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Advances in Applied Science Research, 2012, 3 (1): 508-519

ISSN: 0976-8610
CODEN (USA): AASRFC

Analysis of kernel function by the transformation of field resistivity data

J. C. EGBAI

Department of Physics, Delta State University, Abraka, Nigeria

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ABSTRACT

The interpretation process has been a major problem in geophysical investigation of the subsurface of the earth in terms of lithological variations with depth. The resistivity kernel function performs as an intermediary in the process of interpreting apparent measurements in terms of lithological variations with depth. The kernel function cannot be measured in the field but must be obtained from a transformation of measured electric potentials or apparent resistivities. The function is more traceable mathematically than the apparent resistivity function, it has become the basis of a method of quantitative interpretation. In this work, some literature based on kernel function with their peculiar problems were highlighted. The theoretical derivation where the necessary information about the configuration of the earth were fully explained. The Schlumberger electrode configuration was adopted for data collection in the field. The field data obtained from Ughelli in Delta State, Nigeria were matched with theoretical curves from kernel function. The results obtained from kernel function agreed very well with well logs interpretation in Ughelli.

INTRODUCTION

The resistivity kernel function performs as an intermediary in the process of interpreting apparent measurements in terms of lithological variations with depth. It depends solely on the resistivities and thickness of an earth which is considered to be locally stratified in homogeneous and isotropic layers. It is independent of the electrode configuration unlike the apparent resistivity function. The kernel function cannot be measured in the field but must be obtained from a transformation of measured electric potentials or apparent resistivities. The interpretation process involves matching practical field curves with theoretically generated curves from resistivity kernel functions (Egbai, 1997).

The resistivity kernel function has undergone a series of modifications. It has developed into the resistivity transform function which is similar in shape and magnitude to the apparent resistivity function from which it is derived. This transform function has been used as the basis for an analogy with electric filter theory which permits data to be exchanged readily between the apparent resistivity and transform domains.

The natural kernel function method has been the linear filter which is able to exchange data efficiently between the apparent resistivity and resistivity transform domains.

Linear filter theory provides a rapid means of calculating resistivity transforms and apparent resistivities, thereby facilitating resistivity sounding interpretation. The coefficients associaited with the method previously was inadequate for reflection coefficient approaching minus one. The new coefficient of Schlumberger electrode configuration significantly reduce the inadequacy thereby rendering the linear method an accurate and rapid means of calculating apparent resistivity computations (Neil,1975).

It has been found that the wiener-Hoft least squares method is a very successful tool for the determination of resistivity sounding filters. The values of the individual filter coefficients differs quite appreciably from those obtained by the Ghosh procedure. These differences in the filter coefficients, however , have only a negligible effect on the output of the filter. It seems that these differences in the coefficient correspond to a filter function of a rather narrow frequency band around the Nyquist frequency, which is only very weakly present in the input functions (Koefoed and Dirks, 1979).

The 'raised' kernel function in an intermediate function in the interpretation of resistivity sounding data and the methods have been described both for the determination of the raised kernel function from the apparent resistivity function, and the determination of the layer distribution from the raised kernel function (Koefoed, 2006).

The procedure is described by which the second step in the interpretation method result in the determination of layer distribution of layer distribution from the raised kernel function is considerably accelerated.

A power series expansion can be used to obtained the kernel from apparent resistivities for an arbitrary electrode configuration. Three types of function are most appropriate for this purpose. The expansion coefficient can be by a least-square method. In this case, orthonomalization of the functions is of great advantage. (Kohlbeck,2006).

The linear filter method has, in fact, revolutionized the interpretation of resistivity soundings. It provides a simple, rapid and accurate means of calculating theoretical resistivity sounding curves for any assumed earth model comprising homogeneous and electrically isotropic horizontal layers (Egbai, 1997).

Electrical prospecting is an active component of geophysical exploration, and its scope is constantly widening and now encompasses surface measurements, single-borehole measurements, crosshole measurements, sea-button, tunnel, or gallery measurements (Straub, 1995a). The kernel function, as a function of the depth coordinate, is the solution of a 1-d differential equation. The conventional procedure for the calculation of the kernel function consists in applying a recursive scheme. This procedure according to Straub is effective from a computational point of view but becomes cumbersome from an analytical point of view, especially in the case of an arbitrary number of layers for arbitrary positions of the source and measurement points.

