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Chemical reaction and radiation effects on MHD free convection flow along a streching surface with viscous dissipation and heat generation

S Mohammed Ibrahim

Department of Mathematics, Priyadarshini college of Engineering and Technology, Nellore-524004, A.P., India.

ABSTRACT

The effects of chemical reaction and radiation on a steady two-dimensional flow of a viscous incompressible electrically conducting fluid near an isothermal vertical stretching sheet, in the presence of viscous dissipation and heat generation are studied. The governing equations are transformed by using similarity transformation and the resultant dimensionless equations are solved numerically using the Runge – Kutta fourth order method with shooting technique. The effects of various governing parameters on the velocity, temperature, concentration, skin-friction coefficient, Nusselt number and Sherwood number are computed and discussed in detail. Comparison of the obtained numerical results is made with previously published results. It is found that, the velocity and concentration decreases with increasing the Schmidt number (Sc), chemical reaction parameter (K_r), and the velocity as well as temperature decreases with increasing the radiation parameter (F), while the velocity as well as temperature increases with increasing the heat generation parameter (Q).

Keywords: chemical reaction; MHD; Radiation; Free convection; Steady flow; viscous dissipation; heat generation.

INTRODUCTION

Sakiadas [1] first presented boundary layer flow over a continuous solid surface moving with constant speed. Erickson et al.[2] extended Sakiadas problem to include blowing or suction at the moving surface and investigated its effects on the heat and mass transfer in the boundary layer. Danberg and Fansber [3] investigated the nonsimilar solution for the flow in the boundary layer past a wall i.e. stretched with a velocity proportional to distant along the wall. Gupta and Gupta [4] studied the heat and mass transfer corresponding to similarity solution for the boundary layer over an isothermal stretching sheet subject to blowing or suction. Elbashbeshy [5] investigated heat transfer over a stretching surface with variable and uniform surface heat flux subject to injection and suction. Samad and Mobebujjaman [6] reported the MHD heat and mass transfer free convection flow along a vertical stretching sheet in the presence of magnetic field with heat generation.

At high operating temperatures, radiation effect can be quite significant. Many processes in engineering areas occur at high temperatures and knowledge of radiation heat transfer becomes very important for the design of the pertinent equipment. Nuclear power plants, gas turbines and various propulsion devices for aircraft, missiles, satellites and space vehicles are example of such engineering areas. Takhar et al. [7] studied the radiation effects on MHD free convection flow for nongray-gas past semi-infinite vertical plate. Ghaly and Elbarbary [8] reported the effect of radiation on free convection flow on MHD along a stretching surface with uniform free stream. Anjali Devi and Kayalvizhi [9] presented analytical solution of MHD flow with radiation over a stretching sheet embedded in a porous medium.

In all the studies mentioned above the heat due to viscous dissipation is neglected. Gebharat [10] has shown the importance of viscous dissipative heat in free convection flow in the case of isothermal and constant heat flux at the plate. Israel-Cookey et al [11] investigated the influence of viscous dissipation and radiation on unsteady MHD free convection flow past an infinite heated vertical plate in a porous medium with time dependent suction. Suneetha et al [12] investigated radiation and mass transfer effects on MHD free convection flow past an impulsively started isothermal vertical plate with viscous dissipation. Ganeswara Reddy and Bhaskar Reddy [13] presented soret and dufour effects on steady MHD free convection flow past a semi-infinite moving vertical plate in a porous medium with viscous dissipation. Mohammed Ibrahim and Bhaskar Reddy [14] studied the radiation and mass transfer effects on MHD free convection flow along a stretching surface with viscous dissipation and heat generation.

The heat source/sink effects in thermal convection, are significant where there may exist a high temperature differences between the surface (e.g. space craft body) and the ambient fluid. Heat Generation is also important in the context of exothermic or endothermic chemical reaction. Vajravelu and Hadjinicolaou [15] studied the heat transfer characteristics in the laminar boundary layer of a viscous fluid over a stretching sheet with viscous dissipation or frictional heating and internal heat generation. Hossain et al [16] studied problem of the natural convection flow along a vertical wavy surface with uniform surface temperature in the presence of heat generation/absorption. Kesavaiah et.al [17] reported that the effects of the chemical reaction and radiation absorption on unsteady MHD convective heat and mass transfer flow past a semi-infinite vertical permeable moving plate embedded in porous medium with heat source and suction.

Combined heat and mass transfer problems with chemical reaction are of importance in many processes and have, therefore, received a considerable amount of attention in recent years. In processes such as drying, evaporation at the surface of a water body, energy transfer in a set cooling tower and the flow in a desert cooler, heat and the mass transfer occur simultaneously. Possible applications of this type of flow can be found in many industries, For example, in the power industry, among the methods of generating electric power is one in which electrical energy is extracted directly from a moving conducting fluid. Many practical diffusive operations involve the molecular diffusion of a species in the presence of chemical reaction within or at the boundary. There are two types of reactions; A homogeneous reaction is one that occurs uniformly throughout a give phase. The species generation in a homogeneous reaction is analogous to internal source of heat generation. In contrast, A heterogeneous reaction takes place in a restricted region or within the boundary of a phase. It can therefore be treated as a boundary condition similar to the constant heat flux condition in heat transfer. So the study of heat and mass transfer with chemical reaction is of great practical importance to engineers and scientists because of its almost universal occurrence in many branches of science and engineering. The chemical reactive species in a laminar boundary layer flow over a flat plate was demonstrated by Chambre and Young [18]. The effect of transfer of chemically reactive species in the laminar flow over a stretching sheet explained by Andersson et al. [19]. Afify [20] explicated the MHD free convective flow of viscous incompressible fluid and mass transfer over a stretching sheet with chemical reaction. Liu [21] studied the momentum, heat and mass transfer of a hydromagnetic flow past a stretching sheet in the presence of uniform transverse magnetic field. Sudhakar Reddy et al. [22], Raja sekhar et al. [23], Kishan and Srinivas [24], Anjalidevi and David [25] and Kishan and Deepa [26] investigated the effects of various parameters on fluid flow quantities.

