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Analysis of Linear stability on double diffusive convection in a fluid saturated anisotropic porous layer with Soret effect

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ABSTRACT

The double diffusive convection in a horizontal anisotropic porous layer saturated with a Boussinesq fluid, which is heated and salted from below in the presence of Soret coefficient is studied analytically using linear stability analysis based on the usual normal mode technique. The generalized Darcy model is employed for the momentum equation. The effect of mechanical anisotropy parameter, thermal anisotropy parameter, Lewis number and Soret parameter on stationary and oscillatory convection are shown graphically.

Keywords: Double diffusive convection, Soret parameter, Anisotropic porous layer, Critical Rayleigh number, Lewis number.

INTRODUCTION

The problem of convection induced by temperature and concentration gradients or by concentration gradients of two species, known as double diffusive convection, has attracted considerable interest in the last several decades. If gradients of two stratifying agencies having different diffusivities are simultaneously present in a fluid layer, a variety of interesting convective phenomena can occur that are not possible in single component fluids. The double diffusive convection in porous media has also become important in recent years because of its many applications in geophysics, particularly in saline geothermal fields where hot brines remain beneath less saline, cooler ground waters. A comprehensive review of the literature concerning double diffusive convection in a binary fluid saturated porous medium may be found in the book by Nield and Bejan [7]. Excellent review articles on double diffusive convection in porous media include those by Mojtabi and Charrier-Mojtabi ([3], [4]) and Mamou [12].

In a system where two diffusing properties are present, instabilities can occur only if one of the component is destabilizing. If the cross diffusion terms are included in the species transport equations, then the situation will be quite different. Due to the cross diffusion effect, each property gradient has a significant influence on the flux of the other property. A flux of salt caused by a spatial gradient of temperature is called the Soret effect.

There are many studies available on the onset of double diffusive convection in a porous medium with and without cross diffusion effects (see e.g. Nield and Bejan, [7]. Thermal convection in a binary fluid driven by the Soret and DuFour effects has been investigated by Knobloch [8]. He has shown that equations are identical to the thermosolutal problem except for a relation between the thermal and solute Rayleigh numbers. The double diffusive convection in a porous medium in the presence of Soret and DuFour coefficients has been analyzed by Rudraiah and Malashetty [15]. This work has been extended to weak nonlinear analysis by Rudraiah and Siddheshwar [16]. The effect of temperature dependent viscosity on double diffusive convection in an anisotropic porous medium in the presence of Soret coefficient has been studied by Patil and Subramanian [19]. Straughan and Hutter [5] have investigated the double diffusive convection with Soret effect in a porous layer using Darcy-Brinkman model.

Bahloual et al. [1] have carried out an analytical and numerical study of the double diffusive convection in a shallow horizontal porous layer under the influence of Soret effect. Recently, Mansour et al. [2] have investigated the multiplicity of solutions induced by thermosolutal convection in a square porous cavity heated from below and subject to horizontal solute gradient in the presence of Soret effect.

Most of the studies have usually been concerned with homogeneous isotropic porous structures. However during the last one decade, the effect of non-homogeneity and anisotropy of the porous medium have also been studied. The geological and pedagogical processes rarely form isotropic media as is usually assumed in transport studies. In geothermal system with a ground structure composed of many strata of different permeabilities, the overall horizontal permeability may be up to ten times as large as the vertical component. Process such as sedimentation, compaction, frost action, and reorientation of the solid matrix are responsible for the creation of anisotropic natural porous media. Anisotropy can also be a characteristic of artificial porous material like pelleting used in chemical engineering process, fiber materials used in insulating purposes.

There are many investigations available on the thermal convection in a single component fluid saturated anisotropic porous layer heated from below. A theoretical analysis of non-linear thermal convection in an anisotropic porous media is performed by Kvernvold and Tyvand [17]. Nilsen and Storesletten [22] have studied the problem of natural convection in both isotropic and anisotropic porous channels. Tyvand and Storesletten [18] investigated the problem concerning the onset of convection in an anisotropic porous layer in which the principal axes were obliquely oriented to the gravity vector. Natural thermal convection in horizontal anisotropic porous layers heated from below or in vertical cavities filled with an anisotropic porous layer subjected to a constant heat flux, as described in the work of Degan et al. [10]. Some other studies reported the anisotropy and heterogeneous character of porous media, and a summary of these can be found in the book of Nield and Bejan [7].

