Application of Intuitionistic fuzzy and Hamming Distance Measure in Multiple Attribute Decision Making (MADM)

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ABSTRACT

In this paper, we have considered an intuitionistic fuzzy entropy with both the uncertainty and hesitancy degree of IF sets. Based on this IF entropy, a new decision-making method of a multi-attribute decision making problem (MADM) has been introduced in which attribute values are expressed with IF values. In the case of attribute weight, a case with partially known attribute weights is discussed and a method is developed to determine the attribute weights. This method is an extension of ordinary entropy weight method. At last, an air-codition example is used to illustrate the application of the proposed model.

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INTRODUCTION

For multi-attrbute decision problems, it necessary to consider many factors is simultaneously. These factors complicates the problem and it becomes difficult to arrive at a conclusion. We often notice that crisp data is inadequate or insufficient to handle vagueness or fuzziness of realistic decision problem that can not be represented by the crisp numbers. In such cases, fuzzy sets or extended fuzzy sets are proved to be better choice in modelling the human judgement. IF sets firstly proposed by Atanassov [2] is an extension of Zadeh's fuzzy set [16]. IF sets seems more suitable to express the human's satisfaction or dissatisfaction degree which otherwise can not be represented by crisp numbers. Many studies reveal that IF set is a useful tool in handling the imprecise data and vague expressions than rigid mathematical equations. With these things in mind, many IF decision-making multi-attribute (MADM) methods were developed to deal with such situations.

Entropy is an effective measure for depicting the fuzziness of a fuzzy set. Zadeh [16] first introduced the entropy of a fuzzy event. Later on, De Luca and Termini [6] axiomatized it. Since then, many researchers realized the importance of fuzzy entropy and define it from their own viewpoints. As a result, a lot of attention was paid to the research and application of IF entropy. For example Zhang and Jiang [17] defined a fuzzy entropy by generalizing the IF entropy of De Luca and Termini's logrithmic fuzzy entropy [6], Ye [13,14] proposed two IF mesures using triangular function and Verma and Sharma [10,11] defined an exponential entropy by generalizing of Pal and Pal [9]. All of the above mentioned authors considered the derivation of membership and non-membership but not considered the hesitancy degree of the IF set.

Some researchers realized this and they proposed some new IF entropy measures. For example Wei *et al.* [12] proposed an IF entropy using cosine function etc.

In this paper, we consider an IF entropy measure [6], which not only considers the membership and non-membership degree, but also considers the effect of hesitancy degree of the IF set. TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) is one of the important techniques used to deal with MADM problems. It simultaneously considers both the farthest distance from the negative ideal solution (NIS) and the shortest distance from positive ideal solution (PIS). The order of alternatives is ranked according to relative closeness coefficients [6, 3]. TOPSIS has been widely applied to the traditional crisp and fuzzy MADM problems [1.8,15]. Based on this IF entropy measure and TOPSIS, we will give a new MADM decision making method. The subsequent contents of this paper are organised as follows: In Section 2, the basic definitions and notations of IF set are defined and reviewed. In section 3, an intutionistic fuzzy MADM method is suggested in which weights of the attributes are obtained using this IF entropy measure. An example is given in Section 4. Finally, conclusions are given in Section 5.

PRELIMINARIES

Definition 2.1 Suppose that X is a given universal set. A set A is called an IF set, if $A = \{ < x_i, \mu_A(x_i), \nu_A(x_i) > /x_i \in X \},$ (1) where the functions $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$ are the membership and nonmembership degree of x_i and for every $x_i \in X, 0 \le \mu_A(x_i) + \nu_A(x_i) \le 1$. Furthermore, we call $\pi_A(x_i) = 1 - \mu_A(x_i) - \nu_A(x_i)$ an IF index or hesitency degree of x_i . Conveniently,

if there is only one element in X, we call A the IF number, abbreviated as $A = (\mu_A, \nu_A)$.

Definition 2.2 Suppose that

 $A = \{ < x_i, \mu_A(x_i), \nu_A(x_i) > /x_i \in X \} and$ $B = \{ < x_i, \mu_B(x_i), \nu_B(x_i) > /x_i \in X \} are two IF$ sets, then the following operations can be founded in [2]: $A \subseteq B \qquad \text{if} \qquad \text{only} \qquad \text{if}$

• $A \subseteq B$ if and only if $\mu_A(x_i) \le \mu_B(x_i), v_A(x_i) \ge v_B(x_i), \quad \forall x_i \in X$; • A = B if and only if $A \subseteq B$ and $B \subseteq A$; • The complementary set of A denoted by A^c , is $A^c = \{< x_i, v_A(x_i), \mu_A(x_i) > /x_i \in X\}$ • $A^\circ B$ called A less fuzzy than B, i.e, for all $x_i \in X$,

If
$$\mu_B(x_i) \le \nu_B(x_i)$$
, then
 $\mu_A(x_i) \le \mu_B(x_i), \nu_A(x_i) \ge \nu_B(x_i)$,
If $\mu_B(x_i) \ge \nu_B(x_i)$, then $\mu_A(x_i) \ge \mu_B(x_i)$,
then $\nu_A(x_i) \le \nu_B(x_i)$.

