Available online at www.pelagiaresearchlibrary.com

Pelagia Research Library

Advances in Applied Science Research, 2012, 3 (6):3618-3623

Pelagia Research Library

ISSN: 0976-8610
CODEN (USA): AASRFC

Branch and bound technique to two stage flow shop scheduling problem in which setup time is separated from processing time and both are associated with probabilities including job block criteria

Deepak Gupta[1] and Harminder Singh[2]

[1]Dept. of Mathematics, Maharishi Markandeshwar Engineering College, Mullana
[2]Bhai Gurdas Institute of Engineering and Technology, Sangrur

ABSTRACT

This paper presents an algorithm with the help of branch and bound approach for a flowshop scheduling problems consisting of n jobs to be processed on 2 machines in which setup time is separated from processing time and both are associated with probabilities including job block criteria.

Keywords : Processing time ,Elapsed time, Branch and Bound ,Setup time, idle/waiting time operator

INTRODUCTION

Scheduling problems are common occurrence in our daily life e.g. ordering of jobs for processing in a manufacturing plant, programs to be run in a sequence at a computer center etc. Such problems exist whenever there is an alternative choice in which a number of jobs can be done. Now-a-days, the decision makers for the manufacturing plant have interest to find a way to successfully manage resources in order to produce products in the most efficient way. They need to design a production schedule to minimize the flow time of a product. The optimal solution for the problem is to find the optimal or near optimal sequence of jobs on each machine in order to minimize the total elapsed time.Johnson (1954) first of all gave a method to minimise the makespan for n-job, two-machine scheduling problems. The scheduling problem practically depends upon the important factors namely, Transportation time, break down effect, Relative importance of a job over another job etc. These concepts were separately studied by Ignall and Scharge (1965),Maggu and Dass (1981), Temiz and Erol(2004),Yoshida and Hitomi (1979), Lomnicki (1965), Palmer (1965) , Bestwick and Hastings (1976), Nawaz et al. (1983) , Sarin and Lefoka (1993) , Koulamas (1998) , Dannenbring (1977) , etc. We have extended the study made by *Singh T.P., Gupta Deepak* by introducing the application of idle waiting time operator Oi,w as defined by Maggu and Das (1980) in scheduling theory. The paper differs from Maggu and Das (1980) in the sense that here the probabilities are associated with processing time on each machine. The operator technique is an easy approach in economical and computational point of view as in comparison to the heuristic approach. We have developed an algorithm minimizing the utilization time of second machine combined with Johnson's algorithm in order to minimize the rental cost of the machines. Further we are using branch and bound technique to minimize the total elapsed time in which setup time is separated from processing time.

Practical Situation:
Many applied and experimental situations exist in our day-to-day working in factories and industrial production concerns etc. In many manufacturing companies different jobs are processed on various machines. These jobs are

required to process in a machine shop A, B, C, ---- in a specified order. When the machines on which jobs are to be processed are planted at different places, the transportation time (which includes loading time, moving time and unloading time etc.) has a significant role in production concern. The break down of the machines (due to delay in material,changes in release and tails date, tool unavailability, failure of electric current, the shift pattern of the facility, fluctuation in processing times, some technical interruption etc.) have significant role in the production concern

NOTATIONS:

S : Sequence of jobs 1, 2, 3,….,n
S_k :Sequence obtained by applying Johnson's procedure, k = 1, 2, 3, ------ r.
M_j : Machine j= 1,2,
a_{ij}: Processing time of i^{th} job on machine M_j
p_{ij}: Probability associated to the Processing time a_{ij}
A_{ij} : Expected processing time.
S_{ij}: Set up time of ^{ith} job on machine M_j.
q_{ij}: Probability associated to the set up time a_{ij}
S_{ij}: Expected set up time of i^{th} job on machine M_j
$t_{ij}(S_k)$: Completion time of i^{th} job of sequence S_k on machine M_j

THEOREM

Let n jobs 1, 2, 3, …………n are processed through two machines A & B in order AB with processing time a_i & b_i(i = 1, 2, 3, ……….n) on machine A and B respectively.
If $(a_p, b_p)\ O_{i,w}\ (a_q, b_q) = (a_\beta, b_\beta)$
then $a_\beta = a_p + \max(a_q - b_p, 0)$
and $b_\beta = b_q + \max(b_q - a_q, 0)$
where β is the equivalent job for job block (p, q) and p, q ϵ {1, 2, 3, …………..n}.

