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An inverse thermoelastic problem of finite length thick hollow cylinder with internal heat sources

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ABSTRACT

In this paper, by applying Marchi-zgrablich and Laplace integral transform technique .To study inverse unsteady state response of finite thick hollow cylinder with internal heat sources with third kind boundary conditions to determined linear temperature, displacement and stress function. The results are obtained in terms of infinite series and the numerical calculations are carried out by using MATHCAD -7 software and shown graphically.

Keywords: Inverse thermo elastic problem, finite thick hollow cylinder, Marchi-zgrablich and Laplace integral transform

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INTRODUTION

In [3](Sierakowski and Sun, 1968),the direct problem of an exact solution to the elastic deformation of finite length hollow cylinder is consider .In [5] (Grysa and Cialkowski,1980)and [6] (Grysa and kozlowski,1982) ,has determine one dimensional transient thermo elastic problems derived the heating temperature and the heat flux on the surface of the isotropic infinite slab. [7](Wankhede P.C. and Deshmukh, K.C., 1997) has discuss an axisymmetric inverse steady-state problem of thermoelastic deformation of finite length hollow cylinder In [9] (T.T.Gahane ,V.Verghese and Khobragade N.W.,2012),has discuss thermo elastic problem cylinder with heat sources .to determined linear temperature, displacement and stress function with radiation type boundary conditions with constant temp. [10] Warbhe,M.S and Khobragade ,N.W has discuss numerical Study of transient thermoelastic Problem of finite length hollow cylinder

In the present paper, an attempt has been made completely the inverse unsteady state thermoelastic problem of finite thick hollow cylinder with internal heat sources applied for upper plane surface with third kind boundary conditions. To determine the temperature, displacement and thermal stresses on upper plane surface of finite length thick hollow cylinder with internal heat sources.

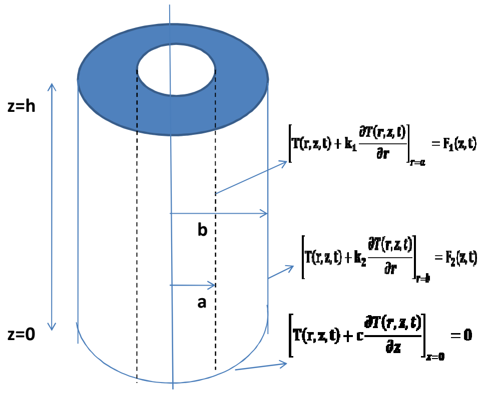

Fig.1 Thick hollow cylinder subject to non homogeneous boundary condition at r=a and r=b

2. STATEMENT OF THE PROBLEM

Consider a thick hollow cylinder of length h occupying the space $D: a \le r \le b, 0 \le z \le h$ (Fig. 1) the thermo elastic displacement function as in [1] is governed by the Poisson's equation

$$\nabla^2 \phi = \frac{(1+\nu)}{(1-\nu)} a_t T \tag{2.1}$$

with $\phi = 0$ at $r = a$ and b (2.2)

Where$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2}$

ν and a_t are the Poisson's ratio and the cylinder linear coefficient of the thermal expansion of the material of the cylinder and T is the temperature of the cylinder satisfying the differential equation.

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial U}{\partial r} + \frac{\partial^2 T}{\partial z^2} + \frac{\Psi(r,z,t)}{k} = \frac{1}{\propto}\frac{\partial T}{\partial t} \tag{2.3}$$

Subject to initial condition

$$T(r,z,0) = 0 \tag{2.4}$$

The boundary conditions

$$\left[T(r,z,t) + k_1 \frac{\partial T(r,z,t)}{\partial r}\right]_{r=a} = F_1(z,t) \tag{2.5}$$

$$\left[T(r,z,t) + k_2 \frac{\partial T(r,z,t)}{\partial r}\right]_{r=b} = F_2(z,t) \tag{2.6}$$

$$\left[T(r,z,t) + c\frac{\partial T(r,z,t)}{\partial z}\right]_{z=0} = 0 \tag{2.7}$$

$$[T(r,z,t)]_{z=h} = g(r,t) \quad \text{(unknown)} \tag{2.8}$$

The interior condition

$$\left[T(r,z,t) + c\frac{\partial T(r,z,t)}{\partial z}\right]_{z=\xi} = f(r,t) \ \text{(known)} \tag{2.9}$$

where $\propto$ the thermal diffusivity of the material of the cylinder, k is thermal conductivity of material and $\Psi(r,z,t)$ is internal heat sources