Definition 2.3 Suppose

 $A = \{ < x_i, \mu_A(x_i), \nu_A(x_i) > /x_i \in X \} and$ $B = \{ < x_i, \mu_B(x_i), \nu_B(x_i) > /x_i \in X \} are two IF$

set, the weight of x_i is w_i , then the weighted Hamming distance measure of A and B is defined as follows:

$$d(A,B) = \frac{1}{2} \sum_{i=1}^{n} v_{i} || \mu_{A}(x_{i}) - \mu_{B}(x_{i})| + |v_{A}(x_{i}) - v_{B}(x_{i}) + |\pi_{A}(x_{i}) - \pi_{B}(x_{i})|$$
(2)

Burillo and Bustince first axiomatized intutionistic fuzzy entropy measure, which is an extension of the De Luca and Termini axioms [6] for fuzzy sets. The axioms of intutionistic fuzzy entropy measure were formulated as follows:

•
$$E(A) = 0$$
 if and only if A is a crisp set;
• $E(A) = 1$ if and only if
 $\mu_A(x_i) = \nu_A(x_i), \forall x_i \in X$;
• $E(A) = E(A^c)$;
• $A^\circ B$, then $E(A) \leq E(B)$.

NEW INTUTIONISTIC FUZZY MADM METHOD BASED ON THE IF

Entropy and Hamming Distance

For a MADM problem, suppose that $A = (A_1, A_2, \dots, A_m)$ is a set of *m* alternatives, $O = (o_1, o_2, \dots, o_n)$ is a set of *n* attributes. Suppose that there exists an alternative set consisting of n non-inferior alternatives from which the most desirable alternative is to be selected. Ratings of alternatives $A_i \in A$ on attributes $o_j \in O$ are expressed with the IFN $\widetilde{a}_{ij} = (\mu_{ij}, v_{ij})$, respectively, where μ_{ij} and v_{ij} are the membership and non-membership degrees of the alternative $A_i \in A$ on the attribute $o_j \in O$ with respect to the fuzzy concept "excellence" given by the decision maker so that the conditions: $0 \le \mu_{ij} \le 1$, $0 \le \nu_{ij} \le 1$ and $0 \le \mu_{ij} + \nu_{ij} \le 1$. (i = 1, 2, ..., m; j = 1, 2, ..., n). In the MADM problems, the IF values are calculated according to Liu and Wang [14] as follows: For the sake of obtaining the degrees to which the alternative A_i satisfies and/or does attribute not satisfy $o_j (i = 1, 2, ..., m; j = 1, 2, ..., n)$, we now use the statistical method. Suppose we invite nexperts to make the judgement. They are expected to answer "yes" or "no" or "I don't know" to the question whether alternative A_j satisfies attribute o_j . We use $n_Y(i, j)$ and $n_N(i,j)$ to denote the number of "yes" and "no", respectively, from n-experts. Then, the degrees to which alternative A_i satisfies and/or does not satisfy attribute O_j can be calculated as:

$$\mu_{ij} = \frac{n_Y(i,j)}{n} \quad \text{and} \quad \nu_{ij} = \frac{n_N(i,j)}{n}$$

Thus, a MADM problem can be expressed with the decision matrix $D = (\widetilde{a}_{ij})_{m \times n}$ as follows:

 $o_1 \quad o_2 \quad o_n$

$$D = (\widetilde{a}_{ij})_{m \times n} = \begin{matrix} A_1 \\ A_2 \\ \vdots \\ A_m \end{matrix} \begin{pmatrix} (\mu_{11}, \nu_{11}) & (\mu_{12}, \nu_{12}) & \dots & (\mu_{1n}, \nu_{1n}) \\ (\mu_{21}, \nu_{21}) & (\mu_{22}, \nu_{22}) & \dots & (\mu_{2n}, \nu_{2n}) \\ \vdots & \vdots & \vdots \\ (\mu_{m1}, \nu_{m1}) & (\mu_{m2}, \nu_{m2}) & \dots & (\mu_{mn}, \nu_{mn}) \\ \end{cases}$$
(3)

Let $w = (w_1, w_2, ..., w_n)^T$ be the weight vector of all attributes, where $0 \le w_j \le 1 (j = 1, 2, ..., n)$ are weights of attributes $o_j \in O$ (j = 1, 2, ..., n), and $\sum_{i=1}^n w_i = 1$

 $\sum_{j=1}^{n} w_j = 1$. The attribute weights information is usually unknown due to the insufficient knoweldge or limitation of time of decision makers in the decision making process. Therefore, the determination of attribute weights is an important issue in MADM problems in which the attribute weights are unknown. In this paper, we propose a method to determine the attribute weights.

