{y}^2} + D_T \frac{\partial^2 \bar{T}}{\partial \bar{y}^2} - K_l(\bar{C} - \bar{C}_\infty) \tag{3}$$

Subject to the initio - boundary conditions as:

$$\left.\begin{aligned} &\bar{u} = 0, \bar{T} = \bar{T}_\infty, \bar{C} = \bar{C}_\infty \text{ , for every } \bar{y} \text{ when } \bar{t} = 0 \\ &\bar{u} = u_0 \cos \bar{\omega} \bar{t}, \ \bar{T} = \bar{T}_{w^*}, \bar{C} = \bar{C}_{w^*} \text{ , at } \bar{y} = 0 \text{ when } \bar{t} > 0 \\ &\bar{u} \to 0, \bar{T} \to \bar{T}_\infty, \bar{C} \to \bar{C}_\infty, \text{ for } \bar{y} \to \infty \text{ when } \bar{t} > 0 \end{aligned}\right\} \tag{4}$$

Assuming that the medium is optically thin with relatively low density and following Cogly – Vincentine – Gillies [29] equilibrium model, the heat flux is quantified as,

$$\frac{\partial \bar{q}_{\bar{y}}}{\partial \bar{y}} = 4I^*\left(\bar{T} - \bar{T}_\infty\right) \tag{5}$$

where, $\mathrm{I}^* = \int_0^\infty (k_{\lambda^{**}})_{w^*} \left(\frac{\partial e_{b\lambda^{**}}}{\partial \bar{T}}\right)_{w^*} d\lambda^{**}$

and $k_{\lambda^{**}}$ is the absorption co-efficient, $e_{b\lambda^{**}}$ is Plank's constant, λ^{**} represents wave length.

On using (5) in (2) gives,

$$\rho c_p \frac{\partial \bar{T}}{\partial \bar{t}} = k \frac{\partial^2 \bar{T}}{\partial \bar{y}^2} - 4I^*\left(\bar{T} - \bar{T}_\infty\right) \tag{6}$$

We now introduce the following non-dimensional quantities as:

$$y = \frac{\bar{y} u_0}{\nu}, \ t = \frac{\bar{t} u^2_0}{\nu}, \ u = \frac{\bar{u}}{u_0}, \ K = \frac{\bar{K} u_0^2}{\nu^2}, \ Gr = \frac{Lg\beta(\bar{T}_{w^*} - \bar{T}_\infty)}{u_0^3}, Gm = \frac{Lg\beta^*(\bar{C}_{w^*} - \bar{C}_\infty)}{u_0^3},$$

$$M = \frac{\sigma B_0^2 \nu}{\rho u_0^2}, \ \omega = \frac{\bar{\omega}\nu}{u_0^2}, \theta = \frac{\bar{T} - \bar{T}_\infty}{\bar{T}_{w^*} - \bar{T}_\infty}, \ \phi = \frac{\bar{C} - \bar{C}_\infty}{\bar{C}_{w^*} - \bar{C}_\infty}, \ \Pr = \frac{\nu \rho c_p}{k}, \ Sc = \frac{\nu}{D_M}, \ Sr = \frac{D_T(\bar{T}_{w^*} - \bar{T}_\infty)}{\nu(\bar{C}_{w^*} - \bar{C}_\infty)},$$

$$R = \frac{4I^*\nu^2}{ku^2_0}, \ \mathrm{F} = \frac{K_l \nu}{u^2_0}.$$

The non-dimensional form of equations (1), (6) and (3) are,

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + Gr\theta + Gm\phi - M_1 u \tag{7}$$

$$\Pr \frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial y^2} - R\theta \tag{8}$$

$$Sc \frac{\partial \phi}{\partial t} = \frac{\partial^2 \phi}{\partial y^2} + SrSc \frac{\partial^2 \theta}{\partial y^2} - FSc\phi \tag{9}$$

where, $M_1 = M + \dfrac{1}{K}$

The corresponding non-dimensional initio-boundary conditions are:

$$\left.\begin{array}{l} u=0,\ \theta=0,\ \phi=0, \text{ for every } y \text{ when } t=0 \\ u=\cos\omega t,\ \theta=1,\ \phi=1, \text{ at } y=0 \text{ when } t>0 \\ u\to 0,\ \theta\to 0,\ \phi\to 0, \text{ for } y\to\infty \text{ when } t>0 \end{array}\right\} \quad (10)$$

3. METHOD OF SOLUTION:

To solve the system of equations (7), (8), (9) subject to (10) in closed form we prefer to use an integral transform method in terms of one - sided Laplace transform. The Laplace transform of the equations (7), (8) and (9) are obtained as:

$$\frac{d^2\tilde{u}}{dy^2}-(s+M_1)\tilde{u}=-Gr\tilde{\theta}-Gm\tilde{\phi} \quad (11)$$

$$\frac{d^2\tilde{\theta}}{dy^2}-(R+s\Pr)\tilde{\theta}=0 \quad (12)$$

$$\frac{d^2\tilde{\phi}}{dy^2}-(F+s)Sc\tilde{\phi}=-SrSc\frac{d^2\tilde{\theta}}{dy^2} \quad (13)$$

Subject to the relevant conditions as,

$$\left.\begin{array}{l} \tilde{u}(0,s)=\dfrac{s}{s^2+\omega^2},\tilde{\theta}(0,s)=\dfrac{1}{s},\tilde{\phi}(0,s)=\dfrac{1}{s} \\ \tilde{u}(\infty,s)=0,\tilde{\theta}(\infty,s)=0,\tilde{\phi}(\infty,s)=0 \end{array}\right\} \quad (14)$$

Solving equations (11) to (13) subject to (14) and for $Pr \neq Sc$ ($Pr, Sc \neq 1$) we get

$$\begin{aligned}\tilde{u}(y,s)&=\frac{s}{s^2+\omega^2}\exp(-y\sqrt{s+M_1})+\frac{Gr}{s(\Pr-1)(H+s)}\left\{\exp(-y\sqrt{s+M_1})-\exp(-y\sqrt{R+s\Pr})\right\}+\\ &\frac{Gm}{s(Sc-1)(I+s)}\left(1+\frac{SrScR}{(\Pr-Sc)(G+s)}\right)\left\{\exp(-y\sqrt{s+M_1})-\exp(-y\sqrt{(F+s)Sc})\right\}+\\ &\frac{SrGm\Pr Sc}{s(\Pr-Sc)(\Pr-1)(H+s)(G+s)}\times\left\{\exp(-y\sqrt{R+s\Pr})-\exp(-y\sqrt{s+M_1})\right\}+\\ &\frac{SrGm\Pr Sc}{s(\Pr-Sc)(Sc-1)(I+s)(G+s)}\times\left\{\exp(-y\sqrt{s+M_1})-\exp(-y\sqrt{(F+s)Sc})\right\}\end{aligned} \quad (15)$$

$$\tilde{\theta}(y,s)=\frac{1}{s}\exp(-y\sqrt{R+s\Pr}) \quad (16)$$

$$\begin{aligned}\tilde{\phi}(y,s)&=\frac{1}{s}\exp(-y\sqrt{(F+s)Sc})+\frac{SrScR}{s(\Pr-Sc)(G+s)}\left\{\exp(-y\sqrt{(F+s)Sc})-\exp(-y\sqrt{R+s\Pr})\right\}\\ &+\frac{SrSc\Pr}{(\Pr-Sc)(G+s)}\left\{\exp(-y\sqrt{(F+s)Sc})-\exp(-y\sqrt{R+s\Pr})\right\}\end{aligned} \quad (17)$$