The kernel function plays an important role in the 1-D problem because of the spectral representation of the electric potential for a stratified model with a point source (Sraub, 1995b).

The kernel function deals with theoretical thickness and theoretical resistivities of earth layers for the aim of drawing theoretical apparent resistivity curves. For purpose of interpretation, these curves are matched with actual field data (Egbai, 1997).

Slichter (1993) and Langer (1993) arrived at a solution to the problem of using surface potential data, $V_{(r)}$, to give directly the conductivity variation with depth, $\sigma(z)$. Assuming conductivity to be continuous and analytic function of depth only, they derive the solution by applying Ohms law and Laplace's equation as:

$$V_{(r)} = \frac{I\rho_1}{2\pi} \int_0^\infty k(\lambda) J_0(\lambda r) d\lambda \tag{1}$$

where $k(\lambda)$ is the Slichiter kernel, J_0 is the zero-order Bessel function of the first kind, r is distance and $\lambda = \left(\rho_t / \rho_l \right)^{1/2} = \left(\sigma_l / \sigma_t \right)^{1/2} =$ anisotropy, ρ is resistivity while $\sigma =$ conductivity. The subscript l means parallel to the layers while t means tangential to the layers. The integral was inverted by a Hankel transform to define the kernel, $k(\lambda)$, in terms of the measured potentials.

$$k(\lambda) = \frac{2\pi\lambda}{I\rho_1}\int_0^{\infty} V_{(r)} J_0(\lambda r) r dr \quad (2)$$

Koefoed (1965a) worked on the resistivity kernel function by removing two particular deficiencies in previous kernel investigations.

(i) The kernel had always been defined in terms of observed potentials, and
(ii) The kernel had been computed by numerical integration. He applied Hankel inversion to the apparent resistivity formula of Stefanesco et al, (1930) to define the kernel function in terms of observed apparent resistivities.

$$\theta(\lambda) = \frac{1}{2\rho_1}\int_0^{\infty} \frac{1}{r}\{\rho_\alpha(r) - \rho_1\} J_1(\lambda r) dr \quad (3)$$

where $J_1 =$ first order Bessel function of the second kind.

Koefoed observed that numerical integration could be made more efficient by redefining the kernel as:

$$\theta(\lambda) = \frac{\rho_n - \rho_1}{2\rho_1} + \frac{1}{2\rho_1}\int_0^{\infty} \frac{1}{r}\{\rho_\alpha(r) - \rho_n\} J_1(\lambda r) dr \quad (4)$$

As r increases, the intgrand approaches zero.

Koefoed (1965b) subsequently abandoned curve-matching in the kernel domain in favour of the graphical method of Perkeris which he restated in terms of a “modified kernel function”.

$$G(\lambda) = \frac{\theta(\lambda)}{1 + \theta(\lambda)} \quad (5)$$

$G(\lambda)$ is asymptotic to $k_1 \exp(-2\lambda h_1)$ at large λ. A plot on log-linear graph paper gives the layer thickness h and the reflection coefficient k_1. The top layer is removed and a reduced $G(\lambda)$ is calculated by recursion to give the parameter of the next layer.

Koefoed, on the significance of kernel function in resistivity interpretation introduced a “raised kernel function” shown as

$$H^*(\lambda) = \theta(\lambda) + \tfrac{1}{2} = \sum \Delta\theta(\lambda) \quad (6)$$

This he defined by:

$$H^*(\lambda) = \frac{1}{2\rho_1}\int_0^{\infty} \frac{1}{r}\rho_\alpha(r) J_1(\lambda r) dr \quad (7)$$

Koefoed (1970) adopted the “resistivity transform function” as a supplement for the “raised kernel function.” The function is shown

$$T(\lambda) = 2\rho_1 H^*(\lambda) \quad (8)$$

This he defined recursively for an assumed earth model. It also depends only on the thickness and resistivities of subsurface layers and is independent of the electrode array used in the field.

It should be stated clearly that no universally accepted kernel function has been fully identified to date. Interpretation are made by matching field curves with one type of kernel function curve or the other.

In this work, the field curves were compared with well log data and a very high positive correlation were achieved.

