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Characterization of Non-oscillatory Motions in Magneto-Rotatory Thermal Convection in Couple-Stress Fluid in a Porous Medium

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ABSTRACT

The thermal instability of a couple-stress fluid heated from below in a porous medium acted upon by uniform vertical magnetic field and rotation is investigated. Following the linearized stability theory and normal mode analysis, the paper mathematically established the conditions for characterizing the oscillatory motions which may be neutral or unstable for rigid boundaries at the top and bottom of the fluid. It is established that all non-decaying slow motions starting from rest, in a couple-stress fluid of infinite horizontal extension and finite vertical depth, which is acted upon by uniform vertical magnetic field and rotation, and a constant vertical adverse temperature gradient, are necessarily non-oscillatory in the regime

$$\varepsilon\left(\frac{Qp_2\pi^2}{(2\pi^2-1)}\right)+\frac{P_lT_A}{(2\pi^2F-P_l)}\le 1 ,$$

where T_A *is the Taylor number, Q is the Chandrasekhar number,* ε *is the porosity,* P_l *is the dimensionless medium permeability of the porous medium and F is the couple-stress parameter. The result is important since it holds for all wave numbers and the exact solutions of the problem investigated are not obtainable in closed form, when both the boundaries are perfectly conducting and rigid.*

Key Words: Thermal convection; Couple-Stress Fluid; Rotation; Magnetic Field; PES; Rayleigh number; Chandrasekhar Number; Taylor number**.**
MSC 2000 No.: *76A05, 76E06, 76E15; 76E07; 76U05.*

INTRODUCTION

Twentieth century has witnessed tremendous advances on the technological front. A detailed account of the theoretical and experimental study of the onset of thermal instability in Newtonian fluids, under varying assumptions of hydrodynamics and hydromagnetics, has been given by Chandrasekhar [6] and the Boussinesq approximation has been used throughout, which states that the density changes are disregarded in all other terms in the equation of motion, except in the external force term. The formation and derivation of the basic equations of a layer of fluid heated from below in a porous medium, using the Boussinesq approximation, has been given in a treatise by Joseph [9]. When a fluid permeates through an isotropic and homogeneous porous medium, the gross effect is represented by Darcy's law. The study of layer of fluid heated from below in porous media is motivated both theoretically and by its practical applications in engineering. Among the applications in engineering disciplines one can name the food processing industry, the chemical processing industry, solidification, and the centrifugal casting of metals. The development of geothermal power resources has increased general interest in the properties of convection in a porous medium. Stommel and Fedorov [26] and Linden [14] have remarked that the length scales characteristic of double-diffusive convecting layers in the ocean may be sufficiently large so that the Earth's rotation might be important in their formation. Moreover, the rotation of the Earth distorts the boundaries of a hexagonal convection cell in a fluid through porous medium, and this distortion plays an important role in the extraction of energy in geothermal regions. The forced convection in a fluid saturated porous medium channel has been studied by Nield et al [16]. An extensive and updated account of convection in porous media has been given by Nield and Bejan [15].

The effect of a magnetic field on the stability of such a flow is of interest in geophysics, particularly in the study of the earth's core, where the earth's mantle, which consist of conducting fluid, behaves like a porous medium that can become conductively unstable as result of differential diffusion. Another application of the results of flow through a porous medium in the presence of magnetic field is in the study of the stability of convective geothermal flow. A good account of the effect of rotation and magnetic field on the layer of fluid heated from below has been given in a treatise by Chandrasekhar [6] .

MHD finds vital applications in MHD generators, MHD flow-meters and pumps for pumping liquid metals in metallurgy, geophysics, MHD couplers and bearings, and physiological processes such magnetic therapy. With the growing importance of non-Newtonian fluids in modern technology and industries, investigations of such fluids are desirable. The presence of small amounts of additives in a lubricant can improve bearing performance by increasing the lubricant viscosity and thus producing an increase in the load capacity. These additives in a lubricant also reduce the coefficient of friction and increase the temperature range in which the bearing can operate.

