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Bi-objective scheduling on parallel machines with uncertain processing time

Deepak Gupta[1], Sameer Sharma[2] and Shefali Aggarwal[1]

[1]Department of Mathematics, M. M. University, Mullana, Ambala, India
[2]Department of Mathematics, D.A.V. College, Jalandhar, Punjab, India

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ABSTRACT

In this paper we consider bi-criteria scheduling problem on parallel machines which optimizes the number of tardy jobs with the condition of primary criterion of T_{max} remains optimized. The processing times of jobs are uncertain that is not known exactly and only estimated values are given. This leads to the use of fuzzy numbers for representing these imprecise values. Here, we use triangular fuzzy numbers to describe the processing times. The membership functions of fuzzy processing times denote the grades of satisfaction with respect to completion times with jobs. The objective of the paper is to develop an algorithm for the bi-objective problem which optimizes the number of tardy jobs without violating the value of $T_{max.}$ A numerical example demonstrates the computational process of the projected algorithm.

Key words: Fuzzy processing time, Average high ranking <A.H.R.>, Maximum tardiness (T_{max}), Total tardiness, Due date.

INTRODUCTION

Parallel machines scheduling problem is a kind of important multi-machine scheduling problem. It means every machine has same work function and every job can be processed by any machine. All machines have the same function. Since the beginning, most of the research in scheduling has concentrated on a single criterion. Numerous optimal and approximation algorithms have been developed for single-criterion problems. A survey of literature has revealed that little work has been reported on the bicriteria scheduling problems. Chen and Bulfin [2] studied single machine bicriteria problems with various combinations of different criteria. Most of the problems in bicriteria scheduling literature involve minimization of flow time subject to a T_{max} value. Minimization of weighted completion time subject to minimal value of T_{max} was first considered by Smith [13]. Vaiaraktarakis and Chung [15] proposed a branch and bound algorithm to minimize total tardiness subject to minimum number of tardy jobs. Azizogulu [1] used a branch and bound method to solve the total earliness and total tardiness problem for the single machine problem. Tabucanon and Cenna [3] studied bi-criteria scheduling problem in a job shop with parallel processors. Sarin and Hariharan [11] proposed a heuristic approach to optimize number of tardy jobs under the constraint of maximum tardiness. Raymond [7] used a branch and bound approach to solve the problem for steel plants involving single machine bicriteria problem. Pickens and Hoff [6] used Fuzzy goal programming approach for forest harvest scheduling. Sunita and Singh T.P [8, 9] studied the optimization of various parameters on parallel machines in fuzzy environment with a single criteria. As per literature review it has been found that majority of the research work has been focused in the area of the single machine bicriteria problems. Parallel machine bicriteria problems have not gained much attention. Divya Prakash [4] studied the bi-criteria scheduling problems on parallel machines.

Sunita and Singh T. P. [10] studied the bi-objective in fuzzy scheduling on parallel machines in which the problem is divided in two steps, in the primary step maximum tardiness (T_{max}) of jobs is calculated and in secondary step, the number of tardy jobs (NT) is minimized with bi-objective function as NT / T_{max}. The present paper is an attempt to extend their study by introducing a system of four parallel machines with an objective to optimize NT / T_{max} under fuzzy environment.

PROBLEM FORMUALTION

In general, two approaches can be used to tackle bicriteria problems:

1. Both the criteria are optimized simultaneously by using suitable weights for criteria
2. The criteria are optimized sequentially by first optimizing the primary criterion and then the secondary criterion subject to the value obtained for the primary criterion.

In the present paper the second type of the approach is adopted.

The formulation of the bi-criteria problems are similar to that of single criteria problems with some additional constraints requiring that the optimal value of the primary objective is not violated. There are two parts of the formulation.

Primary Objective function

Subject to: Primary problem constraints

Secondary objective function

Subject to:

(a) Secondary problem constraint

(b) Primary objective function value constraint

(c) Primary problem constraint.

Here, we consider the parallel machines bicriteria scheduling problem in which the objective is to schedule jobs on parallel identical machines so as to minimize primary and secondary criteria. So, the problem is solved in two parts. Firstly, we optimize the primary criterion followed by the optimization of secondary criteria subject to Primary objective value.

ASSUMPTIONS

The following assumptions are taken in developing the algorithm for bicriteria problem on parallel machines.

1. The jobs are available at time zero.
2. The jobs are independent of each other.
3. No preemption is allowed.
4. The machines are identical in all respects and are available all the time.