THEORY

The theory on kernel function is based on the work of Egbai (1997). The potential at the surface of an n-layer earth having arbitrary resistivities and thickness could be obtained by applying separation of variables to Laplace's equation in cylindrical coordinates written as:

$$V(r) = \frac{I\rho_1}{2\pi}\left[\frac{1}{r} + 2\int_0^\infty \theta_n(\lambda) J_0(\lambda_r) d\lambda\right] \tag{9}$$

The apparent resistivity for a horizontally stratified homogeneous and isotropic layer earth could be written in terms of Bessel function as

$$\rho_a(r) = \rho_1\left[1 + 4r\int_0^\infty \left\{\theta_n(\lambda)^{-1/2}\theta_n\left(\frac{1}{2}\lambda\right)\right\} J_0(\lambda r_r) d\lambda\right] \tag{10}$$

where $J_0(\lambda r) =$ Bessel function of zero order and first kind

$\theta_n(\lambda) =$ kernel function.
$n =$ number of layers.

By differentiating equation (9), the Schlumberger apparent resistivity over an n-layer earth becomes:

$$\rho_a(r) = \rho_1\left\{1 + 2r^2\int_0^\infty \lambda\theta_n(\lambda) J_1(\lambda r) d\lambda\right\} \tag{11}$$

where $J_1 =$ first order Bessel function of the first kind

Equation (10) could be analyzed or evaluated by writing the kernel function as a ratio of polynomials

$$\theta_n(\lambda) = \frac{\rho_n(\lambda)}{H_n(\lambda) - P_n(\lambda)} \tag{12}$$

Recursively, with $U = e^{-2\lambda}$, we have

$$P_1(U) = P_1(U^{-1}) = 0$$
$$H_1(U) = H_1(U^{-1}) = 1$$

$$H_m(U)=H_{m-1}(U)-k_{m-1}U^{\lambda_{m-1}}H_{m-1}\left(U^{-1}\right)$$
$$P_m(U)=P_{m-1}(U)+k_{m-1}U^{\lambda_{m-1}}\left\{P_{m-1}\left(U^{-1}\right)+H_{m-1}\left(U^{-1}\right)\right\}, m=2,\ldots n$$
$$P_{j+1}(U)=P_j(U)+k_jU^{Zj}H_j\left(U^{-1}\right), j=1,2,\ldots,n-1$$
$$H_{j+1}(U)=H_j(U)+k_jU^{Zj}P_j\left(U^{-1}\right)$$

For $n=2$, we have explicitly

$$\theta_2(\lambda)=\frac{k_1e^{-2h_1\lambda}}{1-k_1e^{-2h_1\lambda}} \tag{13}$$

and $n=3$, we have

$$\theta_3(\lambda)=\frac{k_1e^{-2h_1\lambda}+k_2e^{-2h_2\lambda}}{1-k_1e^{2h_1\lambda}+k_1k_2e^{-2(h_2-h_1)\lambda}} \tag{14}$$

where $k=$ reflection factor, $h=$ depth

If $\rho_a(r)$ is known, the kernel function $\theta_n(\lambda)$ could be obtained from equation (10). Introducing the functions

$$f_{(\lambda)}=\theta_n(\lambda)-\frac{1}{2}\theta_n\left(\frac{1}{2}\lambda\right) \tag{15}$$

and

$$C(\lambda)=\rho_1\{4f(\lambda)+1\} \tag{16}$$

From equation (10), we have

$$\rho_a(r)=\int_0^\infty C(\lambda)J_0(\lambda r)d\lambda \tag{17}$$

Applying Fourier –Bessel integral (Bowman, 1958)

$$F(s)=\int_0^\infty\left\{\int_0^\infty F(x)J_0(xt)\right\}(xt)xdxJ_0(st)tdt \tag{18}$$

If we multiply equation (17) by $J_0(\lambda r)$ and integrating over r, we have

$$\int_0^\infty \rho_a(r)J_0(\lambda r)dr=\int_0^\infty r\left\{\int_0^\infty C(t)J_0(tr)dt\right\}J_0(\lambda r)dr=\frac{C(\lambda)}{\lambda} \tag{19}$$

Thus,

$$C(\lambda) = \lambda\int_0^\infty \rho_a(r)J_0(\lambda r)dr = \int_0^\infty \rho_a\left(\frac{t}{\lambda}\right)J_0(t)dt \tag{20}$$