However the interaction of chemical reaction and radiation effects of an electrically conducting and diffusing fluid past a stretching surface has received little attention. Hence an attempt is made to investigate the radiation effects on a steady free convection flow near an isothermal vertical stretching sheet in the presence of a magnetic field, heat generation/absorption, viscous dissipation and chemical reaction. The governing equations are transformed by using similarity transformation and the resultant dimensionless equations are solved numerically using the Runge-Kutta fourth order method with shooting technique. The effects of various governing parameters on the velocity, temperature, concentration, skin-friction coefficient, Nusselt number and Sherwood number are shown in figures and tables and analyzed in detail.

Mathematical Analysis

A steady two-dimensional free convection flow of a viscous incompressible, electrically conducting, radiating and dissipating fluid adjacent to a vertical sheet with mass transfer and heat generation is considered. The flow is assumed to be in the direction of x'-axis, taken along the vertical plate and the y'- axis normal to the plate. Two equal and opposite forces are introduced along the x'-axis (see Figure A), so that the sheet is stretched keeping the origin fixed. The plate is maintained at a constant temperature T_w', which is higher than the constant temperature T_∞' of the surrounding fluid and a constant concentration C_w', which is greater than the constant concentration C_∞' of the surrounding fluid. A uniform magnetic field is applied in the direction perpendicular to the plate.

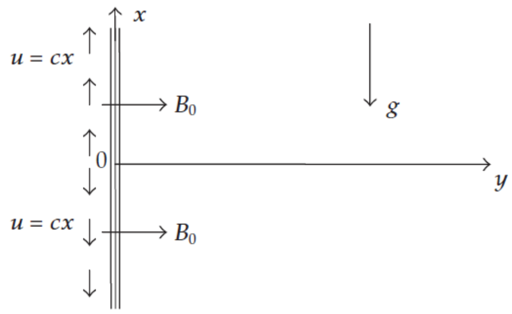

Figure A. Sketch of the physical model.

The fluid is assumed to be slightly conducting, and hence the magnetic field is negligible in comparison with the applied magnetic field. It is further assumed that there is no applied voltage, so that electric field is absent. The fluid is considered to be a gray, absorbing emitting radiation but non-scattering medium and the Rosseland approximation is used to describe the radiative heat flux in the energy equation. It is also assumed that all the fluid properties are constant except that of the influence of the density variation with temperature and concentration in the body force term (Boussinesq's approximation). Then, under the above assumptions, the governing boundary layer equations are

Continuity equation:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

Momentum equation:

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}-\frac{\sigma_0 B_0^2}{\rho}u+g\beta(T-T_\infty)+g\beta^*(C-C_\infty) \tag{2}$$

Energy equation:

$$\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2}-\frac{1}{\rho c_p}\frac{\partial q_r}{\partial y}+Q_0(T-T_\infty)+\frac{\mu}{\rho c_p}\left(\frac{\partial u}{\partial y}\right)^2 \tag{3}$$

Species equation:

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2}-k_r\left(C-C_\infty\right) \tag{4}$$

The boundary conditions for the velocity, temperature and concentration fields are

$$\begin{aligned} &u=cx,\ v=0,\ T=T_w,\ C=C_w \quad \text{at } y=0\\ &u\to u_\infty,\ T\to T_\infty,\ C\to C_\infty \quad \text{as } y\to\infty \end{aligned} \tag{5}$$

where $c>0$, and u_∞ is the free stream velocity.

By using the Rosseland approximation [27], we have

$$q_r=-\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y} \tag{6}$$

By using (6), the energy equation (3) becomes

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2}+\frac{4\sigma^*}{3k^*\rho c_p}\frac{\partial^2 T^4}{\partial y^2}+Q_0(T-T_\infty)+\frac{\mu}{\rho c_p}\left(\frac{\partial u}{\partial y}\right)^2 \tag{7}$$

In order to write the governing equations and the boundary conditions in dimensionless form, the following non-dimensional quantities are introduced.

$$\bar{x}=\frac{cx}{u_\infty}\ ,\quad \bar{y}=\frac{cy}{u_\infty}R\ ,\quad \bar{u}=\frac{u}{u_\infty}\ ,\quad \bar{v}=\frac{v}{u_\infty}R\ ,\ R=\frac{u_\infty}{\sqrt{c\nu}}\quad \theta=\frac{T-T_\infty}{T_w-T_\infty}\ ,$$
$$\phi=\frac{C-C_\infty}{C_w-C_\infty},\ M=\frac{\sigma_0 B_0^2}{\rho c},\ Gr=\frac{g\beta(T_w-T_\infty)}{cu_\infty},\ Gc=\frac{g\beta^*(C_w-C_\infty)}{cu_\infty}, \tag{8}$$
$$\Pr=\frac{\mu c_p}{k},\quad Q=\frac{Q_0}{c\rho c_p},\quad Ec=\frac{u^2_\infty}{c_p(T_w-T_\infty)},\ K_r=\frac{k_r}{c},\ Sc=\frac{\nu}{D},\quad F=\frac{kk^*}{4\sigma^* T_\infty^3},\ r=\frac{T_w-T_\infty}{T_\infty}$$