Darcy's law governs the flow of a Newtonian fluid through an isotropic and homogeneous porous medium. However, to be mathematically compatible and physically consistent with the Navier-Stokes equations, Brinkman [4] heuristically proposed the introduction of the term $\frac{\mu}{\varepsilon}\nabla^2 \vec{q}$, (now known as Brinkman term) in addition to the Darcian term $-\left(\frac{\mu}{k_1}\right)\vec{q}$. But the main effect is through the Darcian term; Brinkman term contributes very little effect for flow through a porous medium. Therefore, Darcy's law is proposed heuristically to govern the flow of this non-Newtonian couple-stress fluid through porous medium. A number of theories of the micro continuum have been postulated and applied (Stokes [25]; Lai et al [12]; Walicka [28]). The theory due to Stokes [25] allows for polar effects such as the presence of couple stresses and body couples. Stokes's [25] theory has been applied to the study of some simple lubrication problems (see e.g. Sinha et al [23]; Bujurke and Jayaraman [5]; Lin [13]). According to the theory of Stokes [25], couple-stresses are found to appear in noticeable magnitudes in fluids with very large molecules. Since the long chain hyaluronic acid molecules are found as additives in synovial fluid, Walicki and Walicka [29] modeled synovial fluid as couple stress fluid in human joints. The study is motivated by a model of synovial fluid. The synovial fluid is natural lubricant of joints of the vertebrates. The detailed description of the joints lubrication has very important practical implications; practically all diseases of joints are caused by or connected with a malfunction of the lubrication. The external efficiency of the physiological joint lubrication is caused by more mechanisms. The synovial fluid is caused by the content of the hyaluronic acid, a fluid of high viscosity, near to a gel. A layer of such fluid heated from below in a porous medium under the action of magnetic field and rotation may find applications in physiological processes. MHD finds applications in physiological processes such as magnetic therapy; rotation and heating may find applications in physiotherapy. The use of magnetic field is being made for the clinical purposes in detection and cure of certain diseases with the help of magnetic field devices.

Sharma and Thakur [21] have studied the thermal convection in couple-stress fluid in porous medium in hydromagnetics. Sharma and Sharma [22] have studied the couple-stress fluid heated from below in porous medium. Kumar and Kumar [11] have studied the combined effect of dust particles, magnetic field and rotation on couple-stress fluid heated from below and for the case of stationary convection, found that dust particles have destabilizing effect on the system, where as the rotation is found to have stabilizing effect on the system, however couple-stress and magnetic field are found to have both stabilizing and destabilizing effects under certain conditions.

Sunil et al. [27] have studied the global stability for thermal convection in a couple-stress fluid heated from below and found couple-stress fluids are thermally more stable than the ordinary viscous fluids. Gupta et al [8] have studied the effect of γ -irradiation on thermal stability of CR-39 polymer where as the effect of thickness of the porous materials on the peristaltic pumping, when the tube wall is provided with non-erodible porous lining has been investigated by Reddy et al [18]. The effect of magnetic field and rotation on thermosolutal convection in Walters B' elastico-viscous fluid has been considered by Kango and Rana [10]. Saravana et al. [19] have considered the heat and mass transfer on the unsteady viscoelastic second order Rivlin-Ericksen fluid past an impulsive started infinite vertical plate in the presence of a foreign mass and constant mass flux on taking into account of viscous dissipative heat at the plate under the influence of a uniform transverse magnetic field. The electrically conducting flow of couple-stress fluid in a vertical porous layer has been investigated by Sreenadh et al [24]. The above studies were helpful in studying porous materials and thermal stability.

Pellow and Southwell [17] proved the validity of PES for the classical Rayleigh-Bénard convection problem. Banerjee et al [2] gave a new scheme for combining the governing equations of thermohaline convection, which is shown to lead to the bounds for the complex growth rate of the arbitrary oscillatory perturbations, neutral or unstable for all combinations of dynamically rigid or free boundaries and, Banerjee and Banerjee [1] established a criterion on characterization of non-oscillatory motions in hydrodynamics which was further extended by Gupta et al. [7]. However no such result existed for non-Newtonian fluid configurations, in general and for couple-stress fluid configurations, in particular. Banyal [3] have characterized the non-oscillatory motions in couple-stress fluid in the presence of magnetic field in a porous medium.

Keeping in mind the importance of non-Newtonian fluids in general and couple-stress fluid in particular, as stated above, this article attempts to characterize the oscillatory motions in the couple-stress fluid heated from below in a porous medium in the presence of uniform vertical magnetic field and rotation with realistic rigid-rigid horizontal boundaries and it has been established that the onset of instability in a couple-stress fluid in a porous medium heated from below, in the presence of uniform vertical magnetic field and rotation, cannot manifest itself as oscillatory motions of growing amplitude if the thermal Taylor number T_A, the Chandrasekhar number Q, the porosity ε, the couple-stress parameter of the fluid F and the medium permeability P_l, satisfy the inequality $\varepsilon\left(\frac{Qp_2\pi^2}{(2\pi^2-1)}\right)+\frac{P_lT_A}{(2\pi^2F-P_l)}\leq 1$, when the bounding surfaces are rigid of infinite horizontal extension.