From equation (16), we have

$$f(\lambda) = \frac{C(\lambda) - \rho_1}{4\rho_1} \tag{21}$$

Hence $f(\lambda)$ can be calculated from equations (20) and (21). From equation (15), $\theta_n(\lambda)$ can be derived as

$$\theta_n(\lambda) = \sum_{k=0}^{\infty}\frac{1}{2^k}\, f\, \frac{1}{2^k} \tag{22}$$

$$\therefore \theta_n(\lambda) = f(\lambda) + \frac{1}{2}\theta_n\left(\frac{\lambda}{2}\right)$$

$$\theta_n\left(\frac{\lambda}{2}\right) = f\left(\frac{\lambda}{2}\right) + \frac{1}{2}\theta_n\left(\frac{\lambda}{4}\right) \tag{23}$$

Thus,

$$\theta_n(\lambda) = f(\lambda) + \frac{1}{2}\left\{f\left(\lambda/2\right) + \frac{1}{2}\theta_n\left(\frac{\lambda}{4}\right)\right\} = f(\lambda) + \frac{1}{2}f\left(\lambda/2\right) + \frac{1}{4}\theta_n\left(\lambda/4\right) \tag{24}$$

and so on. The series in equation (22) is convergent since $f(\lambda) \to$ constant when $\lambda \to 0$. The value for $C(\lambda)$ is given by equation (20). For a two layer earth, we have

$$C(\lambda) = \int_0^\infty\left\{\rho_a\left(\frac{t}{\lambda}\right) - \rho_{end}\left(\frac{t}{\lambda}\right)J_0(t)\right\}dt + \int_0^\infty \rho_{end}\left(\frac{t}{\lambda}\right)J_0(t)dt \tag{25}$$

where $\rho_{end}(r)$ is the apparent resistivity curve at the end for a two-layer earth

$$C_{end}(\lambda) = \int_0^\infty \rho_{end}\left(\frac{t}{\lambda}\right)J_0(t)dt = \rho_{end}\left\{4\theta_{end}(\lambda) - \frac{1}{2}\theta_{end}\left(\frac{\lambda}{2}\right) + 1\right\} \tag{26}$$

where

$$\theta_{end}(\lambda) = \frac{k_{end}e^{-2h_{end}\lambda}}{1 - k_{end}e^{-2h_{end}\lambda}} \tag{27}$$

From equation (25), we have

$$C(\lambda) = \int_0^\infty\left\{\rho_a\left(\frac{t}{\lambda}\right) - \rho_{end}\left(\frac{t}{\lambda}\right)J_0(t)\right\}dt + C_{end}(\lambda) \tag{28}$$

$\rho_a(t) - \rho_{end}(t) = 0$ for $t \geq r_{end}$

For equation (28), we can put

$$T(\lambda)=\int_0^\infty\left\{\rho_a\left(\frac{t}{\lambda}\right)-\rho_{end}\left(\frac{t}{\lambda}\right)J_0(t)\right\}dt=\sum_{j=0}^{\infty}T_j(\lambda) \quad (29)$$

where

$$T_j(\lambda)=\int_{z_j}^{z_{j+1}}\left\{\rho_a\left(\frac{t}{\lambda}\right)-\rho_{end}\left(\frac{t}{\lambda}\right)J_0(t)\right\}dt \quad (30)$$

(Z_j = the j^{th} zero order of $J_0(t)$, $Z_0=0$)

The series in equation (29) will form an alternating series with decreasing terms. Applying Euler transformation to speed up the convergence. For equation (28), $C(\lambda)=C_{end}(\lambda)$ for small values of λ. If we assumed

$$C(\lambda)=C_{end}(\lambda) \text{ for } \lambda\le\lambda_1 \quad (31)$$

Calculation of kernel function $\theta_n(\lambda)$ for $\lambda\le\lambda_1$.

Applying equations (21, 22 and 31), we have

$$\theta_n(\lambda)=\sum_{m=0}^{\infty}\frac{1}{2^m}f\left(\frac{\lambda}{2^m}\right)=\sum_{m=0}^{\infty}\frac{1}{2^m}\frac{C\left(\frac{\lambda}{2^m}\right)-\rho_1}{4\rho_1}=\sum_{m=0}^{\infty}\frac{\lambda}{2^m}\frac{C_{end}\left(\frac{\lambda}{2^m}\right)-\rho_1}{4\rho_1} \text{ for } \lambda\le\lambda_1$$