Also, for $Pr = Sc$ ($Pr, Sc \neq 1$) and $R \neq FSc$ we get,

$$\tilde{u}(y,s)=\frac{s}{s^2+\omega^2}\exp(-y\sqrt{s+M_1})+\frac{Gr}{s(R-M_1)}\left\{\exp(-y\sqrt{s+M_1})-\exp(-y\sqrt{R+s\Pr})\right\}+$$
$$\frac{Gm}{s(F-M_1)}\left(1+\frac{SrScR}{(R-FSc)}\right)\left\{\exp(-y\sqrt{s+M_1})-\exp(-y\sqrt{(F+s)Sc})\right\}+$$
$$\frac{SrGm\Pr Sc}{(R-FSc)(R-M_1)}\times\left\{\exp(-y\sqrt{R+s\Pr})-\exp(-y\sqrt{(M_1+s)})\right\} \tag{18}$$
$$\tilde{\phi}(y,s)=\frac{1}{s}\exp(-y\sqrt{(F+s)Sc})+\frac{SrScR}{s(\lambda-FSc)}\left\{\exp(-y\sqrt{(F+s)Sc})-\exp(-y\sqrt{R+s\Pr})\right\}+$$
$$\frac{SrSc\Pr}{s(R-FSc)}\left\{\exp(-y\sqrt{(F+s)Sc})-\exp(-y\sqrt{R+s\Pr})\right\} \tag{19}$$

On taking the inverse Laplace transform of equations (15), (16) and (17), we obtain for $Pr \neq Sc$, (Pr, $Sc \neq 1$) as:

$$u(\eta,t)=\frac{1}{4}\left(A_{15}\exp(i\omega t)+A_{16}\exp(-i\omega t)\right)+\frac{Gr}{2(\Pr-1)}A_{30}+Gm\left[\frac{1}{2(Sc-1)}A_{42}+\frac{SrScR}{2(\Pr-Sc)}\left\{\frac{1}{Sc-1}(A_{48}-A_{54})+\right.\right.$$
$$\left.\left.\frac{1}{\Pr-1}(A_{60}-A_{63})\right\}+\frac{SrSc\Pr}{2(\Pr-Sc)}\left\{\frac{1}{Sc-1}(A_{64}-A_{65})+\frac{1}{\Pr-1}(A_{66}-A_{67})\right\}\right] \tag{20}$$
$$\theta(\eta,t)=\frac{1}{2}\left[\exp\left(2\eta\sqrt{Rt}\right)erfc\left(\eta\sqrt{\Pr}+\sqrt{\frac{Rt}{\Pr}}\right)+\exp\left(-2\eta\sqrt{Rt}\right)erfc\left(\eta\sqrt{\Pr}-\sqrt{\frac{Rt}{\Pr}}\right)\right] \tag{21}$$
$$\phi(\eta,t)=\frac{1}{2}A_{37}+\frac{SrScR}{2G(\Pr-Sc)}(A_{37}-A_{22}-A_{51}+A_{57})+\frac{SrSc\Pr}{2(\Pr-Sc)}[A_{51}-A_{57}]\ ,(G\neq 0) \tag{22}$$

Also, on taking the inverse Laplace transform of equations (18) and (19) we get for $Pr=Sc$ and $R \neq FSc$ as:

$$u(\eta,t)=\frac{1}{4}[A_{15}\exp(i\omega t)+A_{16}\exp(-i\omega t)]+\frac{Gr}{2(R-M_1)}(A_{19}-A_{22})+\frac{Gm}{2(FSc-M_1)}(A_{19}-A_{37})$$
$$+\frac{SrScGmR}{2(R-FSc)}\times\left[\frac{1}{FSc-M_1}(A_{19}-A_{37})-\frac{1}{R-M_1}(A_{22}-A_{19})\right]+\frac{SrScGm}{2(R-FSc)}\times$$
$$\left[\frac{1}{FSc-M_1}(A_{68}-A_{69})+\frac{1}{R-M_1}(A_{70}-A_{68})\right] \tag{23}$$
$$\phi(\eta,t)=\frac{1}{2}A_{37}+\frac{SrScR}{2(R-FSc)}(A_{37}-A_{22})+\frac{SrSc\Pr}{2(r-FSc)}[A_{69}-A_{70}] \tag{24}$$

***Rate of heat transfer coefficient*:**
The rate of the heat transfer coefficient in terms of non-dimensional Nusselt number is calculated as:

$$Nu=-\frac{1}{\Pr}\left(\frac{\partial\theta}{\partial y}\right)_{y=0}=\frac{1}{\Pr}\left[\sqrt{R}erf\left(\sqrt{\frac{Rt}{\Pr}}\right)+\frac{\sqrt{\Pr}}{\sqrt{\pi t}}\exp\left(-\frac{Rt}{\Pr}\right)\right] \tag{25}$$

***Rate of mass transfer coefficient*:**
The rate of the mass transfer coefficient in terms of non-dimensional Sherwood number is given by:

$$Sh=-\frac{1}{Sc}\left(\frac{\partial\phi}{\partial y}\right)_{y=0}$$

For $Pr \neq Sc$, it gives,

$$Sh = \frac{1}{Sc}(-B_2) + \frac{Sr\Pr}{\Pr - Sc}B_6 + \frac{SrR}{G(\Pr - Sc)}B_7 \quad ,(G \neq 0) \tag{26}$$

For $Pr = Sc$, we obtained as,

$$Sh = \frac{1}{Sc}(-B_2) + \frac{Sr\Pr}{R - FSc}(B_8 - B_9) + \frac{SrR}{R - FSc}B_5 \,,(R \neq FSc) \tag{27}$$

Skin- friction at the plate:

The non-dimensional skin-friction co-efficient at the plate is given by,

$$\tau = \left(\frac{\partial u}{\partial y}\right)_{y=0}$$

For $Pr \neq Sc$(Pr, $Sc \neq 1$), the skin-friction is calculated as:

$$\tau_{xy} = \frac{1}{2}\left[B_{10}\exp(i\omega t) + B_{11}\exp(-i\omega t)\right] + \frac{Gr}{\Pr - 1}B_{18} + \frac{Gm}{Sc - 1}B_{19} + \frac{GmSrScR}{\Pr - Sc}\left[\frac{1}{(Sc-1)}(B_{20} + B_{21}) + \frac{1}{(\Pr - 1)}(B_{22} + B_{23})\right] + \frac{GmSrSc\Pr}{\Pr - Sc} \times \left[\frac{1}{(Sc-1)}B_{24} + \frac{1}{(\Pr - 1)}B_{25}\right] \tag{28}$$

Also, for $Pr = Sc$ and $R \neq FSc$ it is obtained as:

$$\tau_{xy} = \frac{1}{2}\left[B_{10}\exp(i\omega t) + B_{11}\exp(-i\omega t)\right] + \frac{Gr}{R - M_1}(B_{12} - B_1) + \frac{Gm}{F - M_1}(B_{12} - B_2) + \frac{GmSrScR}{R - FSc}\left[\frac{1}{F - M_1} \times (B_{12} - B_2) - \frac{1}{R - M_1}(B_{12} - B_1)\right] + \frac{GmSrSc\Pr}{R - FSc} \times \left[\frac{1}{F - M_1}(B_{26} - B_{27}) + \frac{1}{R - M_1}(B_{28} - B_{26})\right] \tag{29}$$