MADM Problem with Partially Known Attribute Weights Information

In general there are more constraints for the weight vector $w = (w_1, w_2, ..., w_n)$. The set of known weight information is denoted as H. To determine thethe attribute weights for MADM problem with attribute weights partially known under intuitionistic fuzzy enviornment, Xu [21] proposed an optimization model based on the Chen and Tan's score function [7]; Wu and Zhang [20], Wang and Wang [18] determined the attribute weights by establishing a programming model according to the minimum principle. In this paper, we will use the IF entropy measure to determine the attribute weights and the method is similar to Wang and Wang [18]. The specific process is given as follows:

To rank the alternatives according to the $D = (\widetilde{a}_{ij})_{m \times n}$, we propose a method to obtain the weight vector by means of the proposed IF entropy measure. Entropy measures describes the degree of fuzziness and intuitionism. The smaller the intuitionistic fuzzy entropy, the smaller the fuzzy degree of attribute evaluation information thus more the decision-making certainty will be. Hence, we can utilize the prinicple of minimum entropy value to get the weight vector of attribute by computing the following programming:

 $\min E(A_{i}) = \sum_{j=1}^{n} w_{j} E(\widetilde{a}_{ij}) = \frac{-1}{m \log(2)} \sum_{j=1}^{n} w_{j} \left(\sum_{i=1}^{m} \mu_{ij} \ln(\mu_{ij}) + v_{ij} \ln(v_{ij}) - (1 - \pi_{ij}) \ln(1 - \pi_{ij}) - \pi_{ij} \ln(2) \right)$ s.t. $\sum_{j=1}^{n} w_{j} = 1, w \in H.$ (4)

where j = 1, 2, ..., n and $1/m \ln(2)$ is a constant which assures that $0 \le E_{LT}^{IFS} \le 1$. Because each alternative is a fair competition, therefore, the weight coefficient with respect to the same attribute should also be equal, thus we get the following optimization model:

$$\min E = \frac{-1}{m \ln Q} \sum_{i=1}^{m} E(A_i) = \frac{-1}{m \ln Q} \left(\sum_{i=1}^{m} \sum_{j=1}^{n} w_j (\mu_{ij} \ln(\mu_{ij}) + v_{ij} \ln(v_{ij}) - (1 - \pi_{ij}) \ln(1 - \pi_{ij}) - \pi_{ij} \ln Q) \right)$$

$$s.t. \sum_{i=1}^{m} w_{ij} = 1, w \in H.$$

(5) Hence, by solving the equation (5), the optimal solution $w^* = \arg \min E$ is chosen as

the optimal attribute weights.

 $\overline{j=1}$

The New MADM Method Based on the IF Entropy

In this subsection, we put forward the new MADM method based on the above mentioned work and the concept of TOPSIS. The specific calculation steps are given as follows:

Step 1. Calculate the attribute weights according to Subsection 3.1.

Step 2. Determine the positive ideal solution

(PIS) and negative ideal solution (NIS) of the intutionistic fuzzy MADM problem. The PIS is defined as follows:

$$A^{+} = ((\mu_{1}^{+}, \nu_{1}^{+}), (\mu_{2}^{+}, \nu_{2}^{+}), \dots, (\mu_{n}^{+}, \nu_{n}^{+})),$$
(6)

where $(\mu_j^+, \nu_j^+) = (1,0), j = 1,2,...,n$. The NIS is defined as follows:

$$A^{-} = ((\mu_{1}^{-}, \nu_{1}^{-}), (\mu_{2}^{-}, \nu_{2}^{-}), \dots, (\mu_{n}^{-}, \nu_{n}^{-})),$$
(7)

where
$$(\mu_j^-, \nu_j^-) = (0, 1), j = 1, 2, ..., n$$

Step 3. According to the weighted Hamming distance measure in Definition (2.3) the distance

measures between alternative A_i with PIS and NIS are calculated respectively as follows:

$$d(A_i, A^+) = \frac{1}{2} \sum_{j=1}^n w_j (|\mu_{ij} - \mu_j^+| + |\nu_{ij} - \nu_j^+| + |\pi_{ij} - \pi_j^+|)$$

$$=\frac{1}{2}\sum_{j=1}^{n}w_{j}(|1-\mu_{ij}|+|\nu_{ij}|+|1-\mu_{ij}-\nu_{ij}|).$$
(8)

$$d(A_i, A^-) = \frac{1}{2} \sum_{j=1}^n w_j(|\mu_{ij} - \mu_j^-| + |\nu_{ij} - \nu_j^-| + |\pi_{ij} - \pi_j^-|)$$

$$=\frac{1}{2}\sum_{j=1}^{n}w_{j}(|\mu_{ij}|+|1-\nu_{ij}|+|1-\mu_{ij}-\nu_{ij}|).$$
(9)

Step 4. Calculate the relative closeness coefficient of each alternative.