2. FORMULATION OF THE PROBLEM AND PERTURBATION EQUATIONS

Here we consider an infinite, horizontal, incompressible electrically conducting couple-stress fluid layer, of thickness d, heated from below so that, the temperature and density at the bottom surface z = 0 are T_0 and ρ_0 and at the upper surface z = d are T_d and ρ_d respectively, and that a uniform adverse temperature gradient $\beta\left(=\left|\frac{dT}{dz}\right|\right)$ is maintained. The fluid is acted upon by a uniform vertical rotation $\vec{\Omega}(0,0,\Omega)$ and a uniform vertical magnetic field $\vec{H}(0,0,H)$. This fluid layer is flowing through an isotropic and homogeneous porous medium of porosity ε and of medium permeability k_1.

Let ρ, p, T, η, μ_e and $\vec{q}(u,v,w)$ denote respectively the fluid density, pressure, temperature, resistivity, magnetic permeability and filter velocity of the fluid, respectively Then the momentum balance, mass balance, and energy balance equation of couple-stress fluid and Maxwell's equations through porous medium, governing the flow of couple-stress fluid in the presence of uniform vertical magnetic field and rotation are (Stokes [25]; Joseph [9]; Chandrasekhar [6]) are given by

$$\frac{1}{\varepsilon}\left[\frac{\partial \vec{q}}{\partial t}+\frac{1}{\varepsilon}\left(\vec{q}.\nabla\right)\vec{q}\right]=-\nabla\left(\frac{p}{\rho_o}-\frac{1}{2}\left|\vec{\Omega}\times\vec{r}\right|^2\right)+\vec{g}\left(1+\frac{\delta\rho}{\rho_0}\right)-\frac{1}{k_1}\left(\nu-\frac{\mu'}{\rho_0}\nabla^2\right)\vec{q}$$

$$+\frac{\mu_e}{4\pi\rho_o}(\nabla\times\vec{H})\times\vec{H}+\frac{2}{\varepsilon}\left(\vec{q}\times\vec{\Omega}\right), \qquad (1)$$

$$\nabla.\vec{q}=0, \qquad (2)$$

$$E\frac{dT}{dt}+(\vec{q}.\nabla)T=\kappa\nabla^2 T, \qquad (3)$$

$$\nabla.\vec{H}=0, \qquad (4)$$

$$\varepsilon\frac{d\vec{H}}{dt}=(\vec{H}.\nabla)\vec{q}+\varepsilon\eta\nabla^2\vec{H}, \qquad (5)$$

Where $\frac{d}{dt}=\frac{\partial}{\partial t}+\varepsilon^{-1}\vec{q}.\nabla$ stand for the convective derivatives. Here

$E=\varepsilon+(1-\varepsilon)\left(\frac{\rho_s c_s}{\rho_0 c_v}\right)$, is a constant, while ρ_s, c_s and ρ_0, c_v, stands for the density and heat capacity of the solid (porous matrix) material and the fluid, respectively, ε is the medium porosity and $\vec{r}(x,y,z)$.

The equation of state is

$$\rho=\rho_0[1-\alpha(T-T_0)], \qquad (6)$$

Where the suffix zero refer to the values at the reference level z = 0. Here $\vec{g}(0,0,-g)$ is acceleration due to gravity and α is the coefficient of thermal expansion. In writing the equation (1), we made use of the Boussinesq approximation, which states that the density variations are ignored in all terms in the equation of motion except the external force term. The kinematic viscosity ν , couple-stress viscosity μ' , magnetic permeability μ_e, thermal diffusivity κ, and electrical resistivity η, and the coefficient of thermal expansion α are all assumed to be constants.