FUZZY MEMBERSHIP FUNCTION

All information contained in a fuzzy set is described by its membership function. The triangular membership functions are used to represent fuzzy processing times in our algorithm. Figure 1 shows the triangular membership function of a fuzzy set $\tilde{P}$, $\tilde{P}$ = (a, b, c). The membership value reaches the highest point at 'b', while 'a' and 'c' denote the lower bound and upper bound of the set $\tilde{P}$ respectively. The membership value of the x denoted by $\mu(x), x \in R^+$, can be calculated according to the following formula.

$$\mu(x) = \begin{cases} 0; x \le a \\ \dfrac{x-a}{b-a}; a \le x \le b \\ \dfrac{c-x}{c-b}; b < x < c \\ o; x \ge c \end{cases}$$

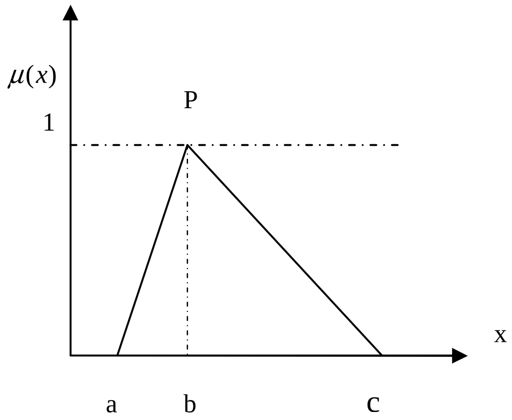

Fig 1: Triangular Fuzzy Membership Function

AVERAGE HIGH RANKING<A.H.R.>

To find the optimal sequence, the processing times of the jobs are calculated by using Yager's [14] average high ranking formula (AHR) = $h(A) = \dfrac{3b+c-a}{3}$.

FUZZY ARITHMETIC OPERATIONS

If $A_1 = (m_{A_1}, \alpha_{A_1}, \beta_{A_1})$ and $A_2 = (m_{A_2}, \alpha_{A_2}, \beta_{A_2})$ be the two triangular fuzzy numbers, then

1. $A_1 + A_2 = (m_{A_1}, \alpha_{A_1}, \beta_{A_1}) + (m_{A_2}, \alpha_{A_2}, \beta_{A_2}) = (m_{A_1} + m_{A_2}, \alpha_{A_1} + \alpha_{A_2}, \beta_{A_1} + \beta_{A_2})$
2. $A_1 - A_2 = (m_{A_1}, \alpha_{A_1}, \beta_{A_1}) - (m_{A_2}, \alpha_{A_2}, \beta_{A_2}) = (m_{A_1} - m_{A_2}, \alpha_{A_1} + \beta_{A_2}, \alpha_{A_2} + \beta_{A_1})$
3. $kA_1 = k(m_{A_1}, \alpha_{A_1}, \beta_{A_1}) = (km_{A_1}, k\alpha_{A_1}, k\beta_{A_1})$; if k>0.
4. $kA_1 = k(m_{A_1}, \alpha_{A_1}, \beta_{A_1}) = (k\beta_{A_1}, k\alpha_{A_1}, km_{A_1})$; if k<0.

NOTATIONS

i	: Represents the i^{th} job, $i = 1, 2, 3, \quad , N$
d_i	: Due date of the i^{th} job.
t_i	: Completion time of i^{th} job
T_i	: Tardiness of the i^{th} job = max (t_i - d_i, 0)
NT	: Number of the tardy jobs
T_{max}	: Maximum tardiness
j	: Location of i^{th} job on machine k
N	: Total number of jobs to be scheduled
k	: Machine on which i^{th} job is assigned at position j

LEMMA 1:
If the jobs are allocated in the early due date (EDD) order first to machine *k* and then to machine *l* and if a job i' allocated to machine *l* is tardy then the job *i* allocated to machine *k* at an identical position to that of the job i' on machine *l* will also be a tardy job.