The radical and axial displacement U and W satisfying the uncoupled the thermoelastic equations are

$$\nabla^2 U - \frac{U}{r^2} + (1-2\nu)^{-1}\frac{\partial e}{\partial r} = 2\frac{(1+\nu)}{(1-2\nu)} a_t \frac{\partial T}{\partial r} \tag{2.10}$$

$$\nabla^2 W \ + (1+2\nu)^{-1}\frac{\partial e}{\partial z} = 2\frac{(1+\nu)}{(1-2\nu)} \frac{\partial T}{\partial z} \tag{2.11}$$

Where $e = \frac{\partial U}{\partial r} + \frac{U}{r} + \frac{\partial W}{\partial z}$ is the volume dilation and

$$U = \frac{\partial \phi}{\partial r} \tag{2.12}$$

$$W = \frac{\partial \phi}{\partial z} \tag{2.13}$$

The stress function are given by

$$\sigma_{rz}(a,z,t) = 0 \quad \sigma_{rz}(b,z,t) = 0 \; \sigma_{rz}(a,z,0) = 0 \tag{2.14}$$

and

$$\sigma_{rr}(a,z,t) = p_{1\,,} \quad \sigma_r(b,z,t) = -p_0 \quad \sigma_z(r,0,t) = 0 \tag{2.15}$$

Where p_1 and p_0 are the surfaces pressures assumed to be uniform over the boundaries of the cylinder. The boundary conditions for the stress functions (2.14) and (2.15) are expressed in term of the displacement components by the following relations:

$$\sigma_{rr} = (\lambda + 2G)\frac{\partial U}{\partial r} + \lambda\left[\frac{U}{r} + \frac{\partial W}{\partial z}\right] \tag{2.16}$$

$$\sigma_{zz} = (\lambda + 2G)\frac{\partial W}{\partial z} + \lambda\left[\frac{\partial U}{\partial r} + \frac{U}{r}\right] \tag{2.17}$$

$$\sigma_{\theta\theta} = (\lambda + 2G)\frac{U}{r} + \lambda\left[\frac{\partial U}{\partial r} + \frac{\partial W}{\partial z}\right] \tag{2.18}$$

$$\tau_{rz} = G\left[\frac{\partial W}{\partial r} + \frac{\partial U}{\partial z}\right] \tag{2.19}$$

Where $\lambda = \frac{2G\nu}{(1-2\nu)}$ is the lame's constant, G is the shear modulus and U and W are the displacement components. The equation (2.1) to (2.19) constitutes the mathematical formulation of the problem under consideration.

3. SOLUTION OF THE PROBLEM

The finite Marchi-Zgrablich integral transform of order p is defined as

$$\bar{f}_p(n) = \int_a^b x f(x) S_p(\mathrm{k}_1, \mathrm{k}_2, \mu_n x)\,dx$$

And inverse Marchi-Zgrablich integral transform as

$$f(x) = \sum_{n=1}^{\infty} \frac{\bar{f}_p(n) S_p(\mathrm{k}_1, \mathrm{k}_2, \mu_n x)}{C_n}$$

Where

$$S_p(\mathrm{k}_1, \mathrm{k}_2, \mu_n x) = J_p(\mu_n x)\{Y_p(\mathrm{k}_1, \mu_n a) + Y_p(\mathrm{k}_2, \mu_n b)\} - Y_p(\mu_n x)\{J_p(\mathrm{k}_1, \mu_n a) + J_p(\mathrm{k}_2, \mu_n b)\}$$

$$C_n = \frac{b^2}{2}\{{S_p}^2(\mathrm{k}_1, \mathrm{k}_2, \mu_n b) - S_{p-1}(\mathrm{k}_1, \mathrm{k}_2, \mu_n b).S_{p+1}(\mathrm{k}_1, \mathrm{k}_2, \mu_n b)\}$$
$$-\frac{a^2}{2}\{{S_p}^2(\mathrm{k}_1, \mathrm{k}_2, \mu_n a) - S_{p-1}(\mathrm{k}_1, \mathrm{k}_2, \mu_n a).S_{p+1}(\mathrm{k}_1, \mathrm{k}_2, \mu_n a)\}$$