In view of the equation (8), the equations (1), (2), (4) and (7) reduce to the following non-dimensional form (with dropping the bars)

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{9}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{\partial^2 u}{\partial y^2}-Mu+Gr\theta+Gc\phi \tag{10}$$

$$u\frac{\partial\theta}{\partial x}+v\frac{\partial\theta}{\partial y}=\frac{1}{\Pr}\frac{\partial^2\theta}{\partial y^2}+\frac{4}{3F\Pr}\left((1+r\theta)^3\frac{\partial^2\theta}{\partial y^2}+3r(1+r\theta)^2\left(\frac{\partial\theta}{\partial y}\right)^2\right)+Q\theta+Ec\left(\frac{\partial u}{\partial y}\right)^2 \tag{11}$$

$$u\frac{\partial\phi}{\partial x}+v\frac{\partial\phi}{\partial y}=\frac{1}{Sc}\frac{\partial^2\phi}{\partial y^2}-K_r\phi \tag{12}$$

The corresponding boundary conditions are

$$u=x,\quad v=0\ ,\quad \theta=1\ ,\quad \phi=1\qquad \text{at}\quad y=0$$
$$u=1\ ,\quad \theta=0\ ,\quad \phi=0\quad \text{as}\quad y\to\infty \tag{13}$$

Introducing a dimensionless stream function ψ defined in the usual way

$$u=\frac{\partial\psi}{\partial\bar{y}},v=-\frac{\partial\psi}{\partial\bar{x}} \tag{14}$$

the continuity equation (9) is identically satisfied and the momentum equation (10), energy equation (11) and concentration equation (12) becomes

$$\frac{\partial\psi}{\partial y}\frac{\partial^2\psi}{\partial x\partial y}-\frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial y^2}=\frac{\partial^3\psi}{\partial y^3}-M\frac{\partial\psi}{\partial y}+Gr\theta+Gc\phi \tag{15}$$

$$\frac{\partial\psi}{\partial y}\frac{\partial\theta}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial\theta}{\partial y}=\frac{1}{\Pr}\frac{\partial^2\theta}{\partial y^2}+\frac{4}{3F\Pr}\left((1+r\theta)^3\frac{\partial^2\theta}{\partial y^2}+3r(1+r\theta)^2\left(\frac{\partial\theta}{\partial y}\right)^2\right)+Q\theta+Ec\left(\frac{\partial^2\psi}{\partial y^2}\right)^2 \tag{16}$$

$$\frac{\partial\psi}{\partial y}\frac{\partial\phi}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial\phi}{\partial y}=\frac{1}{Sc}\frac{\partial^2\phi}{\partial y^2}-K_r\phi \tag{17}$$

and the boundary conditions (13) become

$$\frac{\partial \psi}{\partial y} = x \ , \quad \frac{\partial \psi}{\partial x} = 0 \ , \quad \theta = 1 \ , \quad \phi = 1 \quad \text{at} \quad y = 0,$$

$$\frac{\partial \psi}{\partial y} \to 1 \ , \ \theta \to 0 \ , \ \phi \to 0 \qquad \text{as} \ y \to \infty \tag{18}$$

Introducing

$$\psi(x, y) = f(y) + xg(y) \ , \tag{19}$$

in equations (15), (16) and (17) and equating coefficient of x^0 and x^1, we obtain the coupled non-linear ordinary differential equations

$$f''' = f'g' - gf'' + Mf' - Gr\theta - Gc\phi \tag{20}$$

$$g''' = (g')^2 - gg'' + Mg' \tag{21}$$

$$(3F + 4(1+r\theta)^3)\theta'' + 3\Pr Fg\theta' + 12r(1+r\theta)^2\theta'^2 + 3F\Pr Q\theta + 3F\Pr Ecf''^2 = 0 \tag{22}$$

$$\phi'' + Scg\phi' - K_r Sc\phi = 0 \tag{23}$$

where a prime denotes differentiation with respect to y .
In view of (19), the boundary conditions (18) reduce to

$$f = 0 \ , \ f' = 0 \ , \ g = 0 \ , \ g' = 1 \ , \ \theta = 1 \ , \ \phi = 1 \quad \text{at} \quad y = 0$$

$$f' \to 1 \ , \ g' \to 0 \ , \ \theta \to 0 \ , \ \phi \to 0 \ \ \text{as} \ \ y \to \infty \tag{24}$$

The physical quantities which are of importance for this type problem are the skin friction coefficient, the Nusselt number and Sherwood number, which are defined by

$$\tau_w = \mu \left(\frac{\partial u'}{\partial y'} \right)_{y'=0} , \qquad Nu = \frac{q_w}{k(T'_w - T'_\infty)} \ , \ Sh = \frac{M_w}{D(C'_w - C'_\infty)} \tag{26}$$

where

$$q_w = -k \left(\frac{\partial T'}{\partial y'} \right)_{y'=0} \ , \ M_w = -D \left(\frac{\partial C'}{\partial y'} \right)_{y'=0} \tag{27}$$

Using (19), the quantities in (26) can be expressed as

$$\tau_w = \mu c R \left(\frac{\partial u}{\partial y} \right)_{y=0} = \mu c R[\, f''(0) + xg''(0)] \quad ,$$

$$Nu = \frac{cR}{u_\infty} \theta'(0) \ , \ \ Sh = \frac{cR}{u_\infty} \phi'(0) \ . \tag{28}$$

Solution of the Problem

The shooting method for linear equations is based on replacing the boundary value problem by two initial value problems and the solutions of the boundary value problem is a linear combination between the solutions of the two initial value problems. The shooting method for the non-linear boundary value problem is similar to the linear case, except that the solution of the non-linear problem cannot be simply expressed as a linear combination of the solutions of the two initial value problems. Instead, we need to use a sequence of suitable initial values for the derivatives such that the tolerance at the end point of the range is very small. This sequence of initial values is given by the secant method, and we use the fourth order Runge-Kutta method to solve the initial value problems.

