Available online at www.pelagiaresearchlibrary.com

Pelagia Research Library

Advances in Applied Science Research, 2014, 5(3):391-396

Pelagia Research Library

ISSN: 0976-8610
CODEN (USA): AASRFC

Anisotropic Bianchi type –I cosmological model with wet dark energy in general theory of relativity

S. D. Deo[1], Gopalkrishna S. Punwatkar[2] and Umesh M. Patil[3]

[1]Department of Mathematics, N. S. Science & Arts College, Bhadrawati, Dist. Chandrapur(M.S), India
[2]Department of Mathematics, Datta Meghe Institute of Engineering, Technology & Research, Sawangi (Meghe), Wardha (M.S), India
[3]Department of Mathematics, Shri Shivaji Science College, Amravati(M.S), India

---

ABSTRACT

Anisotropic Bianchi type –I cosmological model is studied in General Theory of relativity with the matter wet dark energy. To determine the model, we assume that $\frac{\dot{A}}{A}=\frac{\dot{B}}{B}=\frac{\alpha_1}{t^n}$ *and* $\frac{\dot{C}}{C}=\frac{\alpha_2}{t^n}$ *where* α_1 , α_2 *are constant. Further various solutions are obtained and some physical and kinematical properties of the model have also been discussed.*

Keywords: Anisotropic space time, Wet Dark Energy, Shear Scalar, Expansion Scalar.
MSC2010 CLASSIFICATION: *83C05, 83C15.*
PACS: *98.80.-k, 95.30.-k, 98.80.Cq, 04.20.-q.*

INTRODUCTION

Space-times admitting a three parameter group of automorphisms are important in discussing the cosmological models. The case where the group is simply transitive over the 3-dimensional, constant¬ time subspace is particularly useful for two reasons. First, Bianchi has shown that there are only nine distinct sets of structure constants for groups of this type. Therefore, we can use algebra to classify the homogeneous Space -times. The second reason for the importance of Bianchi type Space -times is the simplicity of the field equations.

When we study the Bianchi type models, we observe that the models contain isotropic special cases and they permit arbitrarily small anisotropic levels at some instant of cosmic times.

Bianchi type cosmological models are important in the sense that these models are homogeneous and anisotropic, from which the process of isotropization of the universe is studied through the passage of time. Moreover, from the theoretical point of view, anisotropic universe has a greater generality than isotropic models. The simplicity of the field equations made Bianchi space time useful in constructing models of spatially homogeneous and anisotropic cosmologies.

Hence, these models are to be known as very much suitable models of our universe. Therefore, study of these models create much more interest.
The cosmological constant $(\wedge)$ was introduced by Einstein in 1917 as the universal repulsion to make the universe static in accordance with generally accepted picture of the time. In absence of matter described by the stress energy tensor T_{ij}, $(\wedge)$ must be constant, Since the Bianchi identities vanishing covariant divergence of the Einstein tensor $G^{ij}{}_{;j}=0$, while $g^{ij}{}_{;j}=0$ by definition. If Hubble parameter and age of the universe as measured from high red

shift would be found to satisfy the bound $H_0 t_0 > 1$(index zero labels today's value), it would require a term in the expansion rate equation that acts as a cosmological constant.

Observational data like la supernovae suggest that the universe is dominated by two dark components containing dark energy and dark matter. Dark energy with negative pressure is used to explain the present cosmic accelerating expansion while dark matter is used to explain galactic curves and large scale structure formation.

Origin of the dark energy and dark matter and their natures remains unknown and we hope that large Hadron Collider can gives us these hints.

The equation of state for wet Dark fluid is $p_{WDF} = \gamma(\rho_{WDF} - p_*)$

Where, the parameter γ and p_* taken to be positive restrict ourselves to $0 \le \gamma \le 1$.

And we have energy conservation equation as $\rho_{WDF} + 3H(p_{WDF} + \rho_{WDF}) = 0$.

Using equation of state and $3H = \frac{\dot{V}}{V}$ in the above equation, we get

$$\rho_{WDF} = \frac{\gamma}{1+\gamma}\rho_* + \frac{c}{V(1+\gamma)}$$

Where, c is constant of integration and V is volume expansion.