Recently many authors have studied the effect of anisotropy on the onset of convection in a porous layer (see e.g., Govinder [20], [21]: Malashetty and Swamy [13]; Malashetty and Heera [14]). Although some work on double diffusive convection in an isotropic porous medium is available (Malashetty and Heera [14], attention has not been given to the study of double diffusive convection in an anisotropic porous medium with Soret effect. The main objective of this study is therefore to investigate the effect of Soret coefficient, mechanical and thermal anisotropy on the double diffusive convection in a fluid saturated porous layer using linear analysis.

MATHEMATICAL FORMULATION

A horizontal porous layer held between two walls at $z=0$ and $z=d$ saturated with a Boussinesq fluid, which is heated and salted from below, is considered. The porous medium is assumed to possess isotropy in horizontal plane in both thermal and mechanical properties. A constant gradient of temperature ΔT and salinity ΔS is maintained between the two walls. The generalized Darcy model has been employed for the momentum equation. With these assumptions the basic governing equations of motion are

$$\nabla.\mathbf{q}=0, \tag{1}$$

$$\frac{\rho_0}{\varepsilon}\frac{\partial \mathbf{q}}{\partial t}= -\nabla p-\mu\mathbf{K}.\mathbf{q}+\rho\mathbf{g}, \tag{2}$$

$$\gamma\frac{\partial T}{\partial t}+(\mathbf{q}.\nabla)T=\nabla(D_1.\nabla T), \tag{3}$$

$$\varepsilon\frac{\partial S}{\partial t}+(\mathbf{q}.\nabla)S=\varepsilon D_2\nabla^2 S+\varepsilon D_3\nabla^2 T, \tag{4}$$

$$\rho=\rho_0\left[1-\beta_T\left(T-T_0\right)+\beta_S\left(S-S_0\right)\right], \tag{5}$$

Where, $\mathbf{q}$ is the velocity vector (u,v,w) , ρ is the density, t is time, p is pressure, μ is the dynamic viscosity, K is permeability tensor, g is gravitational acceleration, γ is specific heat ratio, T is temperature, ΔT is temperature difference between the walls, ΔS is salinity difference between the walls, D_1 is thermal diffusivity, ε is the porosity, S is solute concentration, D_2 is solute diffusivity, D_3 is cross diffusion due to T component, β_T is thermal expansion coefficient, β_S is solute expansion coefficient, T_b is the temperature of hot walls, T_0 is the temperature of cold walls.

2.1 Basic state

The basic state of the fluid is assumed to be quiescent and is given by,

$$\mathbf{q}_b=(0,0,0),\;\; p=p_b(z),\, T=T_b(z),\, S=S_b(z),\, \rho=\rho_b(z). \tag{6}$$

Using equation (6), equations (1) to (5) yield

$$\frac{dp_b}{dz}=-\rho_b g,\;\; \frac{d^2T_b}{dz^2}=0,\;\; \frac{d^2S_b}{dz^2}=0,\;\; \rho_b=\rho_0\left[1-\beta_T\left(T_b-T_0\right)+\beta_S\left(S_b-S_0\right)\right]. \tag{7}$$

2.2 Perturbed state

On the basic state we superpose perturbations in the form

$$\begin{aligned}&\mathbf{q}=\mathbf{q}_b+\mathbf{q}'(x,y,z,t),\, T=T_b(z)+T'(x,y,z,t),\, S=S_b(z)+S'(x,y,z,t),\\ &p=p_b(z)+p'(x,y,z,t), \rho=\rho_b(z)+\rho'(x,y,z,t)\end{aligned} \tag{8}$$

where primes indicate perturbations.
We consider only two dimensional disturbances and define stream function ψ by

$$(u',w')=\left(\frac{\partial\psi}{\partial z},\, -\frac{\partial\psi}{\partial x}\right) \tag{9}$$

Introducing (8) in equations (1) - (5) and using basic state equations (7) and the transformations

$$\left(x^*,\, z^*\right)=\left(\frac{x}{d},\frac{z}{d}\right),\;\; t^*=\frac{t}{d^2/D_z},\;\; \psi^*=\frac{\psi}{D_z},\;\; T^*=\frac{T'}{\Delta T},\;\; S^*=\frac{S'}{\Delta S}, \tag{10}$$

where, D_z is the effective thermal diffusivity in vertical direction. To render the resulting equations dimensionless, we obtain (after dropping the asterisks and ε and γ are set equal to unity for simplicity).