An operational property is given by

$$\int_a^b \left[\frac{\partial^2 f}{\partial x^2} + \frac{1}{x}\frac{\partial f}{\partial x} + \frac{p^2 f}{x^2}\right] S_p(\mathrm{k}_1, \mathrm{k}_2, \mu_n x)$$

$$= \frac{b}{k_2} S_p(k_1, k_2, \mu_n b) \left[f + k_2 \frac{\partial f}{\partial x} \right]_{x=b}$$
$$- \frac{a}{k_1} S_p(k_1, k_2, \mu_n a) \left[f + k_1 \frac{\partial f}{\partial x} \right]_{x=a} - \mu_n^2 \bar{f}_p(n)$$

Applying the finite Marchi-Zgrablich integral transform to (2.3), (2.4), (2.7), (2.8), (2.9) and using (2.5), (2.6) one obtains

$$\frac{d^2\bar{T}}{dz^2} - \mu_n^2 \bar{T} + \frac{\bar{\Psi}}{k} = \frac{1}{\propto} \frac{dT}{dt} + Q \tag{3.1}$$

Where $Q = \frac{a}{k_1} S_0(k_1, k_2, \mu_n a) F_1(z, t) - \frac{b}{k_2} S_0(k_1, k_2, \mu_n b) F_2(z, t)$ and p=0

$$\bar{T}(\mu_n, z, 0) = 0 \tag{3.2}$$

$$\left[\bar{T}(\mu_n, z, t) + c \frac{d\bar{T}(\mu_n, z, t)}{dz} \right]_{z=0} = 0 \tag{3.3}$$

$$[\bar{T}(\mu_n, z, t)]_{z=h} = \bar{g}(\mu_n, t) \tag{3.4}$$

$$\left[\bar{T}(\mu_n, z, t) + c \frac{d\bar{T}(\mu_n, z, t)}{dz} \right]_{z=\xi} = \bar{f}(\mu_n, t) \tag{3.5}$$

where $\bar{T}$ denotes the Marchi-Zgrablich integral transform of T and μ_n is parameter.

Applying Laplace transform to (3.1), (3.3), (3.4), (3.5) and using (3.2) one obtains

$$\frac{d^2\bar{T}^*}{dz^2} - q^2 \bar{T}^* = \propto \left(Q^* - \frac{\bar{\Psi}^*}{k} \right) \tag{3.6}$$

Where $q^2 = \mu_n^2 + \frac{s}{\propto}$

$$Q^* = \frac{a}{k_1} S_0(k_1, k_2, \mu_n a) F1^*(z, t) - \frac{b}{k_2} S_0(k_1, k_2, \mu_n b) F_2^*(z, t)$$

$$\left[\bar{T}^*(\mu_n, z, t) + c \frac{d\bar{T}^*(\mu_n, z, t)}{dz} \right]_{z=0} = 0 \tag{3.7}$$

$$[\bar{T}^*(\mu_n, z, t)]_{z=h} = \bar{g}^*(\mu_n, t) \tag{3.8}$$

$$\left[\bar{T}^*(\mu_n, z, t) + c \frac{d\bar{T}^*(\mu_n, z, t)}{dz} \right]_{z=\xi} = \bar{f}^*(\mu_n, t) \tag{3.9}$$

