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**Advances in Applied Science Research, 2011, 2 (6):418-423**

Advances in Applied Science Research

**ISSN: 0976-8610**
**CODEN (USA): AASRFC**

# Application of Vague Soft Sets in students' evaluation

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## ABSTRACT

*In this paper, vague soft sets concept is applied to extend Biswas's method for students' answer scripts evaluation and a hypothetical case study has been taken as an example.*

**Keywords:** Soft sets, Vague soft sets, and Soft Evaluation Knowledge.

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## INTRODUCTION

Out of several generalizations of fuzzy set theory for various objectives, the notion introduced by Gau and Buehrer [11] in defining vague sets (VSs) is interesting and useful. Molodtsov (1999) pointed out that the existing theories, viz., theory of probability, theory of fuzzy sets, theory of intuitionistic fuzzy sets, theory of vague set, theory of interval mathematics and theory of rough sets can be considered as mathematical tools for dealing with uncertainties but all these theories have their own limitations. The reason for this is most possibly the inadequacy of the parameterization tool of the theories. So he developed a new mathematical theory called "Soft Set" for dealing with uncertainties which is free from the above limitations. The absence of any restrictions on the approximate description in soft set theory makes this theory very convenient and easily applicable in practice. Xu et al.[ 12 ] have developed a theoretical study of the

'Vague Soft Set' (VSS).The combination of Vague Set and Soft Set will be more useful in the field of applications wherever uncertainty appear.

In [3], Biswas pointed out that one of the chief aim of educational institutions is to provide students with the evaluation reports regarding their test/examination as sufficient as possible and with the unavoidable error as small as possible and presented a fuzzy evaluation method(*fem*) for applying fuzzy sets in students' answer scripts evaluation. He also modified the fuzzy evaluation method to propose a generalized fuzzy evaluation method (*gfem*) for students' answer scripts evaluation. In [4], Chen and Lee pointed out that the methods presented in [3] have two drawbacks, (i) it would take a large amount of time to deal with the matching operations of the matching function and (ii) two different fuzzy marks might be translated into the same awarded letter grade which would be unfair for students' evaluation. Thus, they presented two methods for evaluating students' answer scripts using fuzzy sets.

A soft set is a parameterized family of subsets of the universal set. We can say that soft sets are neighborhood systems, and that they are a special case of context-dependent fuzzy sets. In soft set theory the problem of setting the membership function, among other related problems, simply does not arise. This makes the theory very convenient and easy to apply in practice. Soft set theory has potential applications in many different fields, including the smoothness of functions, game theory, operations research, Riemann integration, Perron integration, probability theory, and measurement theory. Most of these applications have already been demonstrated in Molodtsov's paper.
In the first section of this paper, we present a new method for students' answer script evaluation using VSS and the second section contains an algorithm of the method. Then in the last section a hypothetical case study is discussed using the proposed method. The proposed method can evaluate students' answer script in a more flexible and more intelligent manner.

## 2. PRELIMINARIES

### 2.1 Soft sets and Fuzzy soft sets

**Definition 2.1[8]**

Let $U$ be a universal set, $E$ a set of parameters and $\boldsymbol{A} \subset E$. Then a pair $(F,A)$ is called soft set over $U$, where $F$ is a mapping from $A$ to $2^U$, the power set of $U$.

**Example 2.1**

Let $X=\{c_1, c_2, c_3\}$ be the set of three cars and $E=\{$costly($e_1$), metallic colour ($e_2$), cheap ($e_3$)$\}$ be the set of parameters ,where $A=\{e_1,e_2\} \subset E$. Then $(F,A)=\{F(e_1)=\{c_1,c_2,c_3\},F(e_2)=\{c_1,c_3\}\}$is the crisp soft set over $X$ which describes the "Attractiveness of the cars" which Mr. S (say) is going to buy.

**Definition 2.2[6]**

Let $U$be a universal set, $E$ a set of parameters and $A \subset E$. Let $\mathrm{F}(X)$ denotes the set of all fuzzy subsets of $U$. Then a pair $(F,A)$ is called fuzzy soft set over $U$, where $F$ is a mapping from $A$ to $\mathrm{F}(U)$.

**Example 2.2**

Let $U=\{c_1,c_2,c_3\}$ be the set of three cars and $E=\{$costly($e_1$), metallic colour($e_2$) cheap($e_3$)$\}$ be the set of parameters ,where $A=\{e_1,e_2) \subset E$.
Then $(F,A)=\{F(e_1)=\{c_1/.6,c_2/.4,c_3/.3\},F(e_2)=\{c_1/.5,c_2/.7,c_3/.8\}\}$ is the fuzzy soft set over $U$ describes the " attractiveness of the cars" which Mr. S(say) is going to buy.
2.2 Vague sets and vague soft sets.