$$\left[\frac{1}{Pr_D}\frac{\partial}{\partial t}\nabla^2+\left(\frac{\partial^2}{\partial x^2}+\frac{1}{\xi}\frac{\partial^2}{\partial z^2}\right)\right]\psi = -R_D\frac{\partial T}{\partial x}+R_S\frac{\partial S}{\partial x}, \tag{11}$$

$$\left[\frac{\partial}{\partial t}-\left(\eta\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}\right)\right]T = -\frac{\partial\psi}{\partial x}+\frac{\partial(\psi,T)}{\partial(x,z)}, \tag{12}$$

$$\left[\frac{\partial}{\partial t}-\frac{1}{Le}\nabla^2\right]S-Sr\frac{R_D}{R_S}\nabla^2 T = -\frac{\partial\psi}{\partial x}+\frac{\partial(\psi,S)}{\partial(x,z)}, \tag{13}$$

where , Pr_D is the Darcy Prandtl number $\left(\frac{vd^2\varepsilon}{K_zD_1}\right)$, ξ is the mechanical anisotropy parameter (K_x/K_z), R_D is Darcy Rayleigh number $\left(\frac{\beta_T g\Delta T d K_z}{vD_1}\right)$, R_S is the solute Rayleigh number $\left(\frac{\beta_T g\Delta S d K_z}{vD_1}\right)$, η is thermal anisotropy parameter $\left(\frac{D_x}{D_z}\right)$, Le is the Lewis number $\left(\frac{D_z}{D_x}\right)$, Sr is the Soret parameter $\left(\frac{\beta_S D_3}{\beta_T D_1}\right)$, ψ is the stream function.

Equations (1) - (13) are solved for stress-free, isothermal, isohaline boundary conditions, namely,

$$\psi=T=S=0 \quad \text{at} \quad z=0,1. \tag{14}$$

3 LINEAR STABILITY ANALYSIS

In this section, we discuss the linear stability analysis, which is very useful in the local non-linear stability analysis discussed in the next section. To make this study we neglect the Jacobian in equations (11) and (13) and assume the solutions to be periodic waves of the form

$$\begin{pmatrix} \psi \\ T \\ S \end{pmatrix} = e^{\sigma t} \begin{pmatrix} \Psi \sin \pi \alpha x \\ \Theta \cos \pi \alpha x \\ \Phi \cos \pi \alpha x \end{pmatrix} \sin \pi z . \tag{15}$$

Substituting equations (15) into the linearized version of equations (11) – (13), we get

$$\left(\frac{\sigma}{Pr_D} a^2 + a_1^2 \right) \Psi = - R_D \pi \alpha \Theta + R_S \pi \alpha \Phi , \tag{16}$$

$$\left(\sigma + a_2^2 \right) \Theta = - \pi \alpha \Psi , \tag{17}$$

$$\left(\sigma + \frac{1}{Le} a^2 \right) \Phi + Sr \frac{R_D}{R_S} a^2 \Theta = - \pi \alpha \Psi , \tag{18}$$

where , σ is growth rate, α is wave number, Ψ is the dimensionless amplitude of stream function, Φ is dimensionless amplitude of concentration perturbation, Θ is dimensionless amplitude of temperature perturbation, $a^2 = \pi^2 \left(\alpha^2 + 1 \right)$, $a_1^{\,2} = \pi^2 \left(\alpha^2 + \frac{1}{\xi} \right)$ and $a_2^2 = \pi^2 \left(\eta \alpha^2 + 1 \right)$.