The equation (3.6) is a second order differential equation whose solution is in form

$$\bar{T}^* = Ae^{qz} + Be^{-qz} + PI \tag{3.10}$$

Where $PI = \frac{\propto \left(Q^* - \frac{\bar{\Psi}^*}{k} \right)}{D^2 - q^2}$, $D \equiv \frac{d}{dz}$ and A, B are constant. Using equation (3.7), (3.9) in (3.10) we obtain the values of A and B substituting these values (3.10) and then apply inverse of Laplace transform and Marchi-Zgrablich integral transform. We obtain

$$T(r, z, t) = \frac{2k\pi}{\xi^2} \sum_{n=1}^{\infty} \frac{S_0(k_1, k_2, \mu_n r)}{C_n(1 - c^2 \lambda_n^2)} \left\{ \sum_{m=1}^{\infty} (-1)^{m+1} m[\sin(\lambda_m z) - c\lambda_m \cos(\lambda_m z)] \right\}$$

$$\int_0^t \left[\bar{f}^*(\mu_n, t) - [PI]_{z=\xi} - c \left[\frac{dPI}{dz} \right]_{z=\xi} \right] e^{-k(\mu_n^2 + \lambda_m^2)(t - t')} dt' -$$
$$\frac{2k\pi}{\xi^2} \sum_{n=1}^{\infty} \frac{S_0(k_1, k_2, \mu_n r)}{C_n(1 - c^2 \lambda_n^2)} \left\{ \sum_{m=1}^{\infty} (-1)^{m+1} m[\sin \lambda_m (z - \xi) - c\lambda_m \cos \lambda_m (z - \xi)] \right\} \times$$
$$\int_0^t \left[-[PI]_{z=0} - c \left[\frac{dPI}{dz} \right]_{z=0} \right] e^{-k(\mu_n^2 + \lambda_m^2)(t - t')} dt'$$

$$+\sum_{n=1}^{\infty}\frac{S_0(k_1,k_2,\mu_n r)}{C_n}\int_0^t[\ L^{-1}\{PI\}]e^{-k(\mu_n^2+\lambda_m^2)(t-t')}dt' \quad (3.11)$$

where $\lambda_m = \frac{\pi m}{\xi}$

$$g(r,t) = \frac{2k\pi}{\xi^2}\sum_{n=1}^{\infty}\frac{S_0(k_1,k_2,\mu_n r)}{C_n(1-c^2\lambda_n^2)}\left\{\sum_{m=1}^{\infty}(-1)^{m+1}m[\sin(\lambda_m h)-c\lambda_m\cos(\lambda_m h)]\right\}$$
$$\int_0^t\left[\bar{f}^*(\mu_n,t)-[PI]_{z=\xi}-c\left[\frac{dPI}{dz}\right]_{z=\xi}\right]e^{-k(\mu_n^2+\lambda_m^2)(t-t')}dt' -$$
$$\frac{2k\pi}{\xi^2}\sum_{n=1}^{\infty}\frac{S_0(k_1,k_2,\mu_n r)}{C_n(1-c^2\lambda_n^2)}\left\{\sum_{m=1}^{\infty}(-1)^{m+1}m[\sin\lambda_m(h-\xi)-c\lambda_m\cos\lambda_m(h-\xi)]\right\}\times$$
$$\int_0^t\left[-[PI]_{z=0}-c\left[\frac{dPI}{dz}\right]_{z=0}\right]e^{-k(\mu_n^2+\lambda_m^2)(t-t')}dt' +$$
$$\sum_{n=1}^{\infty}\frac{S_0(k_1,k_2,\mu_n r)}{C_n}\int_0^t[\ L^{-1}\{PI\}]e^{-k(\mu_n^2+\lambda_m^2)(t-t')}dt' \quad (3.12)$$