RESULTS AND DISCUSSION

A theoretical study of the influence of first order chemical reaction on the free convective flow of a viscous incompressible fluid through an oscillating plate immersed in Darcian porous media with consideration of thermal radiation and thermal diffusion has made. In order to get physical insight into the problem, the discussion in terms of graphs and tables for the velocity and concentration fields, viscous drag, rates of heat and mass transfers at the plate have been made by assigning appropriate numerical values to the parameters M,K, Pr, F, R, Gr, Gm, Sr, Sc, ωt , η, ω, t and y. The numerical simulation involved in the proposed formulations is carried out in MATLAB 7.1 and for coding of erf (z) and $erfc$ (z), the works of Leutenegger [30] has been followed. In the present study, air is considered to be a primary fluid (solvent) and to produce a significant effect on mass diffusion, some fluids, considered as secondary (solute) such as Hydrogen, Water vapor, Oxygen and Ammonia, have been diffused through air. Throughout the discussion, the values of Schmidt number (Sc) of the corresponding secondary species are taken as 0.22, 0.60, 0.74, 0.78 and the Prandtl number (Pr) of the diffused fluids are taken as 0.71, which is same as the Prandtl number of air at temperature 290 k and1 atmosphere of pressure.

Table 1 expresses numerically the variation of heat transfer rate quantified by Nusselt number Nu due to changes in values of thermal radiation parameter R against time t for fixed values of Pr =0.71,M=0.5, K=0.5, ωt=π/2, Sr=0.5, Sc=0.22, Gr=5.0, Gm=2.0, ω= 7.857, F=1.0, η =0. It is observed that an increase in values of R initially increases heat flux near the plate which helps in rising heat transfer rate sharply and thereafter falling steadily far away from the plate. The parametric effect of chemical reaction F on Sherwood number Sh is numerically shown in table 2 for a set of fixed values of Pr =0.71, M=0.5, K=0.5, ωt= π/2, Sr=0.5, Sc=0.22, Gr=5.0, Gm=2.0, ω= 7.857, R=1.0, η =0. Due to the presence of F, the rate of mass flux increases, which results in accelerate the mass transfer rate.

Table1.Numerical values of Nusselt number *Nu* for different values of Radiation parameter *R* against arbitrary values of *t*

t	$R=1$	$R=2$	$R=3$	$R=4$
0.2	0.9965	1.5877	2.0959	2.5384
0.7	1.3918	1.9761	2.4320	2.8137
1.2	1.4508	1.9983	2.4406	2.8171
1.7	1.4477	1.9952	2.4398	2.8169
2.2	1.4352	1.9930	2.4396	2.8169
2.7	1.4248	1.9922	2.4395	2.8169
3.2	1.4179	1.9920	2.4395	2.8169
3.7	1.4137	1.9919	2.4395	2.8169

Table 2.Numerical values of Sherwood number *Sh*for different values of Chemical reaction parameter *F* against arbitrary values of time *t* and for fixed values of

t	$F=2$	$F=3$	$F=4$	$F=5$
0.2	3.6475	4.4842	5.3399	5.5299
0.7	3.1543	4.0009	4.7265	5.0467
1.2	3.0479	3.8033	4.4294	4.8481
1.7	2.9959	3.7074	4.3004	4.7680
2.2	2.9692	3.6632	4.2454	4.7347
2.7	2.9558	3.6433	4.2215	4.7204
3.2	2.9492	3.6342	4.2109	4.7140
3.7	2.9459	3.6301	4.2062	4.7112

In figures 1 and 2 the parametric effects of Schmidt number *Sc* and first order chemical reaction *F* on the non-dimensional species concentration ϕ for different values of normal distance η have been depicted for a set of fixed values of *Sr*= 0.5, *M*=0.5, *K*=0.5, *R*=1.0, ωt= π/2, *t*=0.4, *F*=1.4, *Gr*=10.0, *Gm*=0.0 as well as *Sc*=0.22 (fig. 1) and *Pr* =0.71(fig. 2). It has been observed that the increasing effects of *Sc* and *F* are to decrease the concentration profiles, which is consistent with the fact that increase in both the values of *Sc* and *F* followed by a decrease of mass diffusivity D_M , which results in the decrease of concentration boundary layer thereby reducing the species concentration. Hence concentration of the species is higher for small values of *Sc* and *F*, lower for larger values of *Sc* as well as *F*.

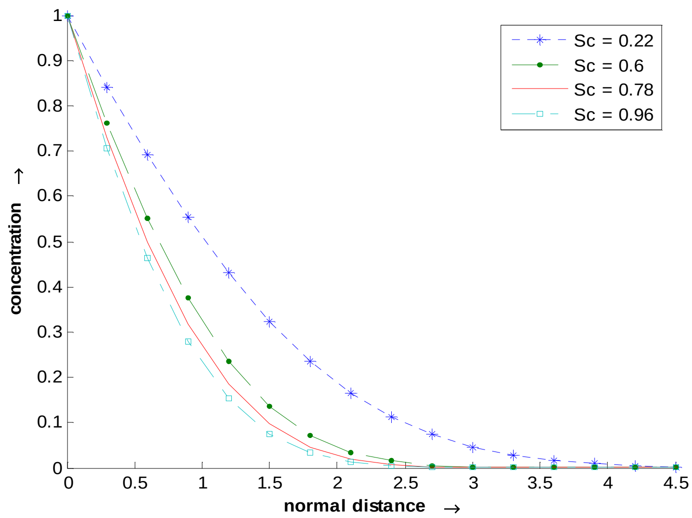

Figure1. Graph of Species concentration (ϕ) against Schmidt number (*Sc*)

Figures3 and 4 demonstrate how the rate of flow in absence of mass buoyancy parameter *Gm* and thermal buoyancy parameter *Gr* are affected by the change of phase angle ωt against normal distance η for an arbitrary set of values of *Pr* =0.71, *Sr* = 0.5, *M*=0.5, *K*=0.5, *R*=1.0, *Sc*=0.22, *t*=0.4, *F*=1.4. It is seen that as the phase angle decreases, the influence of the buoyancy forces on the flow increases, results of which increases the flow rate and thus accelerate the fluid velocity *u*.

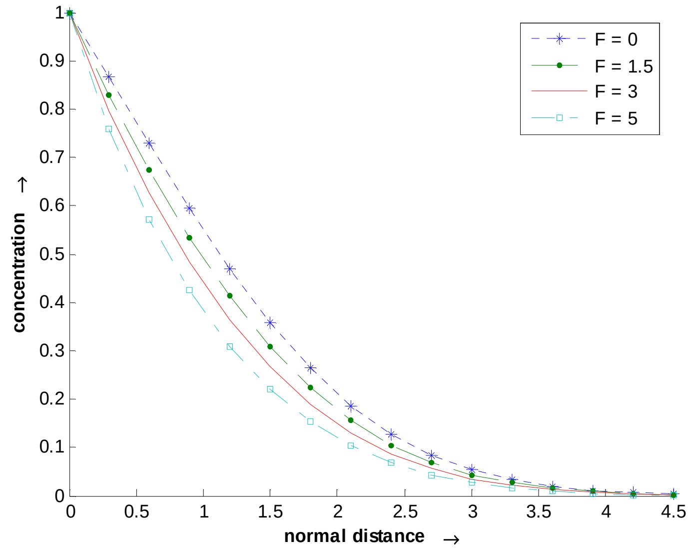

Figure2. Graph of Species concentration (ϕ) against chemical reaction parameter (F)

