Available online at www.pelagiaresearchlibrary.com

Pelagia Research Library

Advances in Applied Science Research, 2012, 3 (6):3551-3557

ISSN: 0976-8610
CODEN (USA): AASRFC

Bingham Plastic characteristic of blood flow through a generalized atherosclerotic artery with multiple stenoses

S. S. Yadav and Krishna Kumar

Narain (P.G.) College, Shikohabad (India)

ABSTRACT

In this paper, we have investigated the effects of length of stenosis, shape parameter, parameter γ on the resistance to flow for a Bingham plastic flow of blood through a generalized artery having multiple stenoses locate at equal distances. Here, the rheology of the flowing blood is characterized by non-Newtonian fluid model. The study reveals that flow resistance decreases as shape parameter and γ increases and it increases as stenosis height and length increase. We have also shown the variation of wall shear stress for different values of parameter γ.

Keywords: Resistance to flow, wall shear stress, stenosis, shape parameter

INTRODUCTION

Hardening of the arteries is a procedure that often occurs with aging. As we grow older, plaque buildup narrows our arteries and makes them stiffer. These changes make it harder for blood to flow through them. Clots may form in these narrowed arteries and block blood flow. Pieces of plaque can also break off and move to smaller blood vessels, blocking them. Either way, the blockage starves tissues of blood and oxygen, which can result in damage or tissue death. This is a common cause of heart attack and stroke. High blood cholesterol levels can cause hardening of the arteries at a younger age. For many people, high cholesterol levels are the result of an unhealthy lifestyle most commonly, eating a diet that is high in fat. Other risk factors are heavy alcohol use, lack of exercise, and being overweight.

Singh [1] observed that flow resistance decreases as shape parameter and γ increases and it increases as stenosis height and length increase. He has also shown that wall shear stress increases up to mid axial distance and then it decreases for increasing values of parameter γ. Singh and Singh [2] discussed the effects of flow behavior indices on non –Newtonian blood flow in small arteries having multiple stenoses and post-stenotic dilatation has been discussed. They observed that as flow index behavior increases, rate of flow resistance ratio increases. Singh et al. [3] assumed the stenosis is mild and radially non-symmetric. They performed the graphical analysis for a single loop of stenosis having maximum depression at different point. They also observed that increasing value of parameter γ shows lower variation. Singh and Shah [4] presented a numerical model to investigate the effect of shape of stenosis on blood flow through an artery using power-law fluid model. They found that the resistance to flow, wall shear stress and apparent viscosity decreases as stenosis shape parameter increases but increases as stenosis size and stenosis length increases. Krishna et al. [5] discussed the effects of various parameters like Darcy number, slip parameter, radius on velocity, frictional force. They obtained expressions for velocity, volume flow rate and frictional forces. Shah [6] presented a numerical model to determine the effect of magnetic field on blood flow through an axially non-symmetric but radially symmetric atherosclerotic artery. She has also shown that in the presence of magnetic field, blood did not drastically change the flow patterns, but caused an appreciable decrease in the shear stresses and a slightly lower resistance to flow.

Mishra and Verma [7] investigated the wall shear stress, resistance parameter and flow rate across mild stenosis situated symmetrically on steady blood flow through blood vessels with uniform or non-uniform cross-section by assuming the blood to be Non-Newtonian, incompressible and homogeneous fluid. Misra and Shit [8] developed a mathematical model for studying the non-Newtonian flow of blood through a stenosed arterial segment. They noticed that the resistance of flow and the skin-friction increase as the stenosis height increases. Bali and Awasthi [9] studied flow of blood through a multistenosed artery under the influence of external applied magnetic field. They observed the effect of magnetic field, height of stenosis, parameter determining the shape of the stenosis on velocity field, volumetric flow rate in stenotic region. Sarojamma et al. [10] studied the influence of body acceleration on the flow of blood through a stenosed catheterized artery. They found that the insertion of a catheter and yield stress of the fluid increase the resistance to flow enormously depending upon the size of catheter. Singh and Singh [11] considered a fully developed one dimensional Bingham plastic flow of blood through a small artery having multiple stenoses and post-stenotic dilatation. They observed that the resistance- to - flow moves nearer to unity as yield stress increases and flux decreases. Variation in viscosities shows no significant change in Resistance –to-flow ratio.