Following Rosenhead [28] and Carnahan et al [29] , the value of *y* at infinity is fixed at 4. The full equations (20) - (23) with the boundary conditions (24) were solved numerically using Runge-Kutta method algorithm with a systematic guessing $f''(0), g''(0), h'(0)\theta'(0)$ and $\phi'(0)$ by the shooting technique until the boundary conditions at infinity $f'(y)$ decay exponentially to one, also $g'(y), h(y), \theta(y)$ and $\phi(y)$ to zero. The functions $f', g', -h$ θ and ϕ are shown in Figures.

RESULTS AND DISCUSSION

As a result of the numerical calculations, the dimensionless velocity, temperature and concentration distributions for the flow under consideration are obtained and their behavior have been discussed for variations in the governing parameters viz., the thermal Grashof number *Gr*, solutal Grashof number *Gc*, magnetic field parameter *M*, Radiation parameter *F*, the parameter of relative difference between the temperature of the sheet and temperature far away from the sheet *r*, Prandtl number *Pr*, Eckert number Ec, heat generation parameter *Q* and Schmidt number *Sc*. In the present study, the following default parametric values are adopted. *Gr* = 2.0, *Gc* = 2.0, *M* = 0.5, *Pr* = 0.71,F = 1.0, r = 0.05, *Q* = 0.1, K_r = 0.5, *Sc* = 0.6, *Ec*=0.01. All graphs therefore correspond to these unless specifically indicated on the appropriate graph.

In order to ascertain the accuracy of our numerical results, the present study is compared with the previous study. The velocity and temperature profiles are compared with available theoretical solution of Ghaly and Elbarbary (Ghaly and Elbarbary 2001) Radiation effects on MHD free convection flow of a gas at a stretching surface with a uniform free stream in Fig (a) and Fig (b). It is observed that the present results are in good agreement with that of Ghaly and Elbarbary.

Velocity components f' and g' as well as the temperature θ and concentration ϕ distribution are presented in Figs.1 - 12 for various values of governing thermo physical parameters.