The basic motionless solution is

$$\vec{q}=(0,0,0), \rho=\rho_0(1+\alpha\beta z), \ \text{p=p(z)}, \ T=-\beta z+T_0, \qquad (7)$$

Here we use the linearized stability theory and the normal mode analysis method. Assume small perturbations around the basic solution, and let $\delta\rho$, δp, θ, $\vec{q}(u,v,w)$ and $\vec{h}=(h_x,h_y,h_z)$ denote respectively the perturbations in density ρ, pressure p, temperature T, velocity $\vec{q}(0,0,0)$ and the magnetic field $\vec{H}=(0,0,H)$. The change in density $\delta\rho$, caused mainly by the perturbation θ in temperature, is given by

$$\rho+\delta\rho=\rho_0[1-\alpha(T+\theta-T_0)]=\rho-\alpha\rho_0\theta, \text{ i.e. } \delta\rho=-\alpha\rho_0\theta. \qquad (8)$$

Then the linearized perturbation equations of the couple-stress fluid reduces to

$$\frac{1}{\varepsilon}\frac{\partial \vec{q}}{\partial t} = -\frac{1}{\rho_0}\nabla\delta p - \vec{g}\,\alpha\theta - \frac{1}{k_1}\left(\nu - \frac{\mu'}{\rho_0}\nabla^2\right)\vec{q} + \frac{\mu_e}{4\pi\rho_0}\left(\nabla\times\vec{h}\right)\times\vec{H} + \frac{2}{\varepsilon}\left(\vec{q}\times\vec{\Omega}\right), \tag{9}$$

$$\nabla.\vec{q} = 0, \tag{10}$$

$$E\frac{\partial\theta}{\partial t} = \beta w + \kappa\nabla^2\theta, \tag{11}$$

$$\nabla.\vec{h} = 0, \tag{12}$$

$$\varepsilon\frac{\partial\vec{h}}{\partial t} = \left(\vec{H}.\nabla\right)\vec{q} + \varepsilon\eta\nabla^2\vec{h}. \tag{13}$$

3. NORMAL MODE ANALYSIS

Analyzing the disturbances into two-dimensional waves, and considering disturbances characterized by a particular wave number, we assume that the Perturbation quantities are of the form

$$\left[w,\theta,h_z,\varsigma,\xi\right] = \left[W(z),\Theta(z),K(z),Z(z),X(z)\right]\exp\left(ik_x x + ik_y y + nt\right), \tag{14}$$

Where k_x, k_y are the wave numbers along the x- and y-directions, respectively, $k = \left(k_x^2 + k_y^2\right)^{\frac{1}{2}}$, is the resultant wave number, n is the growth rate which is, in general, a complex constant and, $\varsigma = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$ and $\xi = \frac{\partial h_y}{\partial x} - \frac{\partial h_x}{\partial y}$ denote the z-component of vorticity and current density respectively, $W(z), K(z), \Theta(z), Z(z)$ and $X(z)$ are the functions of z only.

Using (14), equations (9)-(13), within the framework of Boussinesq approximations, in the non-dimensional form transform to

$$\left(D^2 - a^2\right)\left[\frac{F}{P_l}\left(D^2 - a^2\right) - \left(\frac{\sigma}{\varepsilon} + \frac{1}{P_l}\right)\right]W = Ra^2\Theta + T_A DZ - Q\left(D^2 - a^2\right)DK, \tag{15}$$

$$\left[\frac{F}{P_l}\left(D^2 - a^2\right) - \left(\frac{\sigma}{\varepsilon} + \frac{1}{P_l}\right)\right]Z = -DW - QDX, \tag{16}$$

$$\left(D^2 - a^2 - Ep_1\sigma\right)\Theta = -W, \tag{17}$$

$$\left(D^2 - a^2 - p_2\sigma\right)K = -DW, \tag{18}$$

and

$$\left(D^2 - a^2 - p_2\sigma\right)X = -DZ, \tag{19}$$

Where we have introduced new coordinates (x', y', z') = (x/d, y/d, z/d) in new units of length d and $D = d/dz'$. For convenience, the dashes are dropped hereafter. Also we have substituted $a = kd, \sigma = \frac{nd^2}{\nu}$, $p_1 = \frac{\nu}{\kappa}$, is the thermal Prandtl number; $p_2 = \frac{\nu}{\eta}$, is the magnetic Prandtl number; $P_l = \frac{k_1}{d^2}$ is the dimensionless medium permeability, $F = \frac{\mu^{'}/(\rho_0 d^2)}{\nu}$, is the dimensionless couple-stress viscosity parameter; $R = \frac{g\alpha\beta d^4}{\kappa\nu}$, is the thermal Rayleigh number; $Q = \frac{\mu_e H^2 d^2}{4\pi\rho_0\nu\eta\varepsilon}$, is the Chandrasekhar number and $T_A = \frac{4\Omega^2 d^4}{\nu^2\varepsilon^2}$, is the Taylor number. Also we have Substituted $W = W_{\oplus}$, $\Theta = \frac{\beta d^2}{\kappa}\Theta_{\oplus}$, $Z = \frac{2\Omega d}{\nu\varepsilon}Z_{\oplus}$, $K = \frac{Hd}{\varepsilon\eta}K_{\oplus}$, $X = \left(\frac{Hd}{\varepsilon\eta}\right)\left(\frac{2\Omega d}{\varepsilon\nu}\right)X_{\oplus}$ and $D_{\oplus} = dD$, and dropped $(\oplus)$ for convenience.