Wet dark fluid (WDF) has two components: - one be haves as cosmological constant and other as standard fluid with equation of state $p = \gamma\rho$.

If we take c>0 then this fluid will not violet the strong energy condition $p + \rho \ge 0$;

$$(p_{WDF} + \rho_{WDF}) = (1+\gamma)\rho_{WDF} - \gamma\rho_*$$

$$(p_{WDF} + \rho_{WDF}) = (1+\gamma)\frac{c}{V(1+\gamma)}$$

The wet dark fluid has been used as dark energy in homogenous isotropic FRW model by Holman and Naidu [9]. The Bianchi type –I universe with dark fluid has been studied by Singh and Chaubey [10]. And also Deo S. D. [12] studied Bianchi type –I universe with wet dark energy in Bimetric relativity. Deo and Singh [13] studied Bianchi type – I Wet dark universe in Bimetric relativity.

In this paper, we have studied Bianchi type – I cosmological models with Wet dark energy in general theory of relativity. Some physical and kinematical properties of the model are also discussed.

1. THE METRIC AND FIELD EQUATION

We consider the spatially homogenous and anisotropic Bianchi type –I metric in the form

$$ds^2 = dt^2 - A^2dx^2 - B^2dy^2 - C^2dz^2 \tag{2.1}$$

Where A, B, and C are functions of t alone.

$T_i^{\ j}$ is the energy momentum tensor of the source wet dark energy is denoted by

$$T_i^{\ j} = (p_{WDF} + \rho_{WDF})u_i u^j - p_{WDF}\delta_i^j \tag{2.2}$$

Where, p_{WDF} is the isotopic pressure and ρ_{WDF} is the matter density and u^i is the flow vector of the fluid. The flow of the matter is taken orthogonal to the hyper surface of the homogeneity, so that

$$g_{ij}u_i u^j = 1$$
(2.3)

In a comoving system of coordinates, from (2.2) we find

$$T_1^1 = T_2^2 = T_3^3 = -p_{WDF} \text{ And } T_4^4 = \rho_{WDF} \quad (2.4)$$

The Einstein field equation with time dependent G and $\wedge$ are

$$R_i^j - \frac{1}{2}Rg_i^j = -8\pi G T_i^j + \wedge g_i^j \quad (2.5)$$

For the line element (2.1) and energy momentum tensor (2.2) in comoving coordinate system, the field equation (2.5) proceeds a set of four independent equations

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} = 8\pi G p_{WDF} + \wedge \quad (2.6)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} = 8\pi G p_{WDF} + \wedge \quad (2.7)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} = 8\pi G p_{WDF} + \wedge \quad (2.8)$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC} = -8\pi G \rho_{WDF} + \wedge \quad (2.9)$$

An additional equation for time dependent of G and $\wedge$ is obtained by the divergence of Einstein tensor

$$8\pi G\left[\dot{\rho}_{WDF} + (\rho_{WDF} + p_{WDF})(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C})\right] + 8\pi \dot{G}\rho_{WDF} + \dot{\wedge} = 0$$
(2.10)

The usual energy conservation equation $T_{i;j}^j = 0$ gives

$$\left[\dot{\rho}_{WDF} + (\rho_{WDF} + p_{WDF})(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C})\right] = 0 \quad (2.11)$$

The expression for scalar expansion θ and shear scalar σ are

$$\theta = \frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \quad (2.12)$$

$$\sigma^2 = \frac{1}{2}\left[\frac{\dot{A}^2}{A^2} + \frac{\dot{B}^2}{B^2} + \frac{\dot{C}^2}{C^2}\right] - \frac{1}{6}\theta^2 \quad (2.13)$$

And Hubble parameter H and volume expansion V are defined as

$$H = \frac{\theta}{3} = \frac{\dot{V}}{V} \quad (2.14)$$

$$V = (ABC)^{\frac{1}{3}} \quad (2.15)$$

2. SOLUTION OF FIELD EQUATIONS

We assume the solution of the system of the equation in the form

$$\frac{\dot{A}}{A}=\frac{\dot{B}}{B}=\frac{\alpha_1}{t^n},\ \frac{\dot{C}}{C}=\frac{\alpha_2}{t^n} \tag{3.1}$$

Where α_1, α_2 are constant.