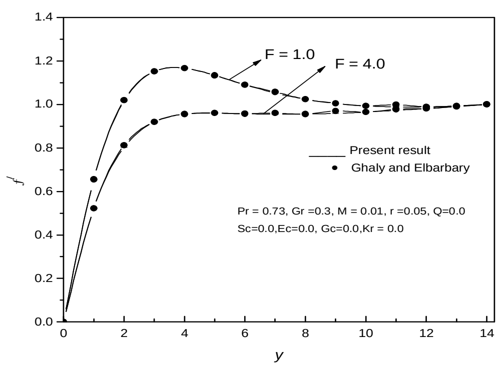

Fig.(a) comparison of the velocity component f' with *F*

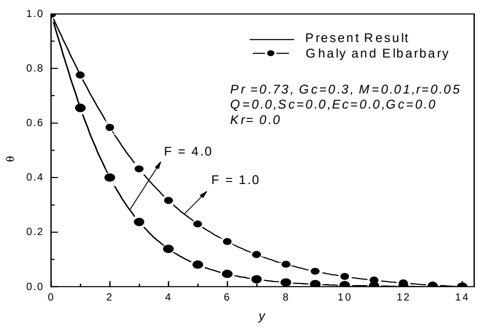

Fig.(b) comparison of the temperature with *F*

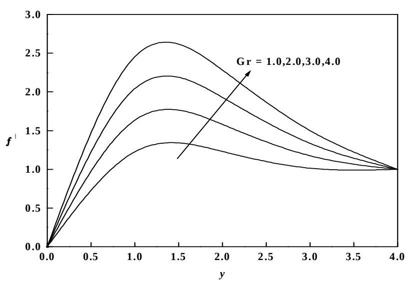

Fig.1(a) variation of the velocity component f' with *Gr*

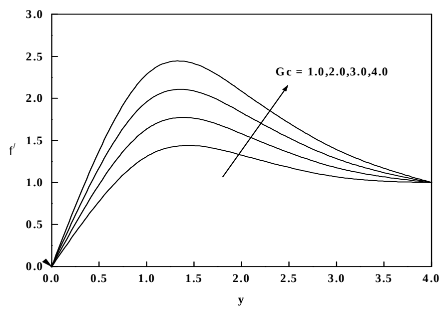

Fig.1(b) variation of the velocity component f' with *Gc*

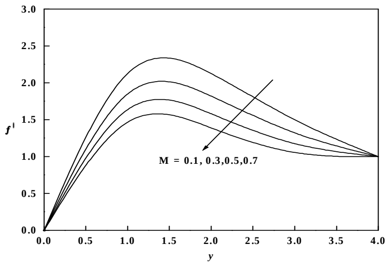

Fig.2(a) variation of the velocity component f' with *M*

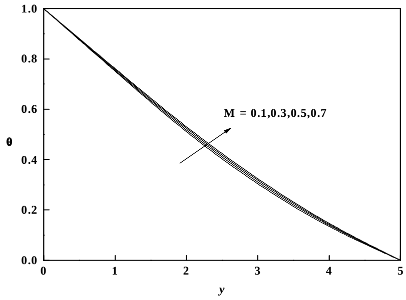

Fig.2(b) variation of temperature with *M*

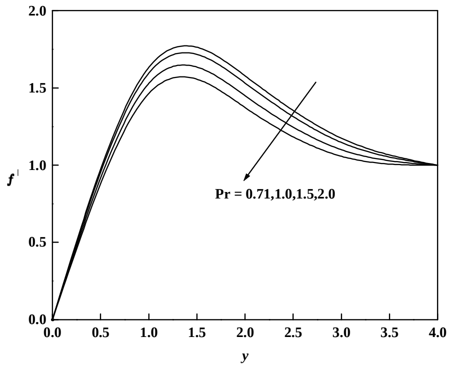

Fig.3(a) variation of the velocity component f' with *Pr*

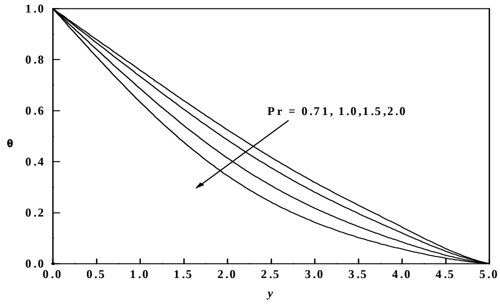

Fig.3(b) variation of the temperature with *Pr*

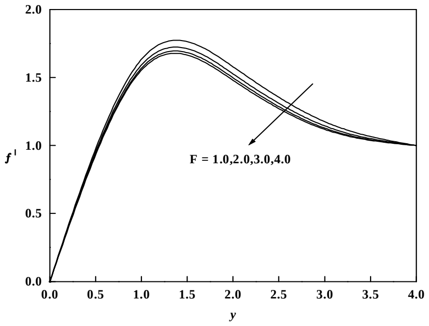

Fig.4(a) variation of the velocity component f' with *F*

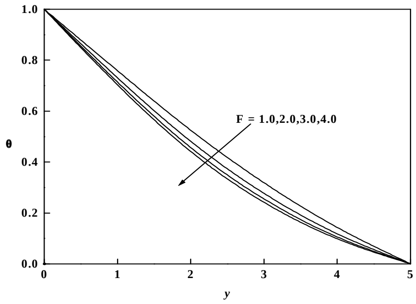

Fig.4(b) variation of the temperature with *Pr*

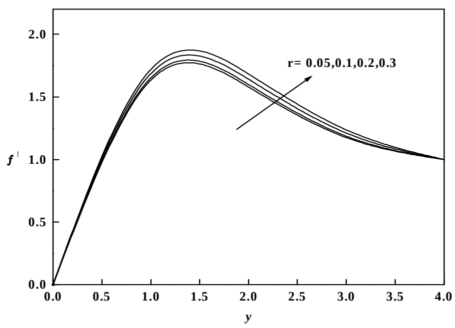

Fig.5(a) variation of the velocity component f' with *r*

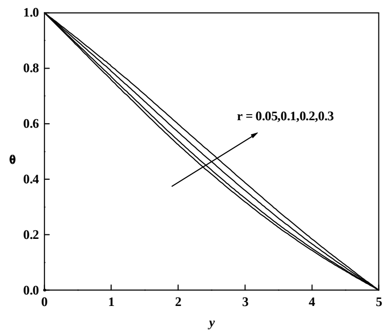

Fig.5(b) variation of the temperature with *r*

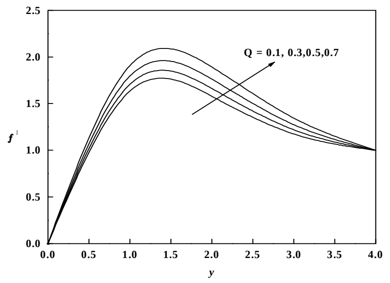

Fig.6(a) variation of the velocity component f' with Q

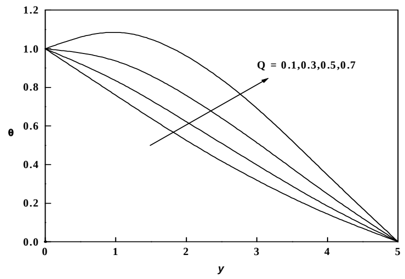

Fig.6(b) variation of the temperature with Q

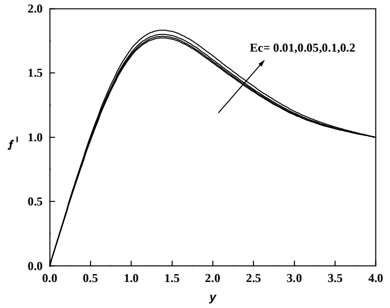

Fig.7(a) variation of the velocity component f' with Ec

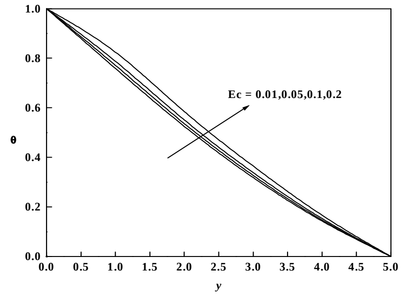

Fig.7(b) variation of the temperature with Ec

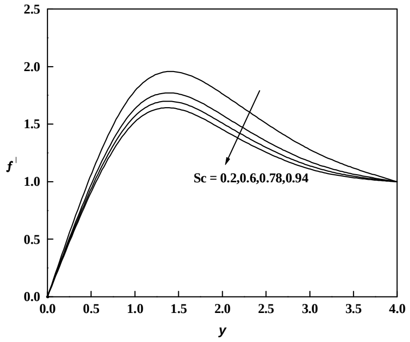

Fig.8(a) variation of the velocity component f' with Sc

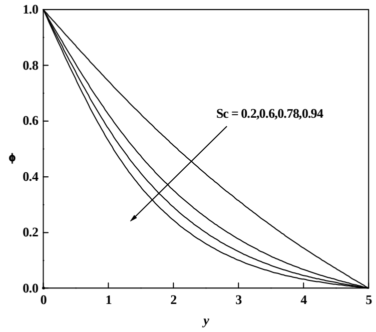

Fig.8(b) variation of the concentration with Sc

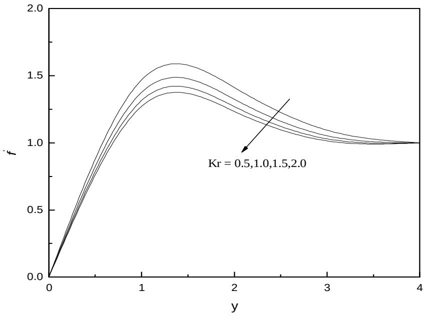

Fig.9(a) variation of the velocity component f' with K_r

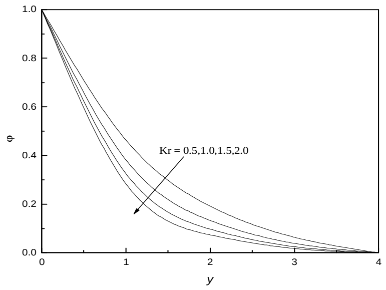

Fig.9(b) variation of the concentration with K_r

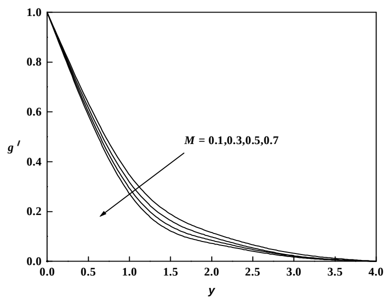

Fig.10 variation of the velocity component g' with *M*

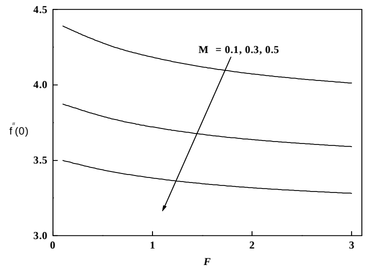

Fig.11 variation of $f''(0)$ with *F* and *M*

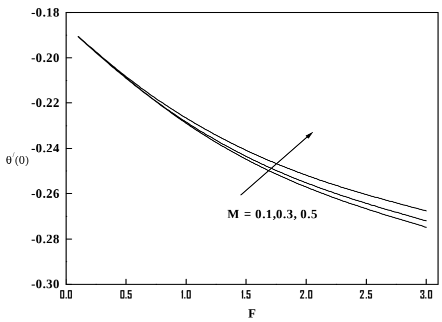

Fig.12 variation of the heat flux $\theta'(0)$ with *F* and *M*

Fig.1(a). shows the variation of the dimensionless velocity component f' for several sets of values of thermal Grashof number *Gr*. As expected, it is observed that there is a rise in the velocity due to enhancement of thermal buoyancy force. Here, the positive values of *Gr* correspond to cooling of the plate. Also, as *Gr* increases, the peak values of the velocity increases rapidly near the plate and then decays smoothly to the free stream velocity.