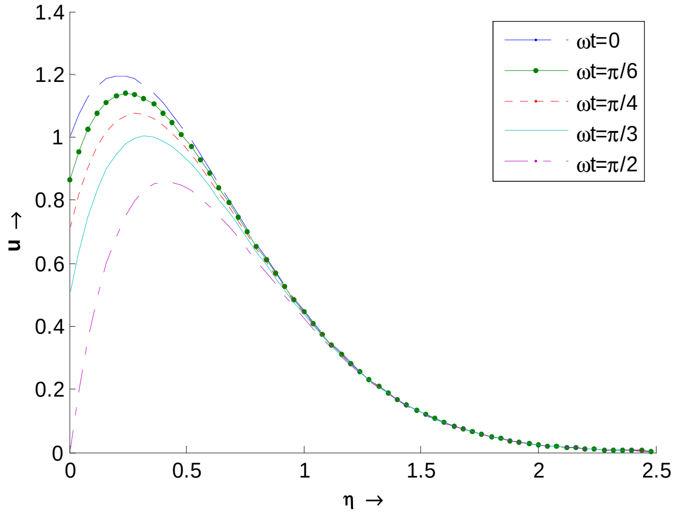

Figure3. Graph of Velocity (u) against phase angle for Gr=10.0, Gm=0.0

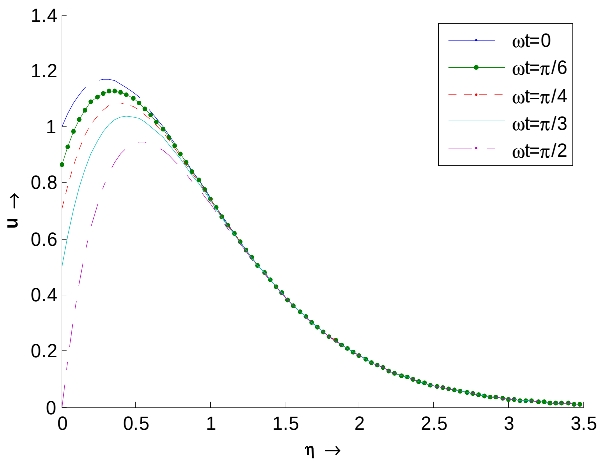

Figure4. Graph of Velocity (u) against phase angle for Gr=0.0, Gm=10.0

The parametric effects of Prandtl number *Pr* and thermal radiation parameter *R* on fluid velocity *u* for various values of η are shown in figures 5 and 6 for a set of fixed values of *Sr* = 0.5, *Sc*=0.22, *M*=0.5, *K*=0.5, *R*=1.0 (fig. 5), *Pr*=0.71(fig. 6), *t*=0.4, *F*=1.4, *Gr*=5.0, *Gm*=2.0, ωt= π/2. The presence of both *Pr* and *R* is seen to retard the motion. Due to the increase in values of *Pr*, viscous forces become prominent which results in the decrease of flow rate thus reducing the value of *u*. The thermal radiation parameter*R* reduces the thermal buoyancy force by minimizing the thickness of the thermal boundary layer, which results in decreasing the velocity of the flow.

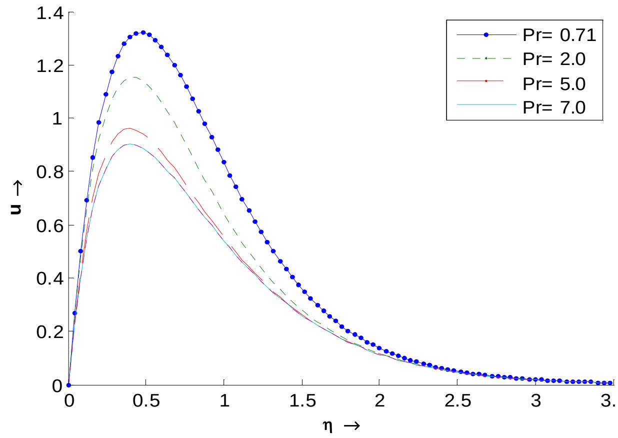

Figure5. Graph of Velocity (*u*) against Prandtl number (*Pr*)

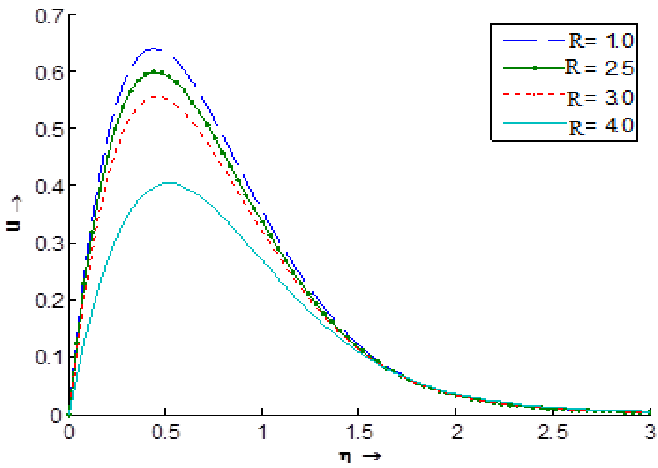

Figure6. Graph of Velocity (*u*) against thermal radiation parameter (*R*)

The schematic representation of the effects of Soret number *Sr*, Schmidt number *Sc*, chemical reaction parameter *F* and magnetic parameter *M* on the non-dimensional velocity *u* for different values of η have been shown in figures 7 to 10 for fixed values of *Pr* =0.71, *R*=1.0,

M=0.5, *K*=0.5, ωt= π/2, *Sc*=0.22, *t*=0.4, *F*=1.4, *Gr*=10.0, *Gm*=5.0 as well as Sr=0.5(figs 8-10), *Sc* =0.22 (figs 7, 9, 10), F=1.4(figs 7, 8, 10) M=0.5(figs 7- 9). It is observed from these figures that the velocity of the fluid gets accelerated by the rise in values of Soret number, but retarded due to increase in values of Schmidt number, magnetic and chemical reaction parameters. The intensification in values of *Sr* produces a raise in the mass buoyancy force which results an increase in the value of *u*.

Whereas, due to enhancement in the values of *Sc* and *F*, the concentration near the plate gets decline which results in reducing the thickness of the concentration boundary layer, thereby minimizing the mass buoyancy force and affecting the flow rate by diminishing the value of u.