Proof: Starting by the equivalent job block criteria theorem for β = (p, q) given by Maggu & Das [6], we have:
$a_\beta = a_p + a_q - \min(b_p, a_q)$ …(1)
$b_\beta = b_p + b_q \min(b_p, a_q)$ …(2)

Now, we prove the above theorem by a simple logic:
Case I: When $a_q > b_p$
$a_q > b_p > 0$
$\max\{a_q > b_p, 0\} = a_q > b_p$ …(3)
and
$b_p > a_q < 0$
$\max\{b_p > a_q, 0\} = 0$ …(4)
(1) $a_\beta = a_p + a_q - \min(b_p, a_q)$
$= a_p + a_q - b_p$ as $a_q > b_p$
$= a_p + \max\{a_q - b_p, 0\}$ (using (3) …(5)
(2) $b_\beta = b_p + b_q - \min(b_p, a_q)$
$= b_p + b_q - b_p$ as $a_q > b_p$
$= b_q + (b_p - b_p)$
$= b_q + 0$
$= b_q + \max(b_p - a_q, 0)$ (using (4) … (6)

Case II : When $a_q < b_p$
$a_q - b_p < 0$
$\max(a_q - b_p, 0) = 0$ …(7)
and
$b_p - a_q > 0$
$\max(b_p - a_q, 0) = b_p - a_q$ …(8)
(1) $a_\beta = a_p + a_q - \min(b_p, a_q)$
$= a_p + a_q - a_q$ as $a_q > b_p$
$= a_p + 0$

$=a_p + \max(a_q - b_p, 0)$ (using (7)) …(9)
(2) $b_\beta = b_p + b_q - \min(b_p, a_q)$
$= b_p + b_q - a_q$ as $a_q < b_p$
$= b_p + (b_p - a_q)$
$= b_p + \max(b_p - a_q, 0)$ (using (8)) … (10)

Case III : When $a_q = b_p$, $a_q - b_p = 0$
Therefore, $\max(a_q - b_p, 0) = 0$ … (11)
Also $b_p - a_q = 0$
Therefore, $\max(b_p - a_q, 0) = 0$ … (12)
(1) $a_\beta = a_p + a_q - \min(b_p, a_q)$
$= b_p + a_q - a_p$ as $b_q = a_p$
$= a_p + 0$
$= a_p + \max(a_q - b_p, 0)$ … (13)
(2) $b_\beta = b_p + b_q - \min(b_p, a_q)$
$= b_p + b_q - b_p$
$= b_q + (b_p - b_p)$
$= b_q + 0$
$=b_q + \max(b_p - a_q, 0)$ (using (12) …(14)
By (5), (6), (9), (10), (13) and (14) we conclude :
$a_\beta = a_p + \max(a_q - b_p, 0)$
$b_\beta = b_p + \max(b_p - a_q, 0)$ for all possible three cases.

The theorem can be generalized for more number of job blocks as stated :
Let n jobs 1, 2, 3, ………..n are processed through two machines A & B in order AB with processing time a_i & b_i (i = 1, 2, 3, ……..n) on machine A & B respectively.

If $(a_{io}, b_{io})\ O_{i,w}\ (ai_1, bi_1)\ O_{i,w}\ (ai_2, bi_2)\ O_{i,w} \ldots\ O_{i,w}\ (a_{ip}, b_{ip}) = (a_\beta, b_\beta)$
Then

$$a_\beta = a_{io} + \sum_{J=1}^{p} \max\{a_{ij} - b_{i(j-1)}\ 0\}$$

and $$b_\beta = b_{ip} + \sum_{J=1}^{p} \max\{b_{i(j-1)} - a_{ij}, 0\}$$

where $i_0, i_1, i_2, i_3, \ldots ip \in \{1, 2, 3 \ldots n\}$ and β is the equivalent job for job block ($i_0, i_1, i_2, i_3, \ldots ip$). The proof can be made using Mathematical induction technique on the lines of Maggu & Das [8].
In the light of above theorem operator $O_{i,w}$ (Idle/Waiting time Operator) is defined as follows :

Definition 1 : Let R_+ be the set of non negative numbers. Let $G = R_+ \times R_+$. Then $O_{i,w}$ is defined as a mapping from G x G → G given by:

$O_{i,w}[(x_1, y_1), (x_2, y_2)] = (x_1, y_1)\ O_{i,w}\ (x_2, y_2)$

$= [x_1 + \max((x_2 - y_1), 0), y_2 + \max((y_1 - x_2), 0)]$, where $x_1, x_2, y_1, y_2 \in R_+$.