For non-trivial solution of Ψ, Θ and Φ, we require

$$\left(\frac{a^2}{Pr_D} \right) \sigma^3 + \left(a_1^2 + \frac{a^4}{Le\, Pr_D} + \frac{a^2 a_2^2}{Pr_D} \right) \sigma^2 + \left(\frac{a^2 a_1^2}{Le} + a_1^{\,2} a_2^2 + \frac{a^4 a_2^2}{Le\, Pr_D} + \pi^2 \alpha^2 \left(R_S - R_D \right) \right) \sigma$$
$$+ \frac{a^2 a_1^2 a_2^2}{Le} + a_2^2 \pi^2 \alpha^2 R_S - R_D \pi^2 \alpha^2 a^2 \left(Sr + \frac{1}{Le} \right) = 0. \tag{19}$$

3.1 Stationary state

For the validity of principle of exchange of stabilities (i.e., steady case), we have $\sigma = 0$ at the margin of stability. Then the Rayleigh number at which marginally stable steady mode exists becomes

$$R_D^{st} = \frac{\left(\eta \alpha^2 + 1 \right) \left[\pi^2 \left(\alpha^2 + 1 \right) \left(\alpha^2 + \xi^{-1} \right) \right] + R_S \alpha^2 Le}{\alpha^2 \left(\alpha^2 + 1 \right) \left(Sr\, Le + 1 \right)} . \tag{20}$$

The minimum value of the Rayleigh number R_D^{st} occurs at the critical wave number $\alpha = \alpha_c$ where $\alpha^2{}_c = x$ satisfies the equation

$$\eta x^4 + 2\eta x^3 + \left(\eta - \frac{1}{\xi} + \frac{R_S Le}{\pi^2} (\eta - 1) \right) x^2 - \frac{2}{\xi} x - \frac{1}{\xi} = 0 . \tag{21}$$

It is important to note that the critical wavenumber α_c depends on the solute Rayleigh number apart from its dependence on Lewis number and anisotropic properties. This result is in contrast to the case of thermally isotropic porous medium.

In the absence of Soret effect, the stationary Rayleigh number given by equation (20) reduces to

$$R_D^{st} = \frac{\left(\eta\alpha^2+1\right)\left[\pi^2\left(\alpha^2+1\right)\left(\alpha^2+\xi^{-1}\right)\right]+R_S\,\alpha^2\,Le}{\alpha^2\left(\alpha^2+1\right)}. \quad (22)$$

Equation (22) coincides with the results of Malashetty and Swamy [22]. In case of single component fluid saturated porous layer, that is, when $R_S = 0$, the stationary Rayleigh number given by equation (22) reduces to

$$R_D^{st} = \frac{\left(\eta\alpha^2+1\right)\left[\pi^2\left(\alpha^2+1\right)\left(\alpha^2+\xi^{-1}\right)\right]}{\alpha^2\left(\alpha^2+1\right)}. \quad (23)$$

Equation (23) coincides with that of Storesletten [17] for the case of single component fluid saturated anisotropic porous layer. Further for isotropic porous medium, $\xi, \eta = 1$, the equation (23) reduces to the classical result

$$R_D^{st} = \frac{\pi^2}{\alpha^2}\left(\alpha^2+1\right)^2, \quad (24)$$

which has the critical value $R_{Dc}^{st} = 4\pi^2$ for $\alpha_c^2 = 1$ obtained by Horton and Rogers [24] and Lapwood [25]. The critical Rayleigh number R_{Dc}^{st} for marginal state is computed from equation (20) for different values of the parameters and the results are discussed in section 4.

3.2 Oscillatory state

We put $\sigma = i\omega$ (ω is real) in equation (19) and rearrange the terms to get the oscillatory Rayleigh number R_D^{osc} at the margin of stability, in the form

$$R_D^{osc} = R_S\left[\frac{Pr_D\,a_1^2}{a^2}+\frac{a^2}{Le}\right]\left(\frac{1}{\frac{Pr_D\,a_1^2}{a^2}+a_2^2-Sr\,a^2}\right)+\frac{1}{\pi^2\alpha^2\left(\frac{Pr_D\,a_1^2}{a^2}+a_2^2-Sr\,a^2\right)} \quad (25)$$
$$\left(\frac{Pr_D\,a_1^4}{Le}+\frac{Pr_D\,a_1^4\,a_2^2}{a^2}+\frac{2\,a^2\,a_1^2\,a_2^2}{Le}+\frac{a^4\,a_1^2}{Le^2}+\frac{a^6\,a_2^2}{Le^2\,Pr_D}+a_1^2a_2^4+\frac{a^4a_2^4}{Le\,Pr_D}\right),$$

with the non-dimensional frequency ω^2 in the form

$$\omega^2 = \frac{1}{\left[a_1^2+\frac{a^4}{Le\,Pr_D}+\frac{a^2\,a_2^2}{Pr_D}\right]}\left(\frac{a^2a_1^2\,a_2^2}{Le}+a_2^2\,\pi^2\,\alpha^2\,R_S-R_D^{osc}\,\pi^2\alpha^2a^2\left(Sr+\frac{1}{Le}\right)\right). \quad (26)$$

The critical Rayleigh number R_{Dc}^{osc} for oscillatory state is computed from equation (25) for different values of the parameters and the results are discussed in section 4.