MATHEMATICAL FORMULATION

Let us consider an artery with stenosis symmetrical about the axis but non-symmetrical with respect to radial co-ordinates. The mathematical expression for geometry is given by

$$\frac{R}{R_0} = 1 - \varepsilon\left[L_0^{\,s-1}\{\gamma z - nd - (n-1)l_0\} - \{\gamma z - nd - (n-1)L_0\}^s \right]$$

$$; n(d + L_0) - L_0 \le \gamma z \le n(d + L_0)$$

$$= 1 \text{ ; Otherwise} \qquad (1)$$

Where R, R_0 are tube radius (with, without stenosis), $s \ge 2$ is a shape parameter determining stenosis shape, L_0 is stenosis length, d indicates its location. n is the number of stenosis in the artery, γ is a positive number greater and equal than 1.

$$\varepsilon = \frac{\delta}{R_0} \frac{s^{s/s-1}}{L_0^{\,s}(s-1)} \qquad (2)$$

δ be the maximum height of the stenosis at

$$z = \frac{nd + (n-1)L_0 + L_0 / s^{1/s-1}}{\gamma}$$

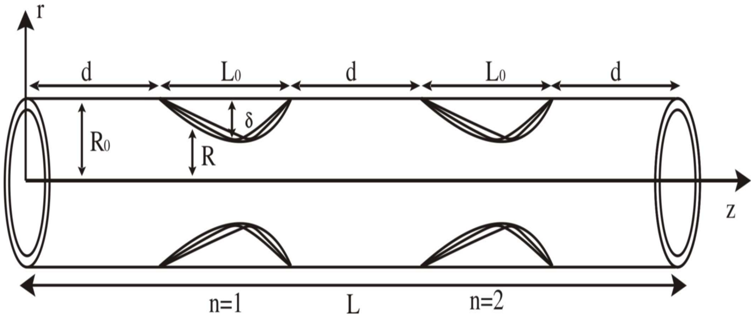

Fig.1: Geometry of Artery with multiple Stenoses

The constitutive equations for Bingham plastic fluid are

$$\dot{\beta} = f(\tau) = -\frac{du}{dr} = \begin{cases} \dfrac{\tau - \tau_0}{\mu} & \tau \geq \tau_0 \\ 0 & \tau \leq \tau_0 \end{cases} \quad (3)$$

The flux Q through the artery is given by

$$Q = \int_0^R 2\pi r u dr \quad (4)$$

Integrating (4) and using the no-slip boundary condition $u = 0$ when $r = R$

$$Q = \pi \int_0^R r^2 \left(-\frac{du}{dr} \right) dr \quad (5)$$

Applying (3) in (5) to obtain

$$Q = \pi \int_0^R r^2 f(\tau) dr \quad (6)$$

The expressions for τ and τ_R (the shear at the wall i.e. when $r = R$) is given by

$$\tau = -\frac{r}{2}\frac{dp}{dz} \text{ and } \tau_R = -\frac{R}{2}\frac{dp}{dz} \quad (7)$$

Where p is the pressure. From equations (3) and (6), we get

$$Q = \frac{\pi R^3}{\tau_R{}^3} \int_0^{\tau_R} \tau^2 f(\tau) dr \quad (8)$$

Substitution of equation (3) and rearrangement gives the result

$$Q = \frac{\pi R^3}{\mu} \left[\frac{\tau_R{}^4}{4} - \tau_0 \frac{\tau_R{}^3}{3} \right] \Rightarrow \tau_R = \frac{4\mu Q}{\pi R^3} + \frac{4}{3}\tau_0 \quad (9)$$

Using the second result of equation (7)

$$\frac{dp}{dz} = -\frac{2}{R} \left[\frac{4\mu Q}{\pi R^3} + \frac{4}{3}\tau_0 \right] \quad (10)$$

$$\frac{dp}{dz} = -\frac{8\mu Q}{\pi R^4} - \frac{8}{3}\tau_0 \frac{1}{R} \tag{11}$$

Integrating equation (11) with respect to z with the condition that

$p = p_1$ at $z = 0$ and $p = p_0$ at $z = l$

$$p_1 - p_0 = -\frac{8\mu Q}{\pi R_0^{\ 4}} \int_0^L \left(\frac{R}{R_0}\right)^{-4} dz - \frac{8\tau_0}{3R_0} \int_0^L \left(\frac{R}{R_0}\right)^{-1} dz \tag{12}$$

$$\lambda = \frac{p_1 - p_0}{Q} = -\frac{8\mu}{\pi R_0^{\ 4}} \int_0^L \left(\frac{R}{R_0}\right)^{-4} dz - \frac{8\tau_0}{3R_0 Q} \int_0^L \left(\frac{R}{R_0}\right)^{-1} dz \tag{13}$$