Definition 2 : An operation is defined as a specific job on a particular machine.

Definition 3 : Total elapsed time for a given sequence

= Sum of expected processing time on 2 [nd] machine (M_2) + Total idle time on M_2

$= \sum_{i=1}^{n} B'_i + \sum_{i=1}^{n} I_{i2} = \sum_{i=1}^{n} B'_i + \max[P_k]$, where $P_K = \sum_{i=1}^{k} A'_i - \sum_{i=1}^{k-1} B'_i$.

PRACTICAL SITUATION

Various practical situations occur in real life when one has got the assignments but does not have one's own machine or does not have enough money or does not want to take risk of investing huge amount of money to purchase machine. Under such circumstances the machine has to be taken on rent in order to complete the assignments. In his starting career, we find a medical practitioner does not buy expensive machines say X-ray machine, the ultra sound machine etc. but instead takes on rent. The examination branch of a board/institute needs machines as data entry machine, computer, printer etc. on rent for computerizing and compiling examination result for secrecy point of view.

Moreover in hospitals industries concern, sometimes the priority of one job over other is preferred. It may be because of urgency or demand of its relative importance. Hence the job block criteria becomes significant.

PROBLEM FORMULATION

Let n jobs 1,2,……..,n be processed on three machines M_1 , M_2 and M_3 in a way such that no passing is allowed. Let a_{ij} be the processing time of i^{th} job o n j^{th} machine with probabilities p_{ij} and s_{ij} be the set up time of i^{th} job on j^{th} machine with probabilities q_{ij}. Let A_{ij} be the expected processing time and S_{ij} be the expected set up time of i^{th} job on j^{th} machine.

Also consider the following structural relation.

either Min $(A_{i1} - S_{i2}) \geq \max (A_{i2} - S_{i1})$

or Max $(A_{i3} - S_{i2}) \geq \max (A_{i2} - S_{i3})$ or both.

THE MATHEMATICAL MODEL OF THE PROBLEM IN MATRIX FORM CAN BE STATED AS:

Jobs	Machine M_1				Machine M_2				Machine M_3			
I	a_{i1}	p_{i1}	s_{i1}	q_{i1}	a_{i2}	p_{i2}	s_{i2}	q_{i2}	a_{i3}	p_{i3}	s_{i3}	q_{i3}
1	A_{11}	p_{11}	s_{11}	q_{11}	a_{21}	p_{21}	s_{21}	q_{21}	a_{31}	p_{31}	s_{31}	q_{31}
2	A_{12}	p_{12}	s_{12}	q_{12}	a_{22}	p_{22}	s_{22}	q_{22}	a_{32}	p_{32}	s_{32}	q_{32}
3	A_{13}	p_{13}	s_{13}	q_{13}	a_{23}	p_{23}	s_{23}	q_{23}	a_{33}	p_{33}	s_{33}	q_{33}
-	-	-	-	-	-	-	-	-	-	-	-	-
n	A_{1n}	p_{1n}	s_{1n}	q_{1n}	a_{n2}	p_{n2}	s_{n2}	q_{n2}	a_{n3}	p_{n3}	s_{n3}	q_{n3}

Step 2: $O_{i,w}[(x_1, y_1), (x_2, y_2)] = (x_1, y_1)\ O_{i,w}\ (x_2, y_2)$

$= [x_1 + \max ((x_2 - y_1), 0), y_2 + \max ((y_1 - x_2), 0)]$, where $x_1, x_2, y_1, y_2 \in R_+$.

Step 3:

Calculate the lower bounds using the following formula

(i) $l_1 = t(j_r, 1) + \sum_{i \epsilon j_r^i} G_i + min_{i \epsilon j_r^i}(H_i)$

(ii) $l_2 = t(j_r, 2) + \sum_{i \epsilon j_r^i} H_i$

(iii)

Step 4:

Calculate $l = max\ (l_1, l_2)$

Step 5:

We evaluate l first for the n classes of permutations, i.e. for these starting with 1, 2, 3………n respectively, having labelled the appropriate vertices of the scheduling tree by these values.