Integrating equation (3.1), we get

$$A=B=\alpha_3\exp\left[\frac{\alpha_1 t^{-n+1}}{-n+1}\right] \tag{3.2}$$

$$C=\alpha_4\exp\left[\frac{\alpha_2 t^{-n+1}}{-n+1}\right] \tag{3.3}$$

Where α_3, α_4 are constant of integration.

Thus the metric (2.1) reduces to

$$ds^2=dt^2-\alpha_3^2\exp\left[\frac{2\alpha_1 t^{-n+1}}{-n+1}\right]\left(dx^2+dy^2\right)-\alpha_4^2\exp\left[\frac{2\alpha_2 t^{-n+1}}{-n+1}\right]dz^2$$

(3.4)

Where $n\neq 1$

i. SPECIAL CASE: for $n\neq 1$

The Standard fluid with equation of state is expressed as

$$p_{WDF}=\gamma\rho_{WDF}$$

(3.5)

Using eq. (3.1), (3.5) in eq. (2.11), we get

$$\frac{\dot{\rho}_{WDF}}{\rho_{WDF}}=-(1+\gamma)\left[\frac{2\alpha_1+\alpha_2}{t^n}\right] \tag{3.6}$$

Integrating, we get

$$\rho_{WDF}=\alpha_5\exp\left[-(1+\gamma)(2\alpha_1+\alpha_2)\frac{t^{-n+1}}{-n+1}\right] \tag{3.7}$$

Differentiating eq. (3.7), we get

$$\dot{\rho}_{WDF}=-\alpha_5\left[\frac{(1+\gamma)(2\alpha_1+\alpha_2)}{t^n}\right]\exp\left[\frac{-(1+\gamma)(2\alpha_1+\alpha_2)t^{-n+1}}{-n+1}\right] \tag{3.8}$$

Using eq. (3.7) in eq. (3.5), we get

$$p_{WDF}=\gamma\alpha_5\exp\left[-(1+\gamma)(2\alpha_1+\alpha_2)\frac{t^{-n+1}}{-n+1}\right] \tag{3.9}$$

Hubble constant H_i, the expansion scalar θ, shear scalar σ volume expansion V for the model (3.4) are given by

$$\theta = \left[\frac{2\alpha_1 + \alpha_2}{t^n}\right] \tag{3.10}$$

We have $H_1 = H_2 = \frac{\alpha_1}{t^n}$, $H_3 = \frac{\alpha_2}{t^n}$ (3.11)

$$\sigma = \frac{1}{\sqrt{3}}\left[\frac{2\alpha_1 + \alpha_2}{t^n}\right] \tag{3.12}$$

$$V = \left[\alpha_3\alpha_4 \exp\left(2\alpha_1 + \alpha_2\right)\frac{t^{-n+1}}{-n+1}\right]^{\frac{1}{3}} \tag{3.13}$$

Using eq. (3.10) and (3.12) we get

$$\frac{\sigma}{\theta} = \frac{1}{\sqrt{3}} = \text{Constant} \tag{3.14}$$

The model (3.4) starts with a big-bang at t=0 when n>0 and the expansion scalar θ decreases at time t increases. However, when n<0, the expansion in the model increases as the time increases.

As the time t increase, the special volume V decreases.

ii. SPECIAL CASE: for n=1

For n=1, eq. (3.1) becomes,

$$\frac{\dot{A}}{A} = \frac{\dot{B}}{B} = \frac{\alpha_1}{t}, \quad \frac{\dot{C}}{C} = \frac{\alpha_2}{t} \tag{3.15}$$

Integrating, we get

$$A = B = \alpha_6 t^{\alpha_1}, \quad C = \alpha_7 t^{\alpha_2} \tag{3.16}$$

Therefore, the metric eq. (2.1) reduces to

$$ds^2 = dt^2 - \alpha_6^2 t^{2\alpha_1}\left(dx^2 + dy^2\right) - \alpha_7^2 t^{2\alpha_2} dz^2 \tag{3.17}$$

Using eq. (3.15), (3.5) in eq. (2.11), we get

$$\frac{\dot{\rho}_{WDF}}{\rho_{WDF}} = -(1+\gamma)\left[\frac{2\alpha_1 + \alpha_2}{t}\right] \tag{3.18}$$