Applying equation (26) and knowing that

$$\frac{1}{2}\sum_{m=0}^{\infty}\frac{1}{2^m}\frac{k_{end}e^{-h_{end}\frac{\lambda}{2^m}}}{1-k_{end}e^{-h_{end}\frac{\lambda}{2^m}}}=\sum_{m=0}^{\infty}\frac{k_{end}\rho^{-2h_{end}\frac{\lambda}{2^m}}}{1-k_{end}\rho^{-2h_{end}\frac{\lambda}{2m}}}$$

we have

$$\theta_n(\lambda)=\frac{\rho_{end}-\rho_1}{2\rho_1}+\left(\frac{\rho_{end}}{\rho_1}\right)\frac{k_{end}\rho^{-2h_{end}\frac{\lambda}{2^m}}}{1-k_{end}\rho^{-2h_{end}\frac{\lambda}{2^m}}} \quad (32)$$

Calculation of kernel function $\theta_n(\lambda)$ for $\lambda\to\infty$

When $\lambda\to\infty$, $f(\lambda)\to 0$. We find a value $\lambda=\lambda_0$ such that $|f(\lambda)|\le\epsilon_1$ for $\lambda\ge\lambda_0$

We then calculate

$$f\left(\frac{\lambda_0}{2^{k-1}}\right) \text{ for } k=1,2,...$$

until $\lambda_0 / 2^{k_0 - 1} \le \lambda_0$

Let $k = k_0$, then applying equation (32), we obtain the value of $\theta_n\left(\frac{\lambda_0}{2^{k_0-1}}\right)$

From equation (22) we obtain

$$\theta_n\left(\frac{\lambda_0}{2^{k-1}}\right) = f\left(\frac{\lambda_0}{2^{k-1}}\right) + \frac{1}{2}\theta_n\left(\frac{\lambda_0}{2^{k}}\right), k = k_0 - 1, \ldots 1 \qquad (33)$$

The theoretical apparent resistivity curves could be computed with equation (32) for $\lambda \le \lambda_1$ and equation (33) for $\lambda \to \infty$.

METHODOLOGY

Ughelli, an oil town of Delta State, Nigeria lies between latitude $5^o30' N$ and $5^o48' N$ and longitudes $5^o48' E$ and $6^o05' E$ and covers an area of about 150 square kilometers. The vegetation is of the Mangrove and Rain forest. The town is drained by a river which flows from North to South and crosses the centre of the town. The area is made of top white sand, clay followed by alluvial sand, sandstone and clay.

The Schlumberger electrode configuration was utilized for the purpose of data acquisition. The geophysical instrument used was the Abem Terrameter SAS 300B. The current electrode spacing reach a maximum of 400m while the potential electrode spacing were varied as the need arose. A total of 84 vertical electrical soundings were carried out in the six locations. The six locations are as follows:

Location
A – Market square
B – Along Ughelli Patani Road
C – Government College Site
D – Post Office Road
E – Catholic Church area
F – Olori Road, Ughelli.

The apparent resistivity equation for field data is given by

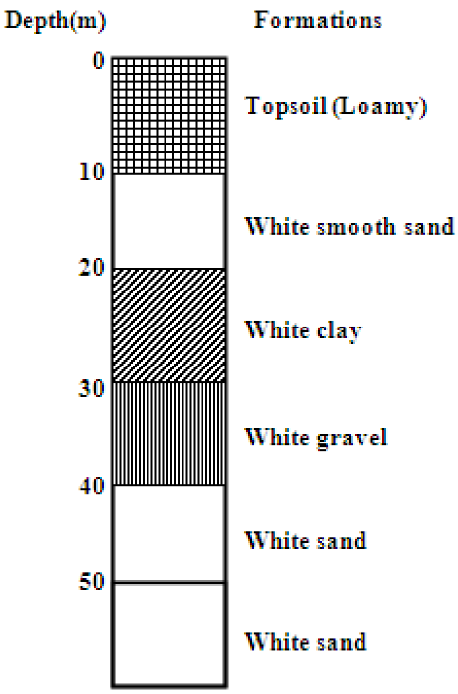
$$\rho_a = \frac{\pi V}{4I}\frac{\left(l^2 - a^2\right)}{a} \qquad (34)$$

If we incorporate equation (32) and (33) into a computer program (Egbai and Ekpekpo, 2003) in line with the basic language for generating automatic field curves and matching it with equation (34), theoretical kernel curves will be obtained.