Suppose $f_1 = \frac{8\mu}{\pi R_0^{\ 4}}$; $f_2 = \frac{8\tau_0}{3R_0 Q}$

$$\lambda = -f_1 \left\{ \int_0^{\frac{n(d+L_0)-L_0}{\gamma}} \left(\frac{R}{R_0}\right)^{-4} dz + \sum_{n=1}^{n=n_{\max}} \int_{\frac{n(d+L_0)-L_0}{\gamma}}^{\frac{n(d+L_0)}{\gamma}} \left(\frac{R}{R_0}\right)^{-4} dz + \int_{\frac{n(d+L_0)}{\gamma}}^{L} \left(\frac{R}{R_0}\right)^{-4} dz \right\}$$

$$-f_2 \left\{ \int_0^{\frac{n(d+L_0)-L_0}{\gamma}} \left(\frac{R}{R_0}\right)^{-1} dz + \sum_{n=1}^{n=n_{\max}} \int_{\frac{n(d+L_0)-L_0}{\gamma}}^{\frac{n(d+L_0)}{\gamma}} \left(\frac{R}{R_0}\right)^{-1} dz + \int_{\frac{n(d+L_0)}{\gamma}}^{L} \left(\frac{R}{R_0}\right)^{-1} dz \right\} \tag{14}$$

Taking $n = 1$, we get

$$\lambda = -(f_1 + f_2)\left(L - \frac{L_0}{\gamma}\right) - (f_1 I_1 + f_2 I_2) \tag{15}$$

Where $I_1 = \int_{\frac{n(d+L_0)-L_0}{\gamma}}^{\frac{n(d+L_0)}{\gamma}} \left(\frac{R}{R_0}\right)^{-4} dz$, $I_2 = \int_{\frac{n(d+L_0)-L_0}{\gamma}}^{\frac{n(d+L_0)}{\gamma}} \left(\frac{R}{R_0}\right)^{-1} dz$

If there is no stenosis i.e. in the normal condition, we have

$$\lambda_N = -(f_1 + f_2)L \tag{16}$$

The resistance to flow is given by

$$\bar{\lambda} = \frac{\lambda}{\lambda_N} = 1 - \frac{L_0}{\gamma L} + \frac{f_1 I_1 + f_2 I_2}{(f_1 + f_2)L} \tag{17}$$

From eq. (7) and (10), the wall shear stress is given by $\tau_R = \left[\frac{4\mu Q}{\pi R^3} + \frac{4}{3}\tau_0 \right]$ (18)

Wall shear stress in normal situation is written as

$$\tau_N = \frac{4\mu Q}{\pi {R_0}^3} \tag{19}$$

The wall shear stress ratio $\overline{\tau}_R$ can be obtained as

$$\overline{\tau}_R = \frac{\tau_R}{\tau_N} = \frac{\pi {R_0}^3}{4\mu Q}\left[\frac{4\mu Q}{\pi R^3} + \frac{4\tau_0}{3} \right] = \frac{1}{\left(\frac{R}{R_0} \right)^3} + \frac{\pi {R_0}^3 \tau_0}{3\mu Q} \tag{20}$$

The wall shear stress ratio at the mid point of the stenosis is

$$\overline{\tau}_{R_1} = \frac{\pi {R_0}^3 \tau_0}{3\mu Q} + \frac{1}{\left(1 - \frac{\delta}{R_0} \right)^3} \tag{21}$$

NUMERICAL RESULTS

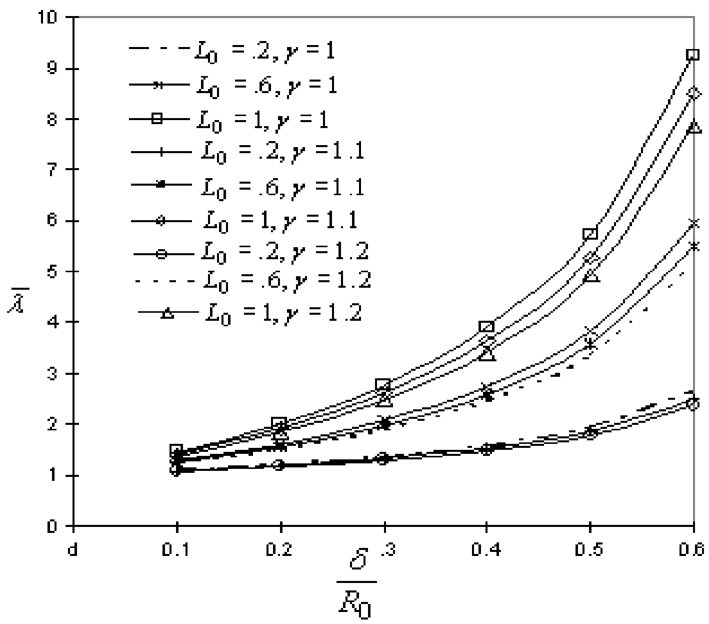

Fig.2. Resistance to flow with stenosis height for different values of stenosis lengths and γ