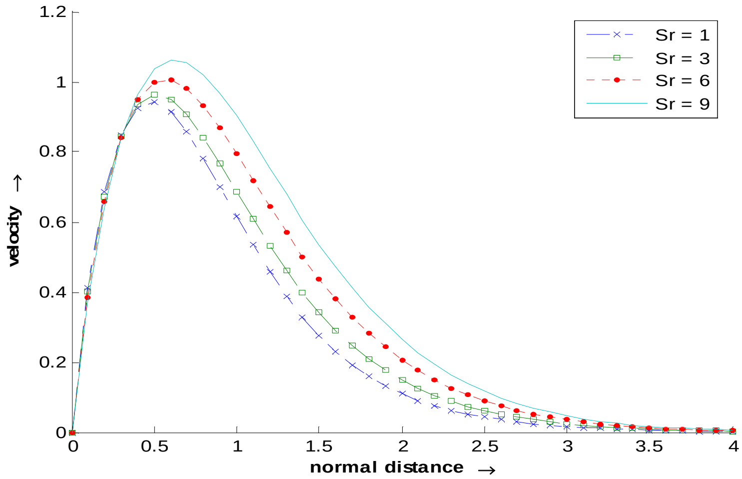

Figure7. Graph of Velocity (*u*) against Soret number (*Sr*)

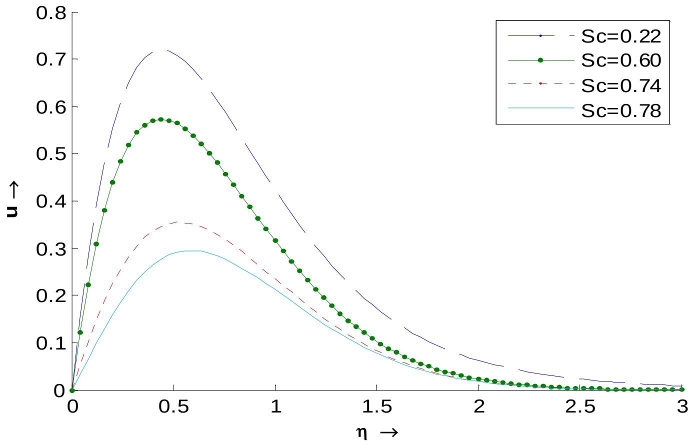

Figure8. Graph of Velocity (*u*) against Schmidt number (*Sc*)

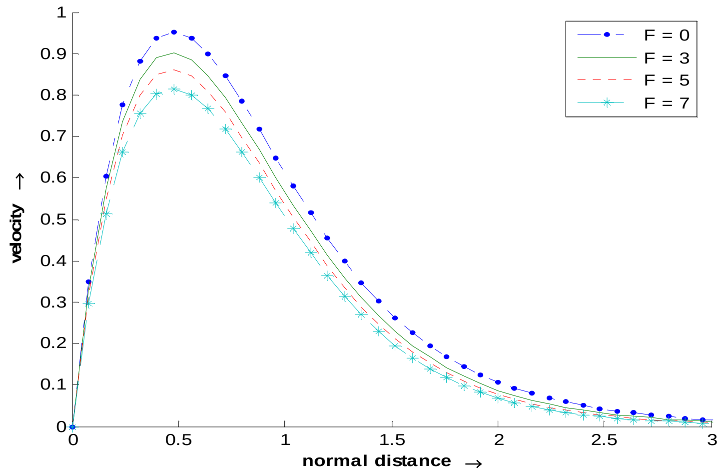

Figure9. Graph of Velocity (*u*) against chemical reaction parameter (*F*)

The presence of magnetic parameter M genarates an electric field in the flow, which produces a resistive force in terms of Lorentz force,thus results in decelerate the flow rate and minimize the value of u.

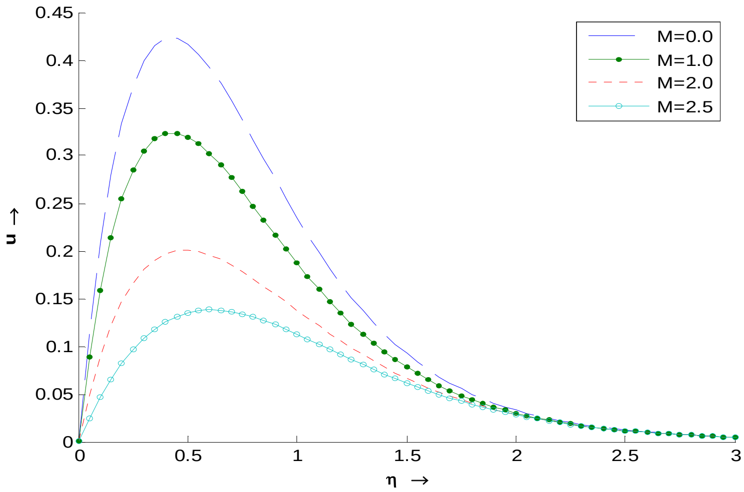

Figure10. Graph of Velocity (*u*) against magnetic parameter (*M*)

Figures11 and12, show graphically the parametric effects of Soret number *Sr* and chemical reaction parameter *F* on the skin – friction τ against time *t* for a set of fixed values of *Pr* =0.71, *R*=1.0, *M*=0.5, *K*=0.5, $\omega t = \pi/2$, *Sc*=0.22, *Gr*=5.0, *Gm*=2.0, ω= 7.857, as well as *Sr*=0.5 (fig.12) and *F*=1.0(fig.11). It is observed that initially when $t \in (0, 0.53]$, the value of τ increases as *Sr* increases, which results in raising the frictional effect and thereafter when $t > 0.53$, the value of τ diminishes i.e the plate friction decreases. It is also observed that, the presence of *F* minimizes the effect of viscous drag and the effect of viscous drag diminishes as time progresses.

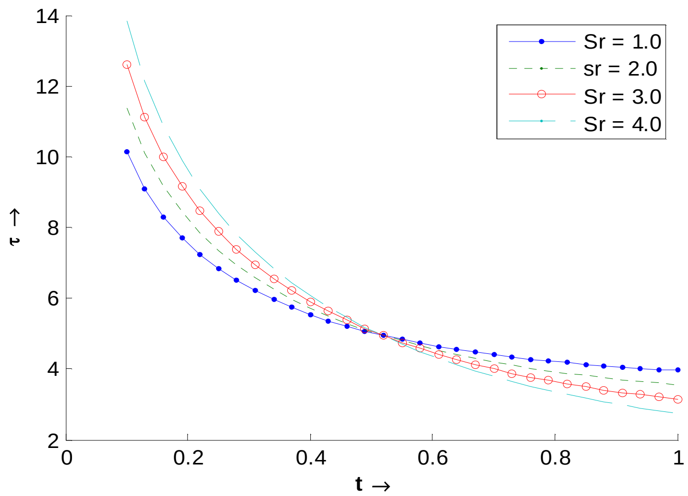

Figure11. Graph of Skin-friction (τ) against Soret number (*Sr*)

The influence of Soret number *Sr* on the mass transfer rate quantified by Sherwood number *Sh* against time *t* is demonstrated in figure13 for fixed values of *Pr* =0.71, *R*=1.0, *F*=1.0, *Sc*=0.22. The presence of *Sr* enhence the rate of interfacial mass flux from the plate towards the fluid region, which results in increasing the values of *Sh* and thus accelerates the mass transfer rate.