RESULTS AND DISCUSSION

Quantitative interpretation was done firstly by curve fitting and matching. Curves of logarithms of apparent resistivities (ρ_α) are normally plotted on the Y-axis against the logarithms of $AB/2$ on X-axis. The results of curve fitting and matching showed a rough estimate of layer resistivities and thickness of the aquifer.

The computer assisted interpretation is based on the algorithm which employ digital linear filters for the fast computation of the resistivity function for a given set of layer parameters (Egbai, 1997). Typical smoothened field curves were used for the iteration data. The six curves obtained are respectively shown in figures 1, 2, 3, 4, 5 and 6.



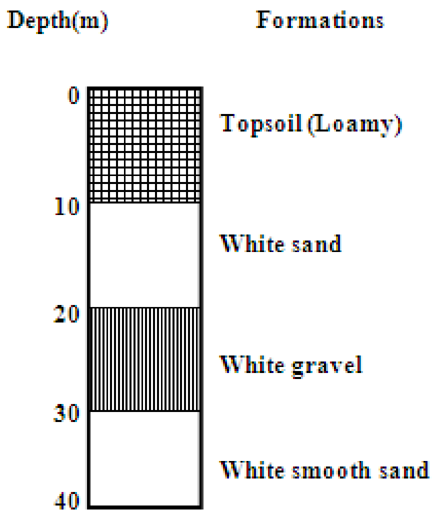


Fig. 8: Driller's log for locations D, E and F

Fig. 7: Driller's log for locations A, B and C.