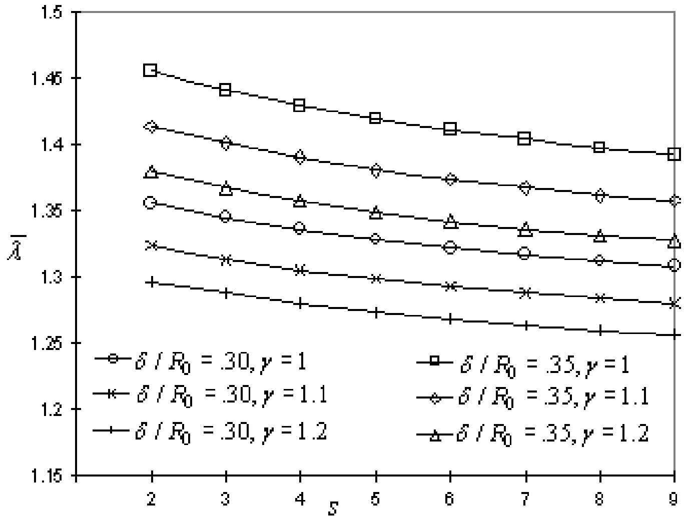

Fig.3. Resistance to flow with stenosis shape for different stenosis heights and parameter γ

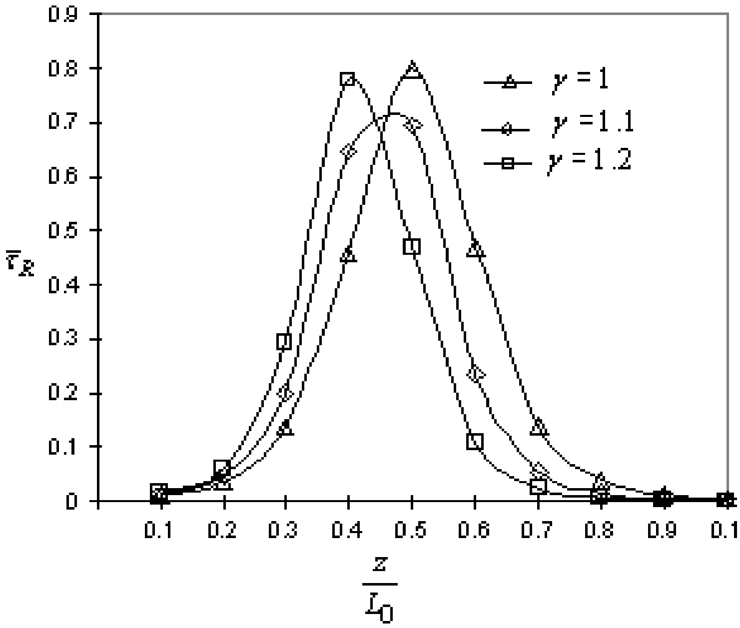

Fig.4. Wall shear stress with axial distance for different value of γ

CONCLUSION

In the present study, we have derived an analytic relation for resistance to flow for Bingham plastic flow of blood through an artery with multiple stenoses situated at the points of equal distance. For enhanced perceptive of the problem the graph has been plotted for a single loop of artery. It is observed that growing of γ shows the minor variations for different values of stenosis height in addition to stenosis length.

REFERENCES

[1] Singh A.K, *Advances in Applied Mathematical Biosciences*, **2012**, 3(1), 41-48
[2] Singh A.K, Singh D.P, *International Journal of Advanced Computer and Mathematical Sciences*, **2012**, 3(3), 305-309
[3] Singh B., Joshi P., Joshi B.K, *Applied Mathematical Sciences*, **2010**, 4(22), 1065-1072
[4] Singh S, Shah R.R, *Advances in Applied Science Research*, **2010**, 1(1), 66-73
[5] Krishna Kumari S.V.H.N, Revankar S.M, Ravi Kumar Y.V.K., *Advances in Applied Science Research*, **2012**, 3(4), 2159-2168
[6] Shah S.R, *International Journal on Pharmaceutical and Biomedical Research*, **2011**, 2(3), 96-106
[7] Mishra B.K, Verma N., *Australian Journal of Basic and Applied Sciences*, **2010**, 4(4), 588-601
[8] Misra J.C, Shit G.C, *International Journal of Engineering Science*, **2006**, 44, 662–671
[9] Bali R., Awasthi U., *Applied Mathematics*, **2012**, 3(5), 436-441
[10] Sarojamma G., Vishali B., Ramana B., *Int. J. of Appl. Math and Mech.*,**2012**, 8 (11), 1-17
[11] Singh A.K., Singh, D.P., *Advances in Applied Science Research*, **2012**, 3(5), 3285-3290