The closeness coefficient C_i represents the distances between the PIS and NIS simultaneously. The closeness coefficient of each alternative is calculated as:

$$C_i = \frac{d(A_i, A^-)}{d(A_i, A^-) + d(A_i, A^+)}.$$
 (10)

Step 5. Rank the alternatives according to the closeness coefficients (C_i) in decreasing order. The best alternative is the closest to the PIS and farthest from the NIS.

NUMERICAL EXAMPLE

To explain the application of MADM method, we give an example as follows:

This is an example of air-condition system selection problem. Suppose there are three aircondition systems: $A_i(i = 1,2,3)$ are to be selected. Evaluation attributes are o_1 (Economical), o_2 (Function),

 O_3 (Operationality). Using statistical methods, we can obtain the membership degree and non-

membership degree V_{ij} for the alternative A_i

satisfying thr attributes o_j respectively. The IF decision matrix provided by experts is shown in Table 1.

Assume the attribute weights are partially known and weights satisfy the set

 $H=\{0.25 \le w_1 \le 0.75, 0.35 \le w_2 \le 0.60, 0.30 \le w_3 \le 0.35\}.$

The calculation steps of the proposed method are given as follows:

Step 1.According to the equation (5),we can use the following programming model:

min $E = 0.7466 \ w_1 + 0.7211 \ w_2 + 0.8822 \ w_3;$ s.t. $\begin{cases} 0.25 \le w_1 \le 0.75 \\ 0.35 \le w_2 \le 0.60 \\ 0.30 \le w_3 \le 0.35 \\ w_1 + w_2 + w_3 = 1 \end{cases}$

We use the MATLAB software to solve this model and get the optimum attribute weight vector as:

$$w = (w_1, w_2, w_3)^T = (0.25, 0.45, 0.30)^T$$
.

Step 2. The PIS (A^+) and NIS (A^-) are respectively given as:

$$A^{+} = ((\mu_{1}^{+}, \nu_{1}^{+}), (\mu_{2}^{+}, \nu_{2}^{+}), (\mu_{3}^{+}, \nu_{3}^{+})) = ((1,0), (1,0), (1,0))$$

$$A^{-} = ((\mu_{1}^{-}, \nu_{1}^{-}), (\mu_{2}^{-}, \nu_{2}^{-}), (\mu_{3}^{-}, \nu_{3}^{-})) = ((0,1), (0,1), (0,1))$$

Step 3. The distance measure of each alternative from PIS and NIS are calculated as:

$$d(A_1, A^+) = 0.3025, d(A_2, A^+) = 0.3590, d(A_3, A^+) = 0.3825$$

$$d(A_1, A^-) = 0.8025, d(A_2, A^-) = 0.7225, d(A_3, A^-) = 0.7750$$

Step 4. The relative closeness coefficients are calculated as:

 $C_1 = 0.7262, C_2 = 0.6681, C_3 = 0.6695.$

Therefore, the ranking order of all alternatives is $A_1 \succ A_3 \succ A_2$ and A_1 is the desireable alternative.

CONCLUSION

IF sets are suitable in describing and dealing with the uncertain and vague information occuring in many MADM problems. In this paper, we take IF entropy which not only considers the membership and non-membership degree but also considers the hesitation degree of the IF sets. Based on this IF entropy measure, a new attribute weight determination method is put forward, which we then use to approach the multi-attribute decision making problem. A numerical example is used to illustrate the feasibility and practicability of the proposed MADM method. The proposed MADM method can be applied to other alternative problems such as the evaluation project investment risk, site selection and credit evaluation.

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Air-condition system	Evaluation attribute		
	<i>O</i> ₁	<i>0</i> ₂	<i>O</i> ₃
A_1	(0.75, 0.10)	(0.60, 0.25)	(0.80, 0.20)
A ₂	(0.80, 0.15)	(0.68, 0.20)	(0.45, 0.50)
A_3	(0.40, 0.45)	(0.75, 0.05)	(0.60, 0.30)

Table 1. Intutionistic Fuzzy Decision Matrix.