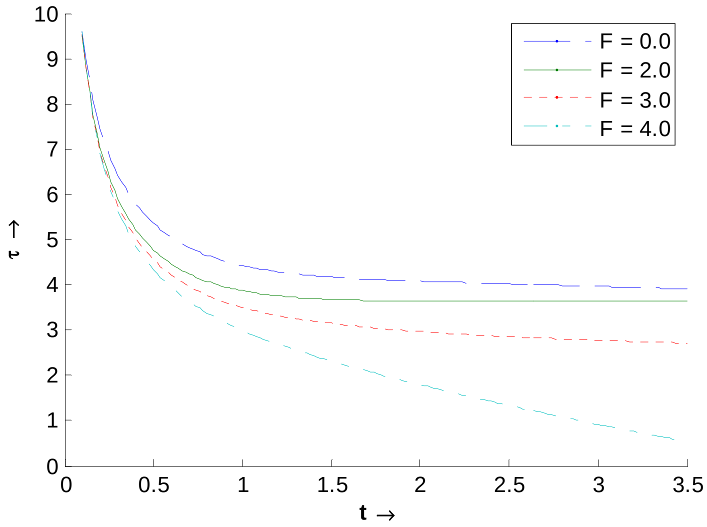

Figure12. Graph of Skin-friction (τ) against chemical reaction parameter (*F*)

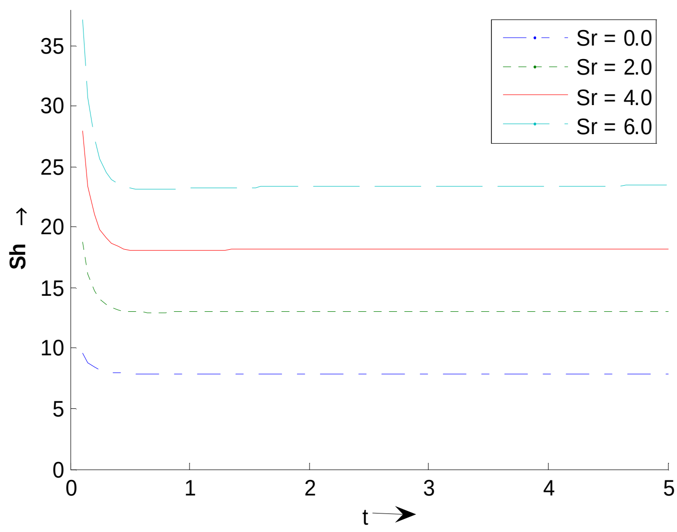

Figure13. Graph of Sherwood number (*Sh*) against Soret number (*Sr*)

Comparison of Results:

To achieve accuracy of the present work, we compare some of the results of the present paper with those available in the literature. As the present paper deals with one - dimensional MHD free convective mass transfer flow through an oscillating plate in presence of thermal radiation, first order chemical reaction and thermal diffusion, the works of Manivannan et al. [23] and Muthucumaraswamy et al. [28] have been chosen for comparison with the present paper. The work of Manivannan et al. [23] concerns with radiation and chemical reaction effects on isothermal vertical oscillating plate with variable mass diffusion, whereas Muthucumaraswamy et al. [28] investigated thermal radiation and MHD effects on flow past a vertical oscillating plate with chemical reaction of first order. The following observations have been made:

In both figure 14 (Manivannan et al. [23]) and figure 15 (present paper) with the fixed parametric values of Sr = 0, M_1 = 0, Sc= 0.6, F=2.0, R=5.0, ωt = π/4, Gr= 5.0, Gm=5.0, Pr = 0.71, it is clearly seen that as time progresses, the fluid velocity increases, attains its maximum value near the plate and steadily declining far away from the plate, indicating buoyancy effects near the plate.

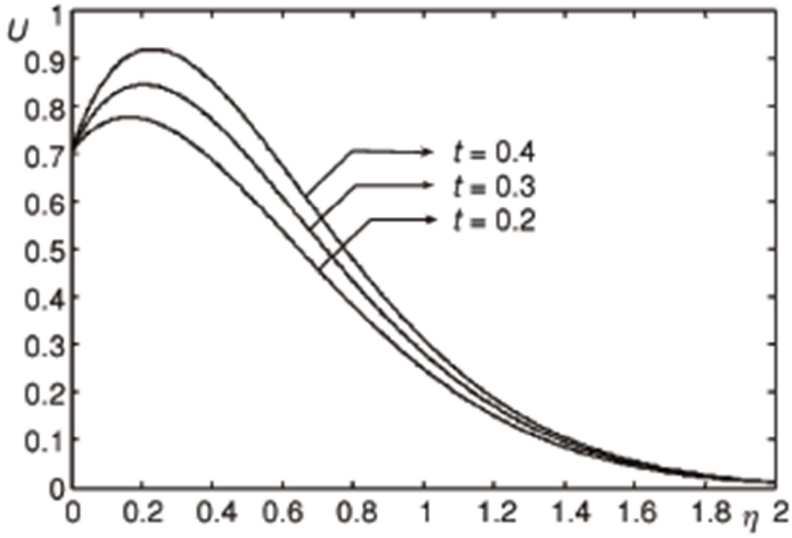

Figure14.Velocity profiles Vs normal distances for different values of time considered by Manivannan et al. [23]

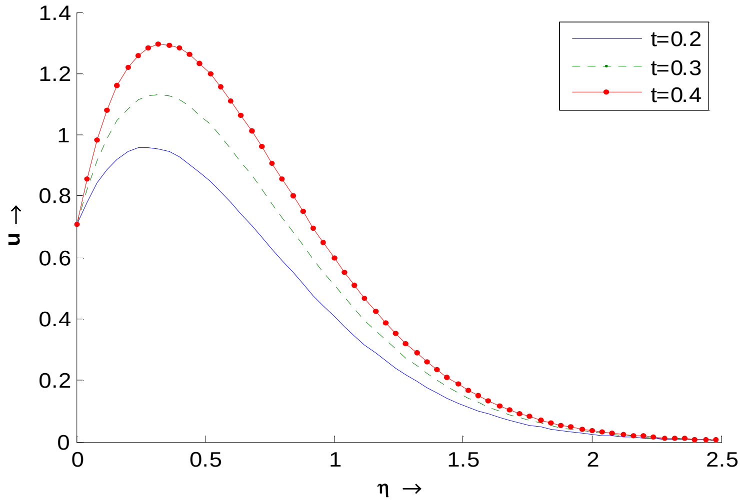

Figure15. Velocity profiles (*u*) Vs normal distances for different values of time (*t*) for the present paper

The figure16 (Muthucumaraswamy et al. [28]) and figure 17(present paper) for a set of fixed values of Sc= 0.6, R=10.0,$\omega t = \pi/4$, Gr= 2.0, Gm=2.0, Pr= 0.71, Sr=0.0, M=0.2, $K \rightarrow \infty$, t=0.6 shows that due to increase in first order chemical reaction parameter (F), the flow rate is retarded, thereby reducing the velocity of the flow, indicating decrease in mass buoyancy effect near the plate.

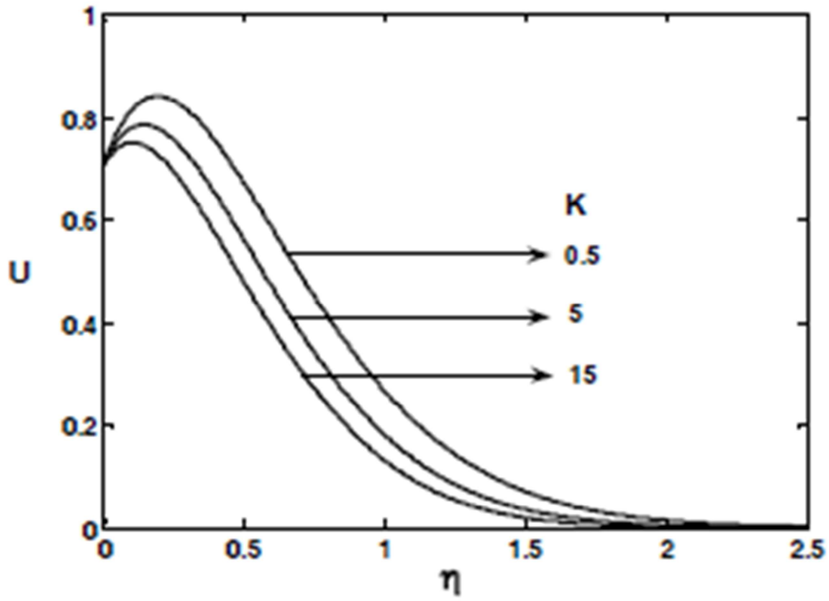

Figure 16.Velocity profiles Vs normal distances for different values of time considered by Muthucumaraswamy et al. [28]