The variation of the dimensionless velocity component f' for several sets of values of solutal Grashof number *Gc* is depicted in Fig.1(b). As expected, the fluid velocity increases and the peak value is more distinctive due to increase in the species buoyancy force. The velocity distribution attains a distinctive maximum value in the vicinity of the plate and then decreases properly to approach the free stream value.

For various values of the magnetic parameter *M*, the dimensionless velocity component f' is plotted in Fig.2(a). It can be seen that as *M* increases, the velocity decreases. As M increases, the Lorentz force, which opposes the flow, also increases and leads to enhanced deceleration of the flow. This result qualitatively agrees with the expectations, since the magnetic filed exerts a retarding force on the free convection flow. Fig.2(b). shows that the dimensionless temperature profiles for different values of magnetic parameter *M*. It is observed that the temperature increases with an increase in the magnetic parameter *M*.

Fig.3(a). illustrates the dimensionless velocity component f' for different values of the Prandtl number *Pr*. The numerical results show that the effect of increasing values of Prandtl number results in a decreasing velocity. From Fig.3(b), it is observed that an increase in the Prandtl number results a decrease of the thermal boundary layer thickness and in general lower average temperature with in the boundary layer. The reason is that smaller values of *Pr* are equivalent to increasing the thermal conductivities, and therefore heat is able to diffuse away from the heated plate more rapidly than for higher values of *Pr*. Hence in the case of smaller Prandtl numbers as the boundary layer is thicker and the rate of heat transfer is reduced.

The effect of the Radiation parameter *F* on the dimensionless velocity component f' and dimensionless temperature are shown in Figs. 4(a) and 4(b) respectively. Fig.4 (a) shows that velocity component f' decreases with an increase in the radiation parameter *F*. From Fig.4(b) it is seen that the temperature decreases as the radiation parameter *F* increases. This result qualitatively agrees with expectations, since the effect of radiation is to decrease the rate of energy transport to the fluid, thereby decreasing the temperature of the fluid.