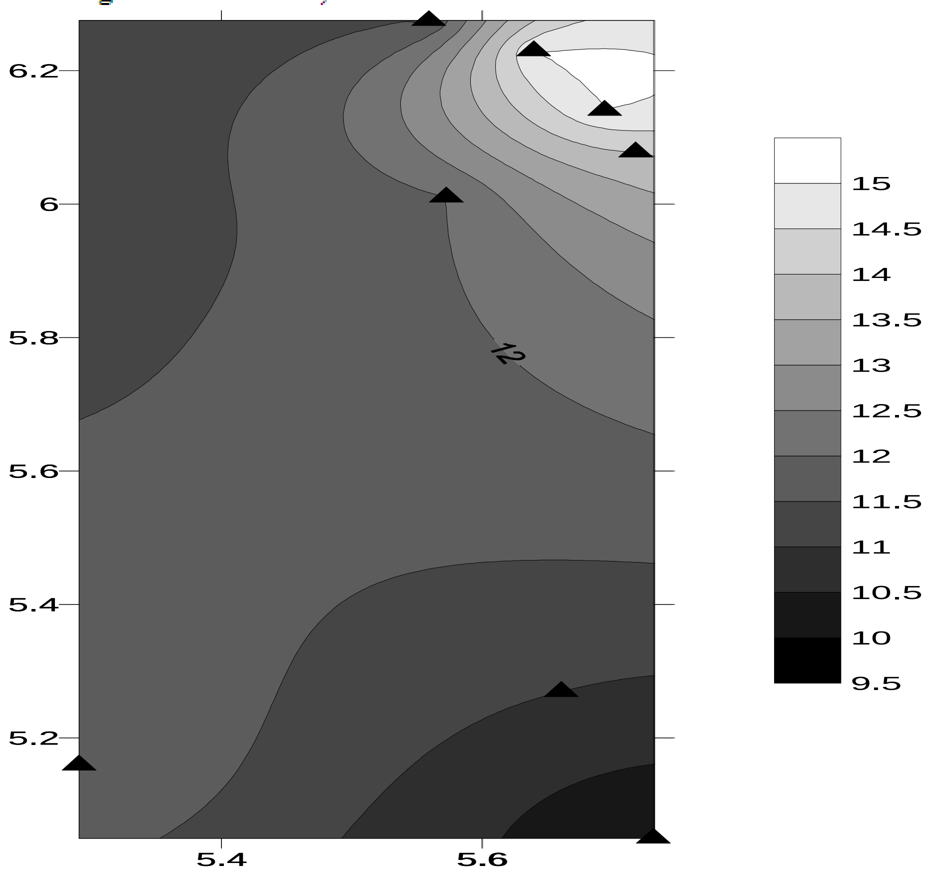


Fig. 8. Contour map of the study Area

Fig. 1: VES Curve for Loc. 1

Fig. 2: VES Curve for Loc. 2

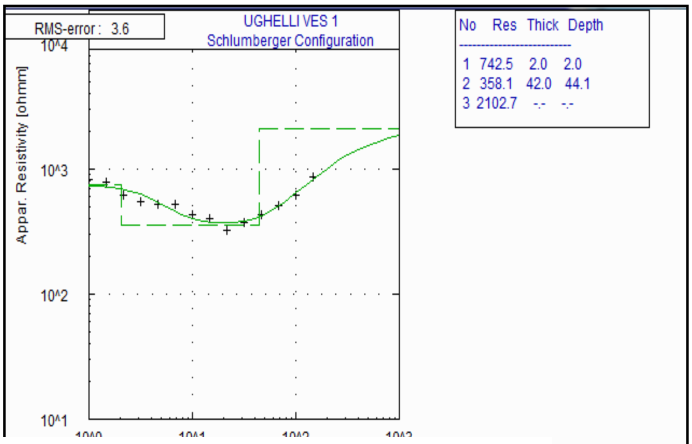


Fig. 3: VES Curve for Loc. 3

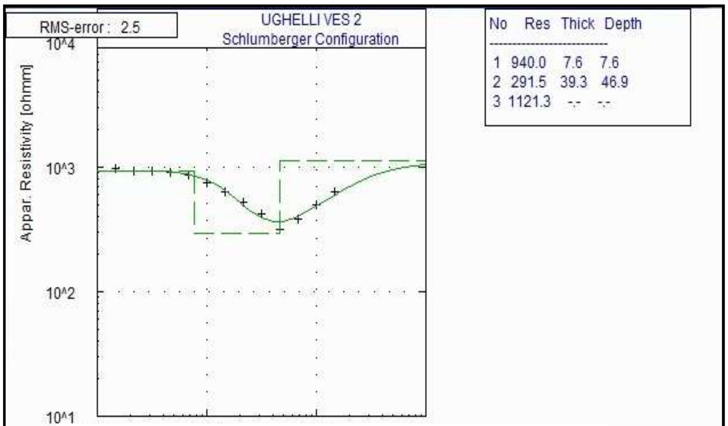


Fig. 4: VES Curve for Loc. 4

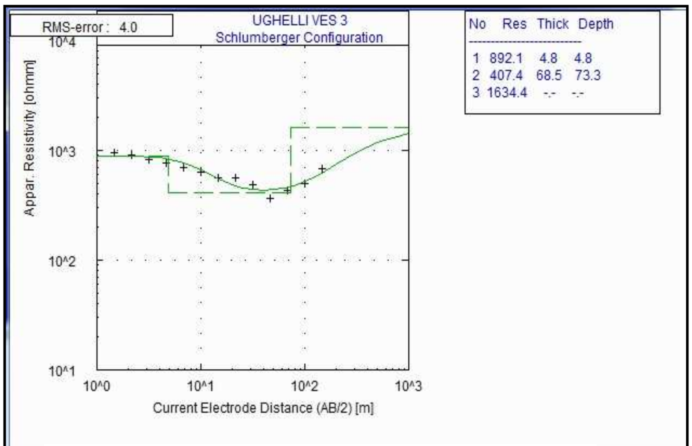


Fig. 5: VES Curve for Loc. 5

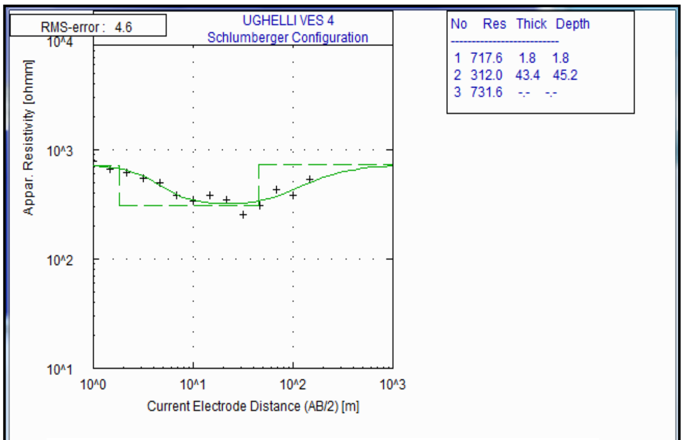
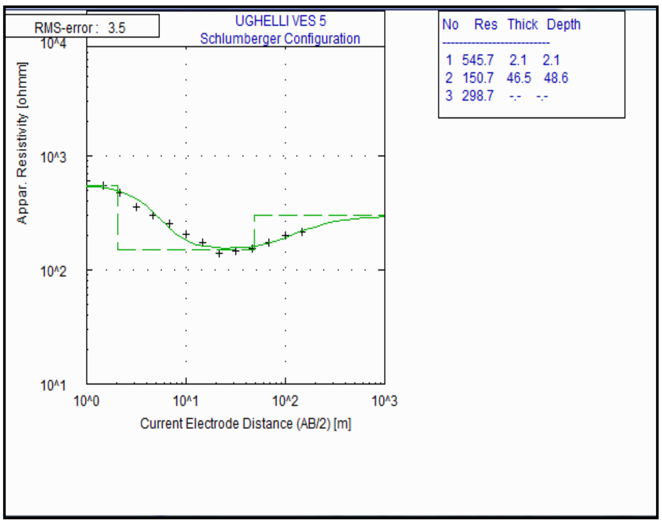
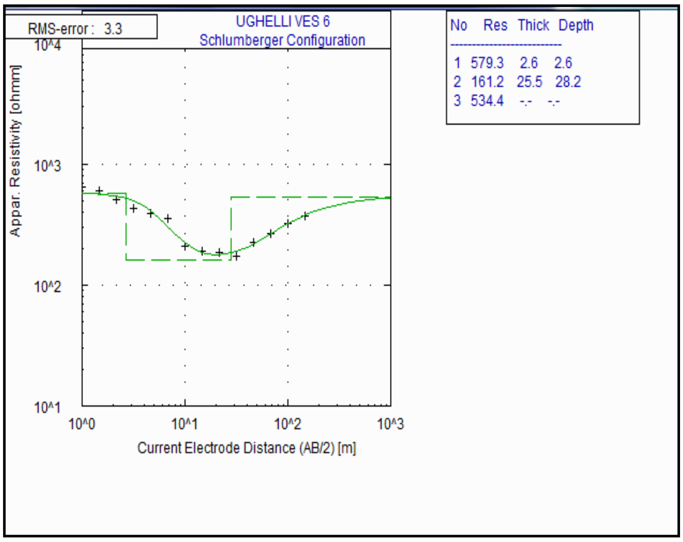


Fig. 6: VES Curve for Loc. 6





The six curves are H-type. The shapes of the curves help in formulating the model parameters got from the theoretical curves using the kernel function. The summary of the results are shown in Table 1. The depth of the aquifers are located within the second layer of the various formations and is within the range of 10 to 20m. The various resistivities, thickness and depth all locations show in the model parameters of the Table 1. The curves of all location have low percentage errors as shows in Table1 and figure 1 to 6.

The driller's log results and hand-dug wells in these six locations at Ughelli confirm the results of surface vertical resistivity sounding (VES) in terms of aquifer depth and thickness.