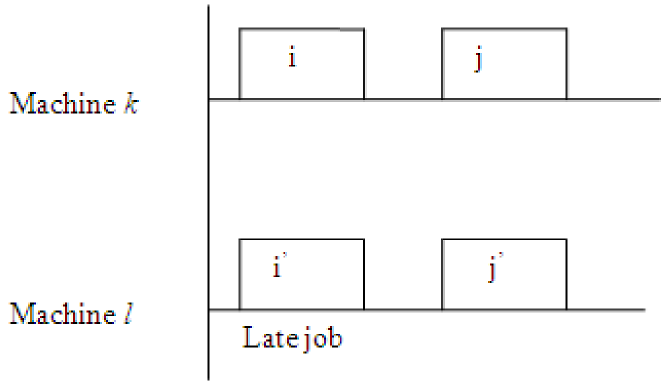

Fig. 2: Jobs located at identical locations on different machines

Proof: Since the jobs are allocated in the EDD first to machine *k* and then to machine *l*. Therefore, we have $d_i^{'} \geq d_i$. Also, the processing times of all the jobs are equal. So, it indicates that, if a job is late on a machine and there are other jobs ending in the identical positions on earlier machines, then all these jobs on previous machines are also late jobs on that particular machine.

THEOREM 1
A sequence of jobs S following EDD is the optimal sequence with minimum number of tardy jobs.

Proof: Let if possible sequence S of jobs is not optimal. Let there exist a better sequence of jobs S'(say) .i.e. a late job of schedule S (say i^{th} job) is early in S'. Let the job *j* replaces the job *i* in sequence S'. As the jobs are in EDD order and processing time of all jobs are equal. Therefore if the job *j* is early in S' then job *i* can not be late in S, which contradicts the assumption that job *i* is late in S. Hence the sequence S following EDD is the optimal sequence with minimum number of tardy jobs (NT).

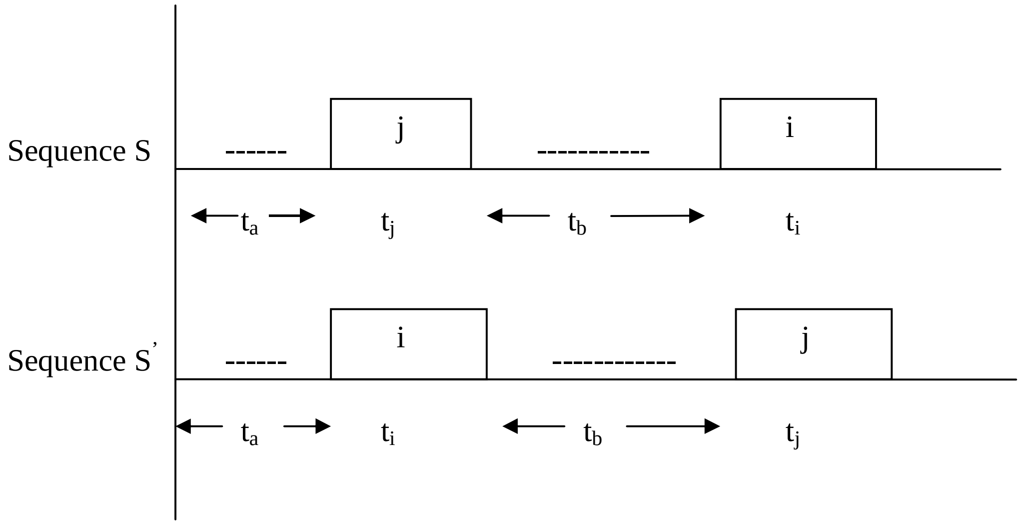

Fig. 3: Jobs located at identical locations on different machines with processing times

THEOREM 2

A Sequence S following EDD is an optimal sequence for bi-criteria problem NT / T_{max}.

Proof: Let if possible the sequence S is not optimal. Let S' is a sequence of jobs better than S, .i.e. there is atleast one job say *i* which is tardy in S and early in S'. Let the job *j* replaces the job *i* in the sequence S'. Let T_{max} be the maximum tardiness of sequence S and T'_{max} be the maximum tardiness of sequence S'.

Let t_a is the processing time of jobs before j^{th} job and t_b is the processing time between j^{th} job and i^{th} job in the sequence S. Therefore,

Tardiness of j^{th} job in the sequence S = $T_j = \max(t_a + t_j - d_j, 0)$

Tardiness of j^{th} job in the sequence S' = $T_j' = \max(t_a + t_i + t_b + t_j - d_j, 0)$ = T'_{max}

Tardiness of i^{th} job in the sequence S = $T_i = \max(t_a + t_j + t_b + t_i - d_i, 0)$ = T_{max}

Tardiness of i^{th} job in the sequence S = $T_i' = \max(t_a + t_i - d_i, 0)$

Here, we have $T_j' \geq T_j$ and $T_j' \geq T_i$ as $d_j \leq d_i$.