**Definition 2.3[11]**

Let $U$ be an initial universe set, $U=\{u_1,u_2,\ldots,u_n\}$. A vague set over $U$ is characterized by truth-membership function $t_v$ and a false-membership function $f_v$, $t_v : U \rightarrow [0,1]$,

$f_v : U \to [0,1]$, where $t_v(u_i)$ is a lower bound on the on the grade of membership of $u_i$ derived from the evidence for $u_i$, $f_v(u_i)$ is a lower bound on the negation of $u_i$ derived from the evidence against $u_i$, and $t_v(u_i)+ f_v(u_i)\leq 1$. The grade of membership of $u_i$ in the vague set is bounded to a subinterval $[t_v(u_i),1- f_v(u_i)]$of [0, 1]. The vague value $[t_v(u_i),1- f_v(u_i)]$ indicates that the exact grade of membership $\mu_v( u_i)$ of $u_i$ may be unknown, but it is bounded by $t_v(u_i)\leq \mu_v( u_i) \leq 1- f_v(u_i)$, where $t_v(u_i)+ f_v(u_i) \leq 1$.

**Definition 2.4[12]**

Let $U$ be a universe, $E$ a set of parameters, $V(U)$ the power set of vague sets on $U$, and $A\subset E$. A pair $(F, A)$ is called a vague soft set over $U$, where $F$ is a mapping given by $F:A\to V(U)$ .

**Example 2.3**

Let $U= \{c_1, c_2, c_3\}$ be the set of three cars and $E$ = {costly ($e_1$), metallic colour($e_2$), cheap($e_3$)} be the set of parameters ,where $A=\{e_1,e_2\}\subset E$. Suppose that $F(e_1)$={ [.6,.7]/ $c_1$, [.4,.6]/ $c_2$,[.3,.5]/ $c_3$}, $F(e_2)$= { [.5,.7]/ $c_1$, [.7,.8]/ $c_2$,[.8, 1]/ $c_3$} then the vague soft sets $(F,A)$ is a parameterized family{ $F(e_1)$, $F(e_2)$} of vague sets on $U$ describes the " attractiveness of the cars" which Mr. S(say) is going to buy .

**3. Application of vague soft set in students' evaluation**

In this section, we present an application of vague soft set (VSS) theory in students' answer scripts evaluation following Biswas approach [4]. Assume that there are five satisfaction levels to evaluate the students' answer scripts regarding a question of an examination i.e. excellent ($e_1$), very good ($e_2$), good ($e_3$), satisfactory ($e_4$) and unsatisfactory ($e_5$). Let $X$ be a set of satisfaction level, $X$ = {excellent ($e_1$), very good ($e_2$), good ($e_3$), satisfactory ($e_4$) and unsatisfactory ($e_5$)} and again let $S$ = {0%. 20%, 40%, 60%, 80%, 100%} be the degree of satisfaction of the evaluator for a particular question of the student's answer script. Suppose $Q$ is a set of questions for a particular paper of 100 marks. We first assume $X$ as a universal set and $S$ the set of parameters. Then a VSS is constructed over the $X$, where $F$ is a mapping $F:S\to V(X)$ and $V(X)$ is the power set of vague sets on $X$. This VSS gives a relation matrix, say, $R$, called expert students evaluation matrix. We refer to the matrix $R$ as "Soft Evaluation Knowledge".

Again we construct another VSS $(F_1, X)$ over $Q$, where $F_1$ is a mapping given by $F_1:X\to V(Q)$ and $V(Q)$ is the power set of vague sets on $Q$ . This VSS gives a relation matrix $R_1$, called examination knowledge matrix. Then, we obtain a new relation

$T= R_1 \circ R$ called satisfaction question matrix in which the membership values are given by

$$t_T(Q_i,S_k)=\vee\{t_{R_1}(Q_i,e_j)\wedge t_R(e_j,S_k)\}$$
$$(1\text{-}f_T)(Q_i,S_k)=\vee\{(1\text{-}f_{R_1})(Q_i,e_j)\wedge(1\text{-}f_R)(e_j,S_k)\}$$, where $\vee$ =max and $\wedge$ = min.

Then compute the matrix $S_T$ where $S_T=[(1-\lambda)\times t_{ij}+\lambda\times(1-f_{ij})]$, where $\lambda\in[0,1]$ is the degree of optimism of the evaluator determined by the evaluator for evaluating students' answer script of $[t_{ij},1-f_{ij}]$ of the matrix $T$.