The influence of the parameter of relative difference between the temperature of the sheet and the temperature far away from the sheet *r* on dimensionless velocity f' and temperature profiles are plotted in Figs. 5(a) and 5(b) respectively. Fig.5(a) shows that dimensionless velocity f' increases with an increase in *r*. It is observed that the temperature increases with an increase in *r* (Fig.5 (b)).

Figs. 6(a) and 6(b) depict the dimensionless velocity f' and temperature profiles for different values of the heat generation parameter *Q*. It is noticed that an increase in the heat generation parameter *Q* results in an increase in the dimensionless velocity f' and temperature with in the boundary layer.

The effect of the viscous dissipation parameter i.e., the Eckert number *Ec* on the dimensionless velocity component f' and temperature are shown in Figs. 7(a) and 7(b) respectively. The positive Eckert number implies cooling of the plate i.e., loss of heat from the plate to the fluid. Hence, greater viscous dissipative heat causes a rise in the temperature as well as the velocity, which is evident from Figs. 7(a) and 7(b).

The influence of the Schmidt number *Sc* on the dimensionless velocity f' and concentration profiles are plotted in Figs. 8(a) and 8(b) respectively. As the Schmidt number increases the concentration decreases. This causes the concentration buoyancy effects to decrease yielding a reduction in the fluid velocity. The reductions in the velocity and concentration profiles are accompanied by simultaneous reductions in the velocity and concentration boundary layers. These behaviors are clear from Figs. 8(a) and 8(b). The effects of the chemical reaction parameter K_r on dimensionless velocity component f' and concentration profiles are plotted in Figs. 9(a) and Fig. 9(b). As the chemical reaction parameter number increases the concentration and velocity profiles are decreases. These behaviors are clear from Figs. 9(b) and 9(a).

Fig. 10 describes the behavior of the dimensionless velocity component g' with changes in the values of the magnetic field parameter *M*. It is seen, as expected, that velocity component g' decreases with an increase in the magnetic field parameter *M*.

Figs.(11) and (12) describe the behavior of $f''(0)$ and the heat flux $\theta'(0)$ with changes in the values of the flow parameters *F* and *M*. we observe that the effect of increasing *M* is the decrease in the wall temperature gradient $\theta'(0)$ and $f''(0)$. On the other hand, the magnitude of $\theta'(0)$ increases while that of $f''(0)$ decreases as *F* increases.

Finally, in order to verify the proper treatment of the present problem, we will compare the obtained numerical solution with the exact values of $g''(0)$. The exact solution of (21) ($g(y) = -v$) is given by

$$g(y) = \frac{1}{\sqrt{M+1}}(1 - e^{-\sqrt{M+y}}) \tag{29}$$

In the following table, the given numbers between brackets refer to the exact values and the given numbers without brackets refer to the approximated values. Vajravelu and Hadjinicolaou (Vajravelu and Hadjinicolaou 1997) convective heat transfer in an electrically conducting fluid at stretching surface with uniform free stream, have obtained for $g''(0)$ (M = 0.01) the value -1.0025, while our result is -1.005 and the exact value is -1.00499. Therefore, the present results are in satisfactory agreement with the exact values.

F	*Gr*	*M*	*Pr*	$g''(0)$
1.0	0.5	0.1	0.72	-1.04881 (-1.04881)
1.0	0.5	0.01	0.72	-1.005 (-1.00499)
2.0	0.5	0.1	0.72	-1.04881 (-1.04881)

Table 1 Variation of f'', g'', θ', ϕ' at the plate with *Gr, Gc, M* for *Pr* = 0.71, *F* = 1.0, *r* = 0.05, *Q* = 0.1, *Ec* = 0.01, *Sc* = 0.6 ,Kr = 0.5.

Gr	*Gc*	*M*	$f''(0)$	$g''(0)$	$\theta'(0)$	$\phi'(0)$
2.0	2.0	0.5	3.10574	-1.22491	-0.22901	-0.691255
3.0	2.0	0.5	3.99741	-1.22491	-0.222828	-0.691255
2.0	3.0	0.5	3.71935	-1.22491	-0.225685	-0.691255

2.0	2.0	1.0	2.61222	-1.41424	-0.222342	-0.680714

Table 2 Variation of f'', g'', θ', ϕ' at the plate with *Pr, F, Q, Ec, Sc* for *Gr = 2.0* , *Gc = 2.0, M* = 0.5

Pr	*F*	*Q*	*Ec*	*Sc*	*Kr*	$f''(0)$	$g''(0)$	$\theta'(0)$	$\phi'(0)$
0.71	1.0	0.1	0.01	0.6	0.5	3.10574	-1.22491	-0.22901	-0.691255
1.0	1.0	0.1	0.01	0.6	0.5	3.04696	-1.22491	-0.251104	-0.691255
0.71	2.0	0.1	0.01	0.6	0.5	3.04058	-1.22491	-0.255139	-0.691255
0.71	1.0	0.15	0.01	0.6	0.5	3.14024	-1.22491	-0.204972	-0.691255
0.71	1.0	0.1	0.05	0.6	0.5	3.12343	-1.22491	-0.193933	-0.691255
0.71	1.0	0.1	0.01	0.78	0.5	3.00728	-1.22491	-0.229774	-0.797555
0.71	1.0	0.1	0.01	0.6	1.0	2.96197	-1.22491	-0.230082	-0.889375

The effect of various parameters on the functions f'', g'', θ' and ϕ' at the plate surface is tabulated in Tables 1 and 2 for r = 0.05. It is observed that the magnitude of the wall temperature gradient increases as Prandtl number *Pr* or radiation parameter *F* increases, while it decreases as the magnetic parameter *M* or the heat source/sink parameter *Q* or the Eckert number *Ec* increases. The magnitude of the wall concentration gradient decreases as the magnetic field parameter *M* increases, while it increases with an increase in the Schmidt number *Sc* and chemical reaction parameter *Kr*. Furthermore, the negative values of the wall temperature and concentration gradients, for all values of the dimensionless parameters, are indicative of the physical fact that the heat flows from the sheet surface to the ambient fluid.