We now consider the case where both the boundaries are rigid and perfectly conducting and are maintained at constant temperature, then the perturbations in the temperature are zero at the boundaries. The appropriate boundary conditions with respect to which equations (15)-(19), must possess a solution are

$$W = DW = 0, \Theta = 0, Z=0, K=0 \text{ and } DX = 0, \text{ at } z = 0 \text{ and } z = 1. \tag{20}$$

Equations (15)-(19), along with boundary conditions (20), pose an eigenvalue problem for σ and we wish to characterize σ_i, when $\sigma_r \geq 0$.

We first note that since W, K and Z satisfy $W(0) = 0 = W(1)$, $K(0) = 0 = K(1)$ and $Z(0) = 0 = Z(1)$, in addition to satisfying to governing equations and hence we have from the Rayleigh-Ritz inequality [20]

$$\int_0^1 |DW|^2 dz \geq \pi^2 \int_0^1 |W|^2 dz, \quad \int_0^1 |DK|^2 dz \geq \pi^2 \int_0^1 |K|^2 dz \text{ and } \int_0^1 |DZ|^2 dz \geq \pi^2 \int_0^1 |Z|^2 dz, \tag{21}$$

4. MATHEMATICAL ANALYSIS

We prove the following lemma:

Lemma 1: For any arbitrary oscillatory perturbation, neutral or unstable

$$\int_0^1 \left\{|DK|^2 + a^2|K|^2\right\}dz \leq \frac{\pi^2}{(2\pi^2 - 1)}\int_0^1 |DW|^2 dz$$

Proof: Multiplying equation (18) by K^* (the complex conjugate of K), integrating by parts each term of the resulting equation on the left hand side for an appropriate number of times and making use of boundary conditions on K namely $K(0) = 0 = K(1)$, it follows that

$$\int_0^1 \left\{|DK|^2 + a^2|K|^2\right\}dz + \sigma_r p_2 \int_0^1 |K|^2 dz = \text{Real part of}\left\{\int_0^1 K^* DW dz\right\} \leq \left|\int_0^1 K^* DW dz\right| \leq \int_0^1 |K^* DW| dz,$$

$$\leq \int_0^1 |K^*||DW| dz \leq \frac{1}{2}\int_0^1 \left(|K|^2 + |DW|^2\right)dz, \tag{22}$$

This gives that

$$\int_0^1 |DK|^2 dz \leq \frac{1}{2}\int_0^1 \left(|K|^2 + |DW|^2\right)dz, \tag{23}$$

Inequality (23) on utilizing (21), gives

$$\int_0^1 |K|^2 dz \le \frac{1}{(2\pi^2 - 1)} \int_0^1 |DW|^2 dz, \tag{24}$$

Since $\sigma_r \ge 0$ and $p_2 \rangle 0$, hence inequality (22) on utilizing (24), give

$$\int_0^1 \left(|DK|^2 + a^2 |K|^2\right) dz \le \frac{\pi^2}{(2\pi^2 - 1)} \int_0^1 |DW|^2 dz, \tag{25}$$

Lemma 2: For any arbitrary oscillatory perturbation, neutral or unstable

$$\int_0^1 |Z|^2 dz \le \frac{P_l}{(2\pi^2 F - P_l)} \int_0^1 |DW|^2 dz$$

Proof: Multiplying equation (16) by Z^* (the complex conjugate of Z), integrating by parts each term of the resulting equation on the left hand side for an appropriate number of times on utilizing equation (19) and appropriate boundary conditions (20), it follows that

$$\frac{F}{P_l}\int_0^1 \left\{|DZ|^2 + a^2|Z|^2\right\} dz + \left(\frac{\sigma_r}{\varepsilon} + \frac{1}{P_l}\right)\int_0^1 |Z|^2 dz + Q\int_0^1 \left\{|DX|^2 + a^2|X|^2\right\} dz + Qp_2\sigma_r \int_0^1 |X|^2 dz$$

$$= \text{Real part of } \left\{\int_0^1 DW^* Z dz\right\} \le \left|\int_0^1 DW^* Z dz\right|,$$

$$\le \int_0^1 |DW^* Z| dz \le \int_0^1 |DW^*||Z| dz,$$

$$= \int_0^1 |DW||Z| dz \quad \le \frac{1}{2}\int_0^1 \left(|Z|^2 + |DW|^2\right) dz, \tag{26}$$