Corresponding to each question $Q_i$ of the paper for the matrix $T$ we take the highest value $0.x_i = (1-\lambda)\times t_{ij} + \lambda\times(1-f_{ij})$ (say) which indicates that the degree of satisfaction of the question $Q_i$ is $100x_i\%$. Then the highest score of the question $Q_i$ is $H(Q_i)=100x_i\%$. If $M(Q_i)$ is the mark allotted to the question $Q_i$ then the total score of the student is calculated by the formula

$$= \frac{1}{100}\sum\left\{H(Q_{i)}\times M(Q_i)\right\}.$$

**3.1 Algorithm**:

- input the VSS (F,S) over the set $X$ of satisfaction levels, where $S$ is the set of degree of satisfaction of the particular question paper and also write the soft evaluation knowledge $R$ representing the relation matrix of the VSS (F,S).
- input the VSS $(F_1,X)$ over the set $Q$ of questions of the paper and write its relation matrix $R_1$.
- compute the relation matrix $T = R_1 \circ R$
- compute $S_T$ from the matrix $T$.
- compute the highest score for each question for the matrix T.
- calculate the total score for the student for each paper.

**3.2 Case Study**:

Consider a candidate answer scripts to paper of 100 marks. Assume that in total there were four questions to be answered. Let $X$ be a set of satisfaction level and let $X=\{e_1, e_2, e_3, e_4, e_5\}$where $e_1$, $e_2$, $e_3$, $e_4$ and $e_5$ represents excellent, very good, good, satisfactory and unsatisfactory respectively. Suppose an evaluator is using vague soft grade sheet. Consider $X$ be as the universal set and $S$ = {0%. 20%, 40%, 60%, 80%, 100%} be the set of degree of satisfaction of the evaluator's as the set of parameters. Suppose that

$F(0\%) = \{ [0,.1]/ e_1, [0,.1]/ e_2, [0,.1]/ e_3, [.4,.5]/ e_4, [1, 1]/ e_5 \}$
$F(20\%) = \{ [0,0] /e_1, [0,0] / e_2, [.1,.2] / e_3, [.4,.5] / e_4, [1,1] / e_5\}$
$F(40\%) = \{ [.6,.6] /e_1, [.5,.6] / e_2, [.5,.6] / e_3, [.4,.5] / e_4, [.4,.5] / e_5\}$
$F(60\%) = \{ [.8,.9] /e_1, [.8,.8] / e_2, [.7,.9] / e_3, [.6,.7] / e_4, [.2,.3] / e_5\}$
$F(80\%) = \{ [1,1] /e_1, [.9,.9] / e_2, [.4,.5] / e_3, [.2,.3] / e_4, [0,0] / e_5\}$
$F(100\%) = \{ [1,1] /e_1, [.8,.9] / e_2, [.2,.3] / e_3, [0,1] / e_4, [0,0] / e_5\}$

Than the VSS (F,S) is a parameterized family {*F*(0%), *F*(20%), *F*(40%), *F*(60%), *F*(80%), *F*(100%)} of vague soft sets over the set $X$ and are determined from expert student evaluation documentation. Thus the VSS $(F,S)$ gives an approximate description of the vague soft examination knowledge of the four questions and their level of satisfaction. This VSS $(F,S)$ is represented by matrix $R$, called expert students evaluation matrix and is given by

$$R = \begin{array}{c} \\ e_1 \\ e_2 \\ e_3 \\ e_4 \\ e_5 \end{array}\begin{array}{c} \begin{array}{cccccc} 0\% & 20\% & 40\% & 60\% & 80\% & 100\% \end{array} \\ \begin{bmatrix} [0,.1] & [0,0] & [.6,.6] & [.8,.9] & [1,1] & [1,1] \\ [0,.1] & [0,0] & [.5,.6] & [.8,.8] & [.9,.9] & [.8,.9] \\ [0,.1] & [.1,.2] & [.5,.6] & [.7,.9] & [.4,.5] & [.2,.3] \\ [.4,.5] & [.4,.5] & [.4,.5] & [.6,.7] & [.2,.3] & [0,0] \\ [1,1] & [1,1] & [.4,.5] & [.2,.3] & [0,0] & [0,0] \end{bmatrix} \end{array}$$

Suppose an evaluator is using vague soft grade sheet. Suppose there are four questions $Q_1$, $Q_2$, $Q_3$ and $Q_4$ in the question paper and we consider the set $Q = \{Q_1, Q_2, Q_3, Q_4\}$ as universal set and $S = \{e_1, e_2, e_3, e_4\}$ as the set of parameters respectively. The evaluator's satisfaction level for the student for question with respect to parameters is respectively