RESULTS AND DISCUSSION

The double diffusive convection in a horizontal anisotropic porous layer saturated with Boussinesq fluid, which is heated and salted from below in the presence of Soret effect is studied analytically using linear stability analyses. The effect of mechanical anisotropy parameter, thermal anisotropy parameter, Lewis number and Soret parameter on stationary and oscillatory convection are shown graphically and the results are discussed in this section.

The variation of the critical stationary and oscillatory Rayleigh number R_{Dc} with solute Rayleigh number R_S for different values of the governing parameters is depicted in Figs.1 – 4. The effect of the mechanical anisotropy parameter ξ on the stationary and oscillatory convection is shown in Fig.1. We observe from this figure that an increase in the value of ξ decreases the critical Rayleigh number for both the stationary and oscillatory modes implying that the effect is destabilizing. Further we observe that the effect of anisotropy parameter is insignificant for large solute Rayleigh number.

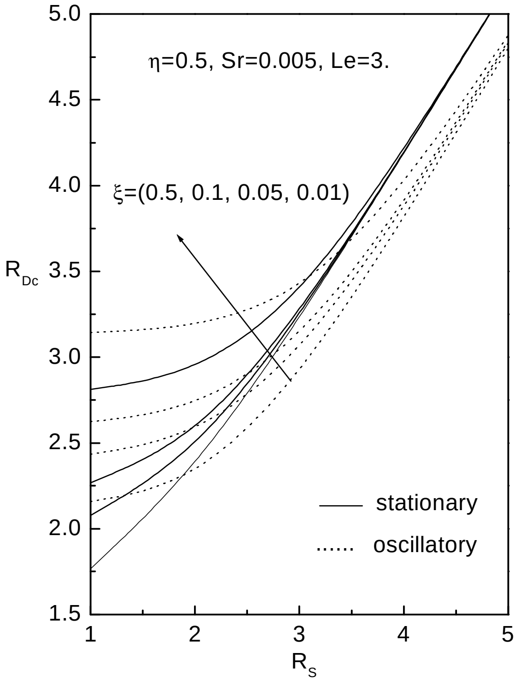

Fig 1: Variation of stationary and oscillatory critical Rayleigh number R_{Dc} with solute Rayleigh number R_S for different values of mechanical anisotropy parameter ξ.

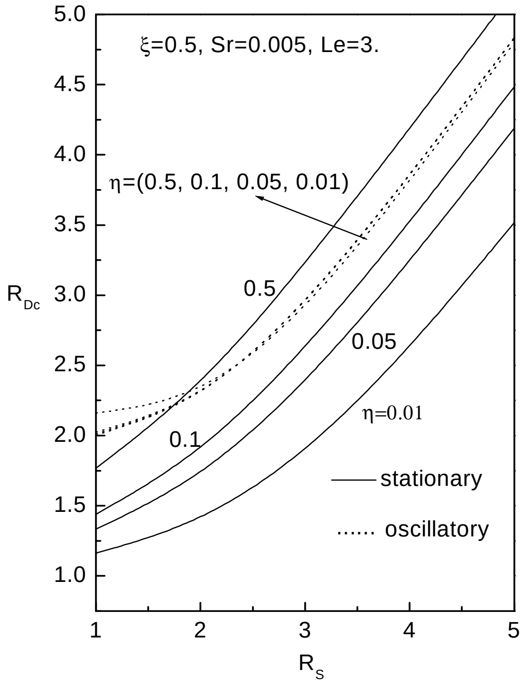

Fig. 2: Fig 1: Variation of stationary and oscillatory critical Rayleigh number R_{Dc} with solute Rayleigh number R_S for different values of thermal anisotropy parameter η.