CONCLUSION

The problem of steady, hydromagnetic, mass transfer, laminar, free convection boundary layer flow along a stretching surface in the presence of thermal radiation, heat generation and viscous dissipation was investigated. A similarity transformation was employed to change the governing partial differential equations into ordinary ones. These equations were solved numerically by fourth order Runge-Kutta method. A wide selection of numerical results have been presented giving the evolution of the velocity, temperature and concentration as well as skin friction coefficient, heat transfer rate and mass transfer rate. The following points are concluded

1. An increase in the radiation parameter leads to decreases in both velocity ad temperatures. This result qualitatively aggresses with expectation, since the effect of radiation is to decrease the rate of energy transport to the fluid, thereby decreasing the temperature of the fluid.
2. An increase in the values of heat generation parameter leads to an increase in both velocity and temperature.
3. An increase in the viscous dissipation leads to an increase in both velocity and temperature.
4. We observe that the effect of increasing *M* is the decrease in the wall temperature gradient $\theta'(0)$ and $f''(0)$.
5. We observe that the magnitude of $\theta'(0)$ increases and $f''(0)$ decreases as radiation parameter increases.
6. We observe that the magnitude of the wall concentration gradient decreses as magnetic fluid parameter *M* increases while it increases with an increase in the Schmidt number *Sc* and chemical reaction parameter *Kr*.

REFERENCES

[1] Sakiadis B C., *American Institute of Chemical Engineers Journal*., **1961**, 7, 26-28.
[2] Erickson L E., Fan L T and Fox V G., *Indust. Eng. Chem*, 5, 19-25.
[3] Danberg J E and Fansber K S, *Quarterly Applied Mathematics*,**1976**, 34, 305-311.
[4]Gupta P S and Gupta A S , *Canadian Journal of Chemical Engineering*, **1977**, 55, 744-746.
[5] Elbashbeshy E M A, *J. Phys. And Appl.Phys*, **1998**, 31, 951-1954.
[6] Samad M A and Mobebujjaman M, *Research Journal of Applied Sciences, Engineering and Technology*,**2009**, 1(3), 98-106.
[7] Takhar H S, Golra R S R and Soundalgekar V M , *International Journal of Numerical. Methods Heat Fluid Flow*, **1996**, 6(2), 77-83.
[8] Ghaly A Y and Elbarbary E M E, *Journal of Applied Mathematics*, **2002**, 2(2), 93-103.
[9] Anjali Devi SP and Kayalvizhi M, *International Journal of Appl. Math and Mech.*, **2010**, 6(7), 82-106.
[10] Gebharat B, *Journal of Fluid Mechanics*, **1996**, 14, 225-232.
[11] Israel-Cookey C Ogulu A and Omubo-Pepple V B, *International Journal Heat Mass Transfer*, **2003**, 46, 2305-2311.
[12] Suneetha S Bhaskar Reddy N and Ramachandra Prasad V, *Thermal Science* ,**2009**, 13(2), 171-181.
[13] Ganeswara Reddy M and Bhaskar Reddy N, *International Journal of Appl. Math and Mech.* **2010**, 6(1), 1-12.
[14] Mohammed Ibrahim S and Bhaskar Reddy N, *International Journal of Appl. Math and Mech*,**2012**, 8(8), 1-21.
[15] Vajravelu K and Hadjinicolaou A, *International Journal. Eng.Sci*, **1997**, 35(12-13), 1237-1244.

[16] Hossain M A Molla M M and Yaa L S, *International Journal of Thermal Science*, **2004**, 43, 157-163.
[17] Kesavaiah D Ch, Satyanarayana P V and Venkataramana S , *International Journal of Appl. Math and Mech.* **2011**, 7(1), 52-69.
[18] Chambre P L and Young J D, *Physics of Fluids*, **1958**, 1, 48-54.
[19] Andersson, H I., Hansen O R and Holmedal B, *International Journal of Heat and Mass Transfer*, **1994**, 37, 659-664.
[20] Afify A, *Heat and Mass Transfer*, **2000**, 40, 495-500.
[21] Liu I.C, *International Communications in Heat and Mass Transfer*, **2005**, 32, 1075-1084.
[22] Sudhakar Reddy T., Siva Prasad Reddy O., Raju M C, and Varma S V K, *Advances in Applied Science Research*, **2012**, 3(6),3482-3490.
[23] Rajasekhar K., Ramana Reddy G V and Prasad B D C N, *Advances in Applied Science Research*, **2012**, 3(5), 2652-2659.
[24] Kishan N and Srinivas M, *Advances in Applied Science Research*, **2012**,3(1), 60-74.
[25] Anjali Devi S P and David A M G, *Advances in Applied Science Research*, **2012**, 3(1), 319-334.
[26] Kishan N and Deepa G, *Advances in Applied Science Research*, 3(1), 430-439.
[27]Rohsenow W N., Hartnett J P, and Cho Y I, Hand book of Heat Transfer,**1998**, 3rd Edition, MC Graw- Hill, New York.
[28] Rosenhead L, Laminar Boundary Layers, *Oxford university press, London.* **1963**, 688.
[29] Carnahan B, Luther H A, James O W, *Applied Numerical Methods, Wiley New York.***1969**.