$F_1(e_1) = \{ [.5,.7]/Q_1 ,[1,1] /Q_2 ,[.5,.7] /Q_3, [.8,.9] /Q_4 \}$
$F_1(e_2) = \{ [.8,.8]/Q_1, [.8,.9] /Q_2, [.6,.7] /Q_3, [.5,.7] /Q_4 \}$
$F_1(e_3) = \{ [.6,.7]/Q_1 ,[.4,.5] /Q_2, [.4,.5] /Q_3, [.1,.3] /Q_4 \}$
$F_1(e_4) = \{ [0,0]/Q_1, [0,0] /Q_2, [.2,.3] /Q_3, [0,0] /Q_4 \}$
$F_1(e_5) = \{ [0,0]/Q_1, [0,0] /Q_2, [0,0] /Q_3, [.5,.6] /Q_4 \}$

Then the VSS $(F_1,X)$ is a parameterized family $\{ F_1(e_1), F_1(e_2), F_1(e_3), F_1(e_4), F_1(e_5)\}$ of all vague set over the set $S$ and are determined from evaluated satisfaction for a particular student. This VSS $(F_1,X)$ gives approximate description of the vague soft examination knowledge of the four question and their level of satisfaction. This VSS $(F_1, X)$ is represented by relation matrix $R_1$, called examination knowledge matrix and given by

$$R_1 = \begin{array}{c} \\ Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \end{array} \begin{array}{c} \begin{array}{ccccc} e_1 & e_2 & e_3 & e_4 & e_5 \end{array} \\ \begin{bmatrix} [.5,.7] & [.8,.8] & [.6,.7] & [0,0] & [0,0] \\ [1,1] & [.8,.9] & [.4,.5] & [0,0] & [0,0] \\ [.5,.7] & [.6,.7] & [.4,.5] & [.2,.3] & [0,0] \\ [.8,.9] & [.5,.7] & [.1,.3] & [0,0] & [.5,.6] \end{bmatrix} \end{array}$$

Then combining the relation matrices

$$T = R_1 \circ R = \begin{array}{c} \\ Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \end{array} \begin{array}{c} \begin{array}{cccccc} 0\% & 20\% & 40\% & 60\% & 80\% & 100\% \end{array} \\ \begin{bmatrix} [0,.1] & [.1,.2] & [.5,.6] & [.8,.8] & [.8,.8] & [.8,.8] \\ [0,.1] & [.1,.2] & [.6,.6] & [.8,.9] & [1,1] & [1,1] \\ [.2,.3] & [.2,.3] & [.5,.6] & [.6,.7] & [.6,.7] & [.6,.7] \\ [.5,.6] & [.5,.6] & [.6,.6] & [.8,.9] & [.8,.9] & [.8,.9] \end{bmatrix} \end{array}$$

Suppose that the index of optimism $\lambda$ determined by the evaluator is $0.60 \in [0,1]$ then $S_T$ can be calculated in the following way, i.e.

$$S_T = \begin{array}{c} \\ Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \end{array} \begin{array}{c} \begin{array}{cccccc} 0\% & 20\% & 40\% & 60\% & 80\% & 100\% \end{array} \\ \begin{bmatrix} .06 & .16 & .68 & .8 & .8 & .8 \\ .06 & .16 & .6 & .86 & 1 & 1 \\ .26 & .26 & .66 & .66 & .66 & .66 \\ .56 & .56 & .86 & .86 & .86 & .86 \end{bmatrix} \end{array}$$

Hence the highest score for $Q_1$ is .8 i.e. it indicates that the degree of satisfaction of the question $Q_1$ of the student's answer script evaluation by the evaluator is 80%. Similarly for $Q_2$ is 100%, $Q_3$ is 66% and $Q_4$ is 86%. Therefore $H(Q_1)$=80, $H(Q_2)$=86, $H(Q_3)$=66 and $H(Q_4)$ = 86. Again suppose that $Q_1$ carries 20 marks, $Q_2$ carries 30 marks, $Q_3$ carries 25 marks and $Q_4$ carries 25 marks.

Therefore the total score of the student

$$= \frac{1}{100} \sum \{H(Q_i) \text{ x } M(Q_i)\}$$

= 1/100 {80 x 20 + 100 x 30 + 66 x 25 + 86 x 25}
= 1/100 {1600 + 3000 + 1650+ 2150}
= 84.

## CONCLUSION

In this paper, we have applied the notion of vague soft sets in evaluating students' answer scripts. A case study has been taken to exhibit the simplicity of the technique.

**Acknowledgements**
The authors would like to thank Dr. B.K.Saikia, Department of Mathematics, Lakhimpur Girls' College, Assam, India for providing very helpful suggestions.

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