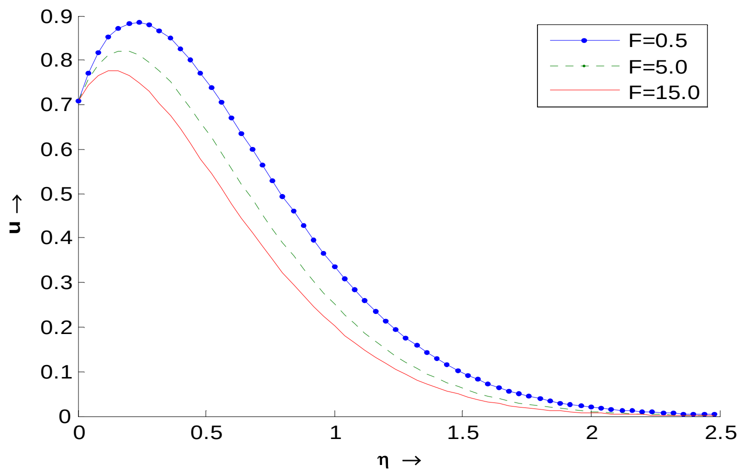

Figure17. Velocity profiles (*u*) Vs normal distances against various values of time (*t*) for the present paper

Hence, a reasonably good agreement of the results obtained by Manivannan et al. [23] and Muthucumaraswamy et al. [28]with that of the present work are established.

CONCLUSION

A theoretical analysis is performed to study the influence of thermal diffusion, thermal radiation and first-order chemical reaction on unsteady MHD free convective flow past a vertical oscillating plate immersed in porous media. One-sided Laplace transform method is applied to solve the governing system of partial differential equations in closed form. The following conclusions can be drawn from the study as:

- Increase in phase angle and Prandtl number retards the flow.
- The fluid velocity is found to accelerate due to increase in Soret number, whereas reverse phenomena are observed under the influence of Schmidt number, magnetic and thermal radiation parameters.
- The thermal diffusion (Soret) causes the skin- friction to rise in a small interval of time and thereafter decreases as time propagates.
- The rate of mass transfer accelerates in presence of Soret number and becomes steady as time propagates.
- Presence of chemical reaction reduces the fluid velocity, plate friction and fluid concentration, while enhances the mass transfer rate. So, it is possible to control the flow rate as well as the surface friction through chemical reaction.

Acknowledgement
The authors are thankful to the reviewers for their constructive suggestions.

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Appendices

$$G = \frac{R - FSc}{\Pr - Sc}, H = \frac{R - M_1}{\Pr - 1}, \ I = \frac{F - M_1}{Sc - 1}, \ A_{11} = \exp\left(2\eta\sqrt{(M_1 + i\omega)t}\right) erfc\left(\eta + \sqrt{(M_1 + i\omega)t}\right)$$

$$A_{12} = \exp\left(-2\eta\sqrt{(M_1 + i\omega)t}\right) erfc\left(\eta - \sqrt{(M_1 + i\omega)t}\right) \ A_{13} = \exp\left(2\eta\sqrt{(M_1 - i\omega)t}\right) erfc\left(\eta + \sqrt{(M_1 - i\omega)t}\right)$$

$$A_{14} = \exp\left(-2\eta\sqrt{(M_1 - i\omega)t}\right) erfc\left(\eta - \sqrt{(M_1 - i\omega)t}\right), A_{15} = A_{11} + A_{12}, A_{16} = A_{13} + A_{14},$$

$$A_{17} = \exp\left(2\eta\sqrt{M_1 t}\right) erfc\left(\eta + \sqrt{M_1 t}\right) \ A_{18} = \exp\left(-2\eta\sqrt{M_1 t}\right) erfc\left(\eta - \sqrt{M_1 t}\right), \ A_{19} = A_{17} + A_{18}$$

$$A_{20} = \exp\left(2\eta\sqrt{Rt}\right) erfc\left(\eta\sqrt{\Pr} + \sqrt{\frac{Rt}{\Pr}}\right) \ A_{21} = \exp\left(-2\eta\sqrt{Rt}\right) erfc\left(\eta\sqrt{\Pr} - \sqrt{\frac{Rt}{\Pr}}\right), A_{22} = A_{20} + A_{21}$$

$$A_{23} = \exp\left(2\eta\sqrt{(M_1 - H)t}\right) erfc\left(\eta + \sqrt{(M_1 - H)t}\right) \ A_{24} = \exp\left(-2\eta\sqrt{(M_1 - H)t}\right) erfc\left(\eta - \sqrt{(M_1 - H)t}\right)$$

$$A_{25} = \exp(-Ht)(A_{23} + A_{24}) \ A_{26} = \exp\left(2\eta\sqrt{(R - H\Pr)t}\right) erfc\left(\eta\sqrt{\Pr} + \sqrt{\frac{(R - H\Pr)t}{\Pr}}\right)$$

$$A_{27} = \exp\left(-2\eta\sqrt{(R - H\Pr)t}\right) erfc\left(\eta\sqrt{\Pr} - \sqrt{\frac{(R - H\Pr)t}{\Pr}}\right), A_{28} = A_{26} + A_{27}, A_{29} = \exp(-Ht)A_{28},$$

$$A_{30} = \frac{1}{H}(A_{19} - A_{22} - A_{25} + A_{29}), A_{31} = \exp\left(2\eta\sqrt{(M_1 - I)t}\right) erfc\left(\eta + \sqrt{(M_1 - I)t}\right)$$

$$A_{32} = \exp\left(-2\eta\sqrt{(M_1 - I)t}\right) erfc\left(\eta - \sqrt{(M_1 - I)t}\right), A_{33} = \exp(-It)(A_{31} + A_{32}),$$

$A_{34} = \frac{1}{I}(A_{19} - A_{33})$, $A_{35} = \exp\left(2\eta\sqrt{FSct}\right)erfc\left(\eta\sqrt{Sc} + \sqrt{Ft}\right)$

$A_{36} = \exp\left(-2\eta\sqrt{FSct}\right)erfc\left(\eta\sqrt{Sc} - \sqrt{Ft}\right)$, $A_{37} = A_{35} + A_{36}$,

$A_{38} = \exp\left(2\eta\sqrt{(F-I)Sct}\right)erfc\left(\eta\sqrt{Sc} + \sqrt{(F-I)t}\right)$ $A_{39} = \exp\left(-2\eta\sqrt{(F-I)Sct}\right)erfc\left(\eta\sqrt{Sc} - \sqrt{(F-I)t}\right)$,

$A_{40} = \exp(-It)(A_{38} + A_{39})$, $A_{41} = \frac{1}{2I}(A_{37} - A_{40})$, $A_{42} = A_{34} - A_{41}$

$A_{43} = \exp\left(2\eta\sqrt{(M_1 - G)t}\right)erfc\left(\eta + \sqrt{(M_1 - G)t}\right)$, $A_{44} = \exp\left(-2\eta\sqrt{(M_1 - G)t}\right)erfc\left(\eta - \sqrt{(M_1 - G)t}\right)$