Integrating eq. (3.18) we get

$$\rho_{WDF} = \alpha_8 t^{-(1+\gamma)(2\alpha_1 + \alpha_2)} \tag{3.19}$$

Using eq. (3.19) in eq. (3.5), we get

$$p_{WDF} = \gamma\alpha_8 t^{-(1+\gamma)(2\alpha_1 + \alpha_2)} \tag{3.20}$$

Hubble constant H_i, the expansion scalar θ, shear scalar σ volume expansion V for the model (3.17) are given by

$$\theta = \left[\frac{2\alpha_1 + \alpha_2}{t}\right] \tag{3.21}$$

We have $H_1 = H_2 = \frac{\alpha_1}{t}$, $H_3 = \frac{\alpha_2}{t}$

$$\sigma = \frac{1}{\sqrt{3}}\left[\frac{2\alpha_1 + \alpha_2}{t}\right] \tag{3.22}$$

$$V = \left(\alpha_4 \alpha_5 t^{2\alpha_1 + \alpha_2}\right)^{\frac{1}{3}} \tag{3.23}$$

Using eq. (3.22) and (3.21), we get

$$\frac{\sigma}{\theta} = \frac{1}{\sqrt{3}} = \text{constant} \tag{3.24}$$

As shear scalar σ , expansion scalar θ, Hubble parameter H are inversely proportional to time. Therefore as the time t increases, shear scalar σ , expansion scalar θ, Hubble parameter H decreases. Volume of the universe V increases as the time t increases.

Since $\frac{\sigma}{\theta} =$ constant therefore model not approach isotropy for large value of time t.

CONCLUSION

In this paper, Anisotropic Bianchi type –I cosmological model is studied in General Theory of relativity with the matter wet dark energy. Here it is observed that when time $t \to 0$, the special volume $V \to \infty$. Also shear scalar σ , expansion scalar θ, Hubble parameter H are inversely proportional to time. Therefore as the time t increases, shear scalar σ , expansion scalar θ, Hubble parameter H decreases. Since $\frac{\sigma}{\theta} =$ constant therefore model not approach isotropy for large value of time t. Finally the solution presented in the paper are useful for better understanding of the evolution of the universe in bianchi type I cosmological model with G and $\wedge$.

REFERENCES

[1] J. Hajj-Boutros, J. Sfeila, *Int.J. Theo.Phys*., **1987**, 26, 97-103.
[2] A. Pradhan, I. Chakrabarty, *Gravitation & Cosmology*, **2001**, Vol.7, No.1 (25), 55-57.
[3] A. Pradhan, V. K. Yadav, *Int.J. Mod. Phys. D*, **2002**, Vol. 11, No. 6, 893-912
[4] A. Pradhan, A.K. Vishwakarma, A New Class of LRS Bianchi Type-I Cosmological Models With Variable G and Λ, **2000**, SUJST, Vol. XII, Sec.B, 42-48.
[5] S. Tripathi, R. K. Dubey, *Indian J. science*, **2011**, Res 2(4): 79-81.
[6] A. K. Yadav, A. Pradhan , A. K. Singh, *Astrophysics space science*, **2012**, vol. 337 no 1, 379-385
[7] A. Pradhan, A. Kumar, *Int. J. Mod. Phys*., **2001**, D Vol.10, No. 3, 291-298, (IUCAA-24/2000).
[8] C.P. Singh, Suresh Kumar, *Int.J.Theo.Phys*., **2009**, 48, 925-936.
[9] Holman, Naidu, dark energy in homogenous isotropic FRW model, arXiv: Astro-phy/0408102(preprint), **2005**.
[10] Singh and Chaubey, *Pramana Journal of physics*, **2008**, vol. 71, no. 3, pp.447-458.
[11] R. K. Tiwari, *Astrophys*., **2010**, Vol.10 no. 4, pp.291-300.
[12] S. D. Deo, Bianchi type –I universe with wet dark energy in Bimetric relativity, oiirj, II (1), **2012**, pp.104-107.
[13] S. D. Deo, S. P. Singh, *Mathematical theory and Modeling*, **2013**, Vol3 (a), 30-32.
[14] S. D. Deo, A. K. Roughe, *Ultra Scientist of physical science*, **2010**, vol.22, no.2, 535-538.