The driller's log results are as shown in fig. 7 and 8 respectively. Fig. 7 shows the driller's log for locations A, B and C while fig. 8 is for locations D, E and F.

Table 1: Summary of Results (Model Parameter)

LOCATIONS	1	2	3	4	5	6
1st Layer model $\ell\alpha$	742.50Ωm	940.00 Ωm	892.1Ωm	717.6.00Ωm	545.7Ωm	579.3Ωm
2nd Layer model $\ell\alpha$	358.10Ωm	291.5Ωm	407.4Ωm	312.0Ωm	150.7Ωm	161.2Ωm
3rd Layer model $\ell\alpha$	2102.7Ωm	1121.3Ωm	1634.4Ωm	731.6Ωm	298.7Ωm	534.4Ωm
1st Layer model thickness	2.0m	7.6m	4.8m	1.8m	2.1m	2.6m
2nd Layer model thickness	42.00m	39.3m	68.5m	43.4m	46.5m	25.6m
Percentage error between field data curves and theoretical curves RMS (%)	3.6	2.5	4.0	4.6	3.5	3.3

CONCLUSION

The field data got from the work has good impression with the kernel function. The result obtained from the kernel function are in agreement with well logs interpretation in Ughelli with low percentage error.

Acknowledgement
I wish to thank Professor M. B. Asokhai for his assistance in the theory aspect of this work and provision of equipment used. Lady Egbai, F. O. is acknowledged for her moral and financial assistance.

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