$A_{45} = \exp(-Gt)(A_{43} + A_{44})$, $A_{46} = \frac{1}{I(G-I)}(A_{19} - A_{33})$, $A_{47} = \frac{1}{G(G-I)}(A_{19} - A_{45})$,

$A_{48} = A_{46} - A_{47}$, $A_{49} = \exp\left(2\eta\sqrt{(F-G)Sct}\right)erfc\left(\eta\sqrt{Sc} + \sqrt{(F-G)t}\right)$

$A_{50} = \exp\left(-2\eta\sqrt{(F-G)Sct}\right)erfc\left(\eta\sqrt{Sc} - \sqrt{(F-G)t}\right)$, $A_{51} = \exp(-Gt)(A_{49} + A_{50})$

$A_{52} = \frac{1}{I(G-I)}(A_{37} - A_{40})$, $A_{53} = \frac{1}{G(G-I)}(A_{37} - A_{51})$, $A_{54} = A_{52} - A_{53}$

$A_{55} = \exp\left(2\eta\sqrt{(R-G\Pr)t}\right)erfc\left(\eta\sqrt{\Pr} + \sqrt{\frac{(R-G\Pr)t}{\Pr}}\right)$ $A_{56} = \exp\left(-2\eta\sqrt{(R-G\Pr)t}\right)erfc\left(\eta\sqrt{\Pr} - \sqrt{\frac{(R-G\Pr)t}{\Pr}}\right)$

$A_{57} = \exp(-Gt)(A_{55} + A_{56})$, $A_{58} = \frac{1}{H(G-H)}(A_{22} - A_{29})$, $A_{59} = \frac{1}{G(G-H)}(A_{22} - A_{57})$,

$A_{60} = A_{58} - A_{59}$, $A_{61} = \frac{1}{H(G-H)}(A_{19} - A_{25})$, $A_{62} = \frac{1}{G(G-H)}(A_{19} - A_{45})$, $A_{63} = A_{61} - A_{62}$,

$A_{64} = \frac{1}{(G-I)}(A_{33} - A_{45})$, $A_{65} = \frac{1}{(G-I)}(A_{40} - A_{51})$, $A_{66} = \frac{1}{(G-H)}(A_{29} - A_{57})$, $A_{67} = \frac{1}{(G-H)}(A_{25} - A_{45})$

, $A_{68} = \frac{y}{t\sqrt{\pi t}}\exp\left(-\left(M_1 t + \frac{y^2}{4t}\right)\right)$, $A_{69} = \frac{y\sqrt{Sc}}{t\sqrt{\pi t}}\exp\left(-\left(Ft + \frac{y^2 Sc}{4t}\right)\right)$

$A_{70} = \frac{y\sqrt{\Pr}}{t\sqrt{\pi t}}\exp\left(-\left(\frac{Rt}{\Pr} + \frac{y^2\Pr}{4t}\right)\right)$, $B_1 = -\left(\sqrt{R}erf\left(\sqrt{\frac{Rt}{\Pr}}\right) + \sqrt{\frac{\Pr}{\pi t}}\exp\left(-\frac{Rt}{\Pr}\right)\right)$

$B_2 = -\left(\sqrt{FSc}\ erf\left(\sqrt{Ft}\right) + \sqrt{\frac{Sc}{\pi t}}\exp(-Ft)\right)$, $B_3 = -\exp(-Gt)\left(\sqrt{(F-G)Sc}\ erf\left(\sqrt{(F-G)t}\right) + \sqrt{\frac{Sc}{\pi t}}\exp(-(F-G)t)\right)$,

$B_4 = -\exp(-Gt)\left(\sqrt{R-G\Pr}\ erf\left(\sqrt{\frac{(R-G\Pr)t}{\Pr}}\right) + \sqrt{\frac{\Pr}{\pi t}}\exp\left(-\frac{(R-G\Pr)t}{\Pr}\right)\right)$, $B_5 = B_1 - B_2$, $B_6 = B_4 - B_3$, $B_7 = B_5 - B_6$,

$B_8 = \frac{\sqrt{\Pr}}{2t\sqrt{\pi t}}\exp\left(-\frac{Rt}{\Pr}\right)$, $B_9 = \frac{\sqrt{Sc}}{2t\sqrt{\pi t}}\exp(-Ft)$, $B_{10} = -\left(\sqrt{M_1 + i\omega}\,erf\left(\sqrt{(M_1 + i\omega)t}\right) + \frac{1}{\sqrt{\pi t}}\exp(-(M_1 + i\omega)t)\right)$,

$B_{11} = -\left(\sqrt{M_1 - i\omega}\ erf\left(\sqrt{(M_1 - i\omega)t}\right) + \frac{1}{\sqrt{\pi t}}\exp(-(M_1 - i\omega)t)\right)$, $B_{12} = -\left(\sqrt{M_1}erf\left(\sqrt{M_1 t}\right) + \frac{1}{\sqrt{\pi t}}\exp(-M_1 t)\right)$,

$B_{13} = -\exp(-Ht)\left(\sqrt{(M_1 - H)}\ erf\left(\sqrt{(M_1 - H)t}\right) + \frac{1}{\sqrt{\pi t}}\exp(-(M_1 - H)t)\right)$ $B_{14} = -\exp(-Ht)\left(\sqrt{R - H\Pr}\ erf\left(\sqrt{\frac{(R-H\Pr)t}{\Pr}}\right) + \sqrt{\frac{\Pr}{\pi t}}\exp\left(-\frac{(R-H\Pr)t}{\Pr}\right)\right)$

$B_{15} = -\exp(-It)\left(\sqrt{(M_1 - I)}\ erf\left(\sqrt{(M_1 - I)t}\right) + \frac{1}{\sqrt{\pi t}}\exp(-(M_1 - I)t)\right)$ $B_{16} = -\exp(-It)\left(\sqrt{(F-I)Sc}\ erf\left(\sqrt{(F-I)t}\right) + \sqrt{\frac{Sc}{\pi t}}\exp(-(F-I)t)\right)$,

$B_{17} = -\exp(-Gt)\left(\sqrt{M_1 - G}\,erf\left(\sqrt{(M_1 - G)t}\right) + \frac{1}{\sqrt{\pi t}}\exp(-(M_1 - G)t)\right)$, $B_{18} = \frac{1}{H}(B_{12} - B_1 - B_{13} + B_{14})$, $B_{19} = \frac{1}{I}(B_{12} - B_{15} - B_2 + B_{16})$,

$B_{20} = \frac{1}{I(G-I)}(B_{12} - B_{15} - B_2 + B_{16})$, $B_{21} = \frac{1}{G(G-I)}(B_{17} - B_{12} + B_2 - B_3)$, $B_{22} = \frac{1}{H(G-H)}(B_1 - B_{14} - B_{12} + B_{13})$,

$B_{23} = \frac{1}{G(G-H)}(B_{12} - B_1 + B_4 - B_{17})$, $B_{24} = \frac{1}{G-I}(B_3 + B_{15} - B_{16} - B_{17})$, $B_{25} = \frac{1}{G-H}(B_{14} - B_4 - B_{13} + B_{17})$,

$B_{26} = \frac{1}{t\sqrt{\pi t}} \exp(-M_1 t)$ $B_{27} = \frac{1}{t\sqrt{\pi t}} \exp(-Ft)$, $B_{28} = \frac{1}{t\sqrt{\pi t}} \exp(-Rt)$, $B_{29} = B_{15} - B_{16}$, $B_{30} = B_{14} - B_{13}$.