Further, if the job *j* is early in S' then the job *i* can not be late in S, which violate the assumption. Also if the job *j* is the late job then $T'_{max} \geq T_{max}$, .i.e. S' violates the value of T_{max}. Also from theorem 1, if the job *j* is late job then the sequence S' violate the minimum value of NT. Hence both of these cases result in a sequence S' that is not better than S. Hence the sequence following EDD is an optimal sequence for bi-criteria problem NT / T_{max}.

ALGORITHM

The algorithm proposed to find the optimal sequence for bi-criteria NT / T_{max} problem is as follows:
Step 1: Find the A.H.R. (Average High Ranking) of the fuzzy processing time.
Step 2: Arrange all the jobs on the available parallel machines in order of early due dates.
Step 3: Find the value of maximum tardiness (T_{max}) and number of tardy jobs (NT).
Step 4: Let L be the set of all late jobs in the current schedule.
Step 5: Select the first late job $i \in L$. Put this late job (*i*) as late as possible without violating the value of T_{max} and count the number of tardy jobs (NT).
Step 6: Repeat the step 4 for all the late jobs. The sequence with minimum NT without violating the value of T_{max} will be the optimal sequence.

NUMERICAL ILLUSTRATION

A number of jobs with processing time in fuzzy environment with due date on four parallel machines in a flow shop problem is given. Optimize the number of tardy jobs with condition of T_{max}.

Table 1: Jobs with fuzzy processing time and due date

Jobs	1	2	3	4	5	6	7
Processing Time	(8, 9 ,10)	(15, 16, 17)	(8, 9 ,10)	(5, 6, 7)	(9, 10, 11)	(9, 10, 11)	(10, 11, 12)
Due date	12	18	17	10	14	8	13

Solution: As per step 1, the A.H.R. of the jobs with fuzzy processing times is

Table 2: Jobs with A.H.R. and due date

Jobs	1	2	3	4	5	6	7
Processing Time	29/3	50/3	29/3	20/3	32/3	32/3	35/3
Due date	12	18	17	10	14	8	13

As per step 2, the schedule of the jobs on the available four parallel machines in order of early due dates is

Table 3: Flow table of the jobs on four parallel machines in EDD order

Jobs	6	4	1	7	5	3	2
M_1	0 – 32/3						32/3 – 82/3
M_2		0 – 20/3			20/3 – 52/3		
M_3			0 – 29/3			29/3 – 58/3	
M_4				0 – 35/3			
d_i	8	10	12	13	14	17	18
T_i	3	-	-	-	10/3	7/3	28/3

Therefore, T_{max} = 28/3 and number of tardy jobs = NT = 4.
As per step 4, the set of late jobs = L = {6, 5, 3, 2}
As per step 5, the first late job in set L is 6^{th} job. To optimize the number of tardy jobs put this 6^{th} job in the last, we have

Table 3: Flow table of the jobs on four parallel machines

Jobs	4	1	7	5	3	2	6
M_1	0 – 20/3				20/3 – 49/3		
M_2		0 – 29/3				29/3 – 79/3	
M_3			0 – 35/3				
M_4				0 – 32/3			32/3 – 64/3
d_i	10	12	13	14	17	18	8
T_i	-	-	-	-	-	25/3	40/3

Therefore, T_{max} = 40/3 which violate the value of T_{max}, hence can not be optimal.
Now, Here L = {2, 6}
On repeating the process to optimize the number of tardy jobs, we have

Table 4: Flow table of the jobs on four parallel machines

Jobs	4	1	7	5	3	6	2
M_1	0 – 20/3				20/3 – 49/3		
M_2		0 – 29/3				29/3 – 61/3	
M_3			0 – 35/3				
M_4				0 – 32/3			32/3 – 82/3
d_i	10	12	13	14	17	8	18
T_i	-	-	-	-	-	37/3	28/3

Therefore, T_{max} = 28/3 and number of tardy jobs = NT = 2. Hence, it optimizes NT keeping T_{max} unchanged.

CONCLUSION

For a given set of jobs initially arranged in EDD order, a late job considered for being exchanged must be with another late job or a job having the same due date so as to improve the value of secondary criteria taken as number of tardy jobs, given the primary criterion of minimizing total tardiness. The study may further be extended by generalizing the number of parallel machines and the Trapezoidal fuzzy member function may be considered to represent the processing time of the jobs.

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