This gives that

$$\frac{F}{P_l}\int_0^1 |DZ|^2 dz \le \frac{1}{2}\int_0^1 \left(|Z|^2 + |DW|^2\right) dz, \tag{27}$$

Inequality (26) on utilizing (21), gives

$$\int_0^1 |Z|^2 dz \le \frac{P_l}{(2\pi^2 F - P_l)} \int_0^1 |DW|^2 dz, \tag{28}$$

This completes the proof of lemma**.**

Now we prove the following theorems:

Theorem 1: If R$\rangle$ 0, F$\rangle$ 0, $Q \rangle 0$, $T_A \rangle 0$, $P_l \rangle 0$, $p_1 \rangle 0$, $p_2 \rangle 0$, $\sigma_r \ge 0$ and $\sigma_i \ne 0$ then the necessary condition for the existence of non-trivial solution (W, Θ, K, Z, X) of equations (15) – (19), together with boundary conditions (20) is that

$$\varepsilon\left(\frac{Qp_2\pi^2}{(2\pi^2 - 1)}\right) + \frac{P_l T_A}{(2\pi^2 F - P_l)} \rangle 1 .$$

Proof: Multiplying equation (15) by W^* (the complex conjugate of W) throughout and integrating the resulting equation over the vertical range of z, we get

$$\frac{F}{P_l}\int_0^1 W^*\left(D^2-a^2\right)^2 Wdz-\left(\frac{\sigma}{\varepsilon}+\frac{1}{P_l}\right)\int_0^1 W^*\left(D^2-a^2\right)Wdz$$
$$=Ra^2\int_0^1 W^*\Theta dz+T_A\int_0^1 W^*DZdz-Q\int_0^1 W^*D\left(D^2-a^2\right)Kdz, \tag{29}$$

Taking complex conjugate on both sides of equation (17), we get

$$\left(D^2-a^2-Ep_1\sigma^*\right)\Theta^*=-W^*, \tag{30}$$

Therefore, using (30), we get

$$\int_0^1 W^*\Theta dz=-\int_0^1 \Theta\left(D^2-a^2-Ep_1\sigma^*\right)\Theta^* dz, \tag{31}$$

Also taking complex conjugate on both sides of equation (16), we get

$$\frac{F}{P_l}\left(D^2-a^2\right)Z^*-\left(\frac{\sigma^*}{\varepsilon}+\frac{1}{P_l}\right)Z^*=-DW^*, \tag{32}$$

Therefore, using (32), we get

$$\int_0^1 W^*DZdz=-\int_0^1 DW^*Zdz=\frac{F}{P_l}\int_0^1 Z^*\left(D^2-a^2\right)Zdz-\left(\frac{\sigma^*}{\varepsilon}+\frac{1}{P_l}\right)\int_0^1 Z^*Zdz+Q\int_0^1 ZDX^*dz, \tag{33}$$

Integrating by parts the third term on left hand side and using equation (19), and appropriate boundary condition (20), we get

$$\int_0^1 W^*DZdz=\frac{F}{P_l}\int_0^1 Z^*\left(D^2-a^2\right)Zdz-\left(\frac{\sigma^*}{\varepsilon}+\frac{1}{P_l}\right)\int_0^1 Z^*Zdz+Q\int_0^1 X\left(D^2-a^2-p_2\sigma\right)X^*dz, \tag{34}$$

Also taking complex conjugate on both sides of equation (18), we get

$$\left[D^2-a^2-p_2\sigma^*\right]K^*=-DW^*, \tag{35}$$

Therefore, equation (35), using appropriate boundary condition (20), we get

$$\int_0^1 W^*D\left(D^2-a^2\right)Kdz=-\int_0^1 DW^*\left(D^2-a^2\right)Kdz=\int_0^1 K\left(D^2-a^2\right)\left(D^2-a^2-p_2\sigma^*\right)K^*dz, \tag{36}$$