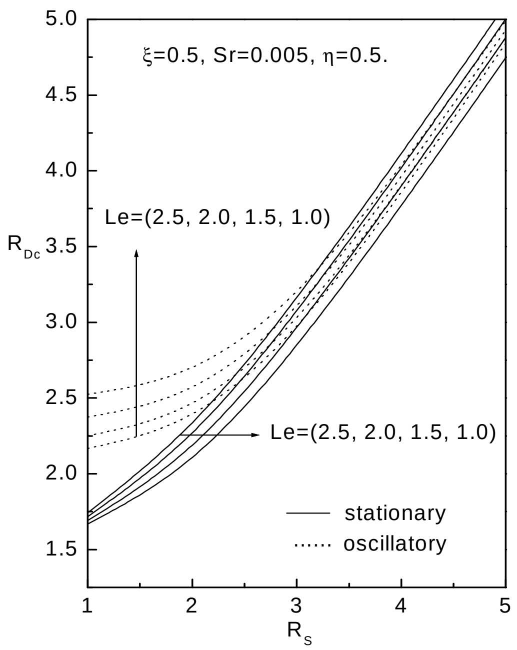

Fig. 3: Variation of stationary and oscillatory critical Rayleigh number R_{Dc} with solute Rayleigh number R_S for different values of Lewis number Le.

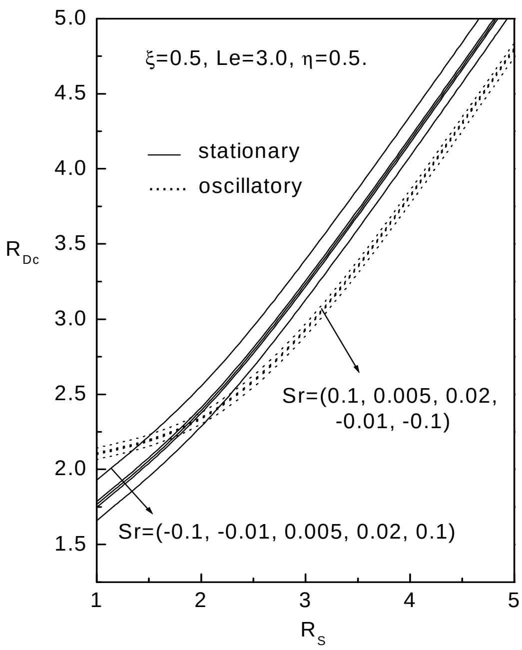

Fig 4: Variation of stationary and oscillatory critical Rayleigh number R_{Dc} with solute Rayleigh number R_S for different values of Soret parameter Sr.

Fig. 2 displays the effect of thermal anisotropy parameter η on both the stationary and oscillatory convection. It is apparent that an increase in the value of thermal anisotropy parameter η increases the critical Rayleigh number for both the stationary and oscillatory modes. Thus the effect of increasing the thermal anisotropy parameter is to stabilize the system.

The effect of Lewis number Le on the stationary and oscillatory convection is shown in Fig. 3. We find that increase in the value of Lewis number Le increases the critical Rayleigh number for stationary and oscillatory modes. Thus the effect of Lewis number is to stabilize the system in the stationary and oscillatory modes

Fig.4 depicts the effect of Soret parameter Sr on the stationary and oscillatory convection. We observe that the negative Soret parameter stabilizes the system while positive Soret parameter destabilizes in the stationary mode. In the oscillatory mode, the negative Soret coefficient has destabilizing effect where as the positive Soret coefficient has a stabilizing effect.

CONCLUSION

An analytical study of double diffusive convection in a fluid saturated anisotropic porous layer with Soret effect is studied using a linear stability analysis. We observe from this study that the value of critical Rayleigh number increases asymptotically with R_S to indicate the stabilizing effect of the solute Rayleigh number on the system in stationary and oscillatory modes. The effect of anisotropic properties is felt only for small values of R_S. In each mode the effect of the mechanical anisotropy parameter ξ is to destabilize the system while the effect of thermal anisotropy parameter η is to stabilize the system. The effect of Le is to stabilize the system in the stationary mode while in the oscillatory mode the trend reverses. The negative Soret parameter stabilizes the system while positive Soret parameter destabilizes in case of stationary mode while its effect reverses in case of oscillatory mode.

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