Step 6:Now explore the vertex with lowest label. Evaluate l for the (n-1) subclasses starting with this vertex and again concentrate on the lowest label vertex. Continuing this way, until we reach at the end of the tree represented by two single permutations, for which we evaluate the total work duration. Thus we get the optimal schedule of the jobs.

Step 7: Prepare in-out table for the optimal sequence obtained in step 3 and get the minimum total elapsed time.
Numerical illustration :let 5 jobs are to be processed on two machines in which processing time and setting times are given with their respectative probabilities

Jobs	Machine M_1				Machine M_2			
i	A_i	p_i	s_i^a	r_i	B_I	Q_i	S_i^b	s_i
1	8	.2	4	.2	15	.1	4	.2
2	15	.1	8	.2	20	.3	2	.4
3	16	.1	6	.2	16	.2	5	.1
4	8	.4	2	.3	10	.2	6	.1
5	14	.2	5	.1	9	.2	8	.2

Our objective the total elapsed time.

Solution:Step1.define expected time

JOBS	MACHINE A	MACHINE B
	A_i	B_i
1	0.8	0.7
2	0.7	4.4
3	1.1	2
4	2.6	1.4
5	1.2	1.3

Step2: : $O_{i,w}[(x_1, y_1), (x_2, y_2)] = (x_1, y_1)\ O_{i,w}\ (x_2, y_2)$
$= [x_1 + \max((x_2 - y_1), 0), y_2 + \max((y_1 - x_2), 0)]$, where $x_1, x_2, y_1, y_2 \in R_+$.

JOBS	MACHINE A	MACHINE B
	A_i	B_i
1	0.8	0.7
α	0.7	3.2
3	1.1	2
5	1.2	1.3

Step3: $l_1 = t(j_r, 1) + \sum_{i \epsilon j_r^i} G_i + min_{i \epsilon j_r^i}(H_i)$
$l_2 = t(j_r, 2) + \sum_{i \epsilon j_r^i} H_i$

Step 4 and **Step 5**:
We have LB(1) = 8,LB(α)= 7.9, LB(3) = 8.3, LB(4) = 8.4
step 6:LB(α,1) = 1.5 , LB(α,2) = 2, LB(α,3) = 7.2 , LB(α ,3,1,5)= 6.1 , LB(α ,3,5,1) = 6.1therefore, sequence is S :(3,2,4,1)

Step 7:

JOBS	MACHINE A	MACHINE B
	IN -OUT	IN -OUT
2	0-1.5	1.5-7.5
4	3.1-6.3	7.5-9.5
3	6.9-7.5	9.5-12.7
1	8.7-10.3	12.7-14.2
5	18.3-21.1	21.1-22.9

So .total elapsed time is 22.9

REFERENCES

[1] Brah, S.A. and Loo, L.L.,**1999**, *European Journal of Operation Research, Vol. 113, No.1, pp.113-122.*
[2] Brown, A.P.G. and Lomnicki, Z.A. **1966**, *Operational Research Quarterly, Vol. 17, pp.173-182.*
[3] Chander Shekharan, K, Rajendra, Deepak Chanderi **1992**, *European Journal of Operation Research Vol. 61,pp. 318-325.*
[4] Futatsuishi, Y., Watanabe, I., and Nakanishi, T. **2002**, *Mathematics and Computers in Simulation,Vol. 59, No. 3, pp.73-79.*

[5] Ignall, E. and Schrage, L. **1965**, *Operations Research, Vol. 13, pp.400-412.*
[6] Johnson, S.M. **1954**, *Naval Research Logistics Quarterly,Vol. 1 No. 1, pp. 61-68.*
[7] Kim, J.S., Kang, S.H.,and Lee, S.M.**1997**,*Omega,Vol. 25, No. 1, pp. 547-555*
[8] Lomnicki, Z.A. **1965**, *Operational Research Quarterly, Vol. 16, pp.89-100.*
[9] Maggu & Das **1981**, *Pure and Applied Mathematika Sciences, 12-16.*
[10]Yang, D.L. and Chern, M.S. **2000** *Computers and Operations Research, Vol.27, No.10, pp.975-985*.
[11]Yoshida, T. and Hitomi, K., **1979**, *AIIE Transactions, Vol. 11, No. 1, pp. 261-273.*
[12] Maggu, P.L. and Das, G**.1980**, *PAMS, Vol. XI, No., pp 1-2.*