Substituting (31), (34) and (36), in the right hand side of equation (29), we get

$$\left(\frac{\sigma}{\varepsilon}+\frac{1}{P_l}\right)\int_0^1 W^*\left(D^2-a^2\right)Wdz-\frac{F}{P_l}\int_0^1 W^*\left(D^2-a^2\right)^2Wdz=Ra^2\int_0^1 \Theta\left(D^2-a^2-Ep_1\sigma^*\right)\Theta^*dz$$
$$-\frac{T_AF}{P_l}\int_0^1 Z(D^2-a^2)Z^*dz+T_A\left(\frac{\sigma^*}{\varepsilon}+\frac{1}{P_l}\right)\int_0^1 Z^*Zdz-Q\int_0^1 K\left(D^2-a^2\right)\left(D^2-a^2p_2\sigma^*\right)K^*dz, \tag{37}$$

Integrating the terms on both sides of equation (37) for an appropriate number of times and making use of the appropriate boundary conditions (20), we get

$$\frac{F}{P_l}\int_0^1\left\{|D^2W|^2+2a^2|DW|^2+a^4|W|^2\right\}dz+\left(\frac{\sigma}{\varepsilon}+\frac{1}{P_l}\right)\int_0^1\left(|DW|^2+a^2|W|^2\right)dz=Ra^2\int_0^1\left(|D\Theta|^2+a^2|\Theta|^2\right)dz$$
$$+Ra^2Ep_1\sigma^*\int_0^1|\Theta|^2dz-\frac{T_AF}{P_l}\int_0^1\left\{|DZ|^2+a^2|Z|^2\right\}dz-T_A\left(\frac{\sigma^*}{\varepsilon}+\frac{1}{P_l}\right)\int_0^1|Z|^2dz-T_AQ\int_0^1\left(|DX|^2+a^2|X|^2\right)dz$$
$$-T_AQp_2\sigma\int_0^1|X|^2dz-Q\int_0^1\left(|D^2K|^2+2a^2|DK|^2+a^4|K|^2\right)dz-Qp_2\sigma^*\int_0^1\left(|DK|^2+a^2|K|^2\right)dz, \tag{38}$$

Now equating imaginary parts on both sides of equation (38), and cancelling $\sigma_i(\neq 0)$, we get

$$\frac{1}{\varepsilon}\int_0^1\left\{|DW|^2+a^2|W|^2\right\}dz=-Ra^2Ep_1\int_0^1|\Theta|^2dz+\frac{T_A}{\varepsilon}\int_0^1|Z|^2dz+Qp_2\int_0^1\left(|DK|^2+a^2|K|^2\right)dz-T_AQp_2\int_0^1|X|^2dz, \tag{39}$$

Now R $\rangle$ 0, $\varepsilon\rangle 0$ and $T_A\rangle$ 0, utilizing the inequalities (26) and (29), the equation (39) gives,

$$\frac{1}{\varepsilon}\left[1-\frac{P_lT_A}{(2\pi^2F-P_l)}-\varepsilon\left(\frac{Qp_2\pi^2}{(2\pi^2-1)}\right)\right]\int_0^1|DW|^2dz+I_1\langle 0 \tag{40}$$

Where $I_1=\frac{a^2}{\varepsilon}\int_0^1|W|^2dz+Ra^2Ep_1\int_0^1|\Theta|^2dz+T_AQp_2\int_0^1|X|^2dz$,

is positive definite and therefore , we must have

$$\varepsilon\left(\frac{Qp_2\pi^2}{(2\pi^2-1)}\right)+\frac{P_lT_A}{(2\pi^2F-P_l)}\rangle 1. \tag{41}$$

Hence, if

$$\sigma_r\geq 0 \text{ and } \sigma_i\neq 0, \text{ then } \varepsilon\left(\frac{Qp_2\pi^2}{(2\pi^2-1)}\right)+\frac{P_lT_A}{(2\pi^2F-P_l)}\rangle 1\ . \tag{42}$$

And this completes the proof of the theorem.

Presented otherwise from the point of view of existence of instability as stationary convection, the above theorem can be put in the form as follow:-

Theorem 2: The sufficient condition for the onset of instability as a non-oscillatory motions of non-growing amplitude in a couple-stress fluid in a porous medium heated from below, in the presence of uniform vertical magnetic field and rotation is that, $\varepsilon\left(\frac{Qp_2\pi^2}{(2\pi^2-1)}\right)+\frac{P_lT_A}{(2\pi^2F-P_l)}\leq 1$, where T_A is the Taylor number, Q is the Chandrasekhar number, ε is the porosity, P_l is the medium permeability and F is the couple-stress parameter, when both the boundaries are perfectly conducting and rigid.

or

The onset of instability in a couple-stress fluid in a porous medium heated from below, in the presence of uniform vertical magnetic field and rotation, cannot manifest itself as oscillatory motions of growing amplitude if the Taylor number T_A, the Chandrasekhar number Q, the porosity ε, the medium permeability P_l and the couple-stress parameter F, satisfy the inequality $\varepsilon\left(\frac{Qp_2\pi^2}{(2\pi^2-1)}\right)+\frac{P_lT_A}{(2\pi^2F-P_l)}\leq 1$, when both the bounding surfaces are perfectly conducting and rigid.

The sufficient condition for the validity of the 'PES' can be expressed in the form:

Theorem 3: If $(W, \Theta, K, Z, X, \sigma)$, $\sigma = \sigma_r + i\sigma_i$, $\sigma_r \geq 0$ is a solution of equations (15) – (19), with R$\rangle$ 0 and,

$$\varepsilon\left(\frac{Qp_2\pi^2}{(2\pi^2 - 1)}\right) + \frac{P_l T_A}{(2\pi^2 F - P_l)} \leq 1,$$

Then $\sigma_i = 0$.

In particular, the sufficient condition for the validity of the 'exchange principle' i.e., $\sigma_r = 0 \Rightarrow \sigma_i = 0$ if $\varepsilon\left(\frac{Qp_2\pi^2}{(2\pi^2 - 1)}\right) + \frac{P_l T_A}{(2\pi^2 F - P_l)} \leq 1$.

In the context of existence of instability in 'oscillatory modes' and that of 'overstability' in the present configuration, we can state the above theorem as follow:-

Theorem 4: The necessary condition for the existence of instability in 'oscillatory modes' and that of 'overstability' in a couple-stress fluid in a porous medium heated from below, in the presence of uniform vertical magnetic field and rotation is that the Taylor number T_A, the Chandrasekhar number Q, the porosity ε, the couple-stress parameter of the fluid F and the medium permeability P_l, must satisfy the inequality $\varepsilon\left(\frac{Qp_2\pi^2}{(2\pi^2 - 1)}\right) + \frac{P_l T_A}{(2\pi^2 F - P_l)} \rangle 1$, when both the bounding surfaces are perfectly conducting and rigid

Special Cases: It follows from theorem 1 that an arbitrary neutral or unstable mode is non-oscillatory in character and 'PES' is valid for:

(i). Thermal convection in couple-stress fluid heated from below i. e. when Q = 0 = T_A, Sharma and Thakur [23].

(ii). Magneto-thermal convection in couple-stress fluid heated from below (T_A=0), if

$$\varepsilon\left(\frac{Qp_2\pi^2}{(2\pi^2 - 1)}\right) \leq 1.$$

(iii). Rotatory-thermal convection in couple-stress fluid heated from below (Q = 0), if

$$\frac{P_l T_A}{(2\pi^2 F - P_l)} \leq 1.$$

CONCLUSION

This theorem mathematically established that the onset of instability in a couple-stress fluid in the presence of uniform vertical rotation, cannot manifest itself as oscillatory motions of growing amplitude if the Taylor number T_A, the Chandrasekhar number Q, the porosity ε, the couple-stress parameter of the fluid F and the medium permeability P_l, satisfy the inequality $\varepsilon\left(\frac{Qp_2\pi^2}{(2\pi^2 - 1)}\right) + \frac{P_l T_A}{(2\pi^2 F - P_l)} \leq 1$, when both the bounding surfaces are perfectly conducting and rigid.

The essential content of the theorem, from the point of view of linear stability theory is that for the configuration of couple-stress fluid of infinite horizontal extension heated form below, having rigid boundaries at the top and bottom of the fluid, in the presence of uniform vertical magnetic field and rotation, parallel to the force field of gravity, an arbitrary neutral or unstable modes of the system are definitely non-oscillatory in character if $\varepsilon\left(\frac{Qp_2\pi^2}{(2\pi^2 - 1)}\right) + \frac{P_l T_A}{(2\pi^2 F - P_l)} \leq 1$, and in particular PES is valid.

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