4. DETERMINATION OF THERMO ELASTIC DISPLACEMENT

Substituting the value of T(r, z t) from (3.11) in (2.1), one obtain the thermo elastic displacement function $\phi(r,z,t)$ as

$$\phi(r,z,t) = \frac{(1+\nu)}{(1-\nu)}a_t\frac{k\pi}{2\xi^2}\sum_{n=1}^{\infty}\frac{r^2S_0(k_1,k_2,\mu_n r)}{C_n(1-c^2\lambda_n^2)}\left\{\sum_{m=1}^{\infty}(-1)^{m+1}m[\sin(\lambda_m z)-c\lambda_m\cos(\lambda_m z)]\right\}$$
$$\times\int_0^t\left[\bar{f}^*(\mu_n,t)-[PI]_{z=\xi}-c\left[\frac{dPI}{dz}\right]_{z=\xi}\right]e^{-k(\mu_n^2+\lambda_m^2)(t-t')}dt'$$
$$-\frac{(1+\nu)}{(1-\nu)}a_t\frac{k\pi}{2\xi^2}\sum_{n=1}^{\infty}\frac{r^2S_0(k_1,k_2,\mu_n r)}{C_n(1-c^2\lambda_n^2)}\left\{\sum_{m=1}^{\infty}(-1)^{m+1}m[\sin\lambda_m(z-\xi)-c\lambda_m\cos\lambda_m(z-\xi)]\right\}\times$$
$$\int_0^t\left[-[PI]_{z=0}-c\left[\frac{dPI}{dz}\right]_{z=0}\right]e^{-k(\mu_n^2+\lambda_m^2)(t-t')}dt'$$
$$+\frac{(1+\nu)}{2(1-\nu)}a_t\sum_{n=1}^{\infty}\frac{r^2S_0(k_1,k_2,\mu_n r)}{C_n}\int_0^t[\ L^{-1}\{PI\}]e^{-k(\mu_n^2+\lambda_m^2)(t-t')}dt' \quad (4.1)$$

Using (4.1) in (2.12) and (2.13) one obtain the radial and axial displacement U and W

$$U = \frac{(1+\nu)}{(1-\nu)}a_t\frac{k\pi}{4\xi^2}\sum_{n=1}^{\infty}\frac{[2rS_0+r^2\mu_n S'_0]}{C_n(1-c^2\lambda_n^2)}\left\{\sum_{m=1}^{\infty}(-1)^{m+1}m[\sin(\lambda_m z)-c\lambda_m\cos(\lambda_m z)]\right\}$$
$$\times\int_0^t\left[\bar{f}^*(\mu_n,t)-[PI]_{z=\xi}-c\left[\frac{dPI}{dz}\right]_{z=\xi}\right]e^{-k(\mu_n^2+\lambda_m^2)(t-t')}dt'$$
$$-\frac{(1+\nu)}{(1-\nu)}a_t\frac{k\pi}{4\xi^2}\sum_{n=1}^{\infty}\frac{[2rS_0+r^2\mu_n S'_0]}{C_n(1-c^2\lambda_n^2)}\left\{\sum_{m=1}^{\infty}(-1)^{m+1}m[\sin\lambda_m(z-\xi)-c\lambda_m\cos\lambda_m(z-\xi)]\right\}\times$$
$$\int_0^t\left[-[PI]_{z=0}-c\left[\frac{dPI}{dz}\right]_{z=0}\right]e^{-k(\mu_n^2+\lambda_m^2)(t-t')}dt'$$
$$+\frac{(1+\nu)}{4(1-\nu)}a_t\sum_{n=1}^{\infty}\frac{[2rS_0+r^2\mu_n S'_0]}{C_n}\int_0^t[\ L^{-1}\{PI\}]e^{-k(\mu_n^2+\lambda_m^2)(t-t')}dt' \quad (4.2)$$

$$W = \frac{(1+\nu)}{(1-\nu)}a_t\frac{k\pi}{4\xi^2}\sum_{n=1}^{\infty}\frac{r^2S_0(k_1,k_2,\mu_n r)}{C_n(1-c^2\lambda_n^2)}\left\{\sum_{m=1}^{\infty}(-1)^{m+1}m[\lambda_m\cos(\lambda_m z)+c\lambda_m^2\sin(\lambda_m z)]\right\}$$
$$\times\int_0^t\left[\bar{f}^*(\mu_n,t)-[PI]_{z=\xi}-c\left[\frac{dPI}{dz}\right]_{z=\xi}\right]e^{-k(\mu_n^2+\lambda_m^2)(t-t')}dt'$$
$$-\frac{(1+\nu)}{(1-\nu)}a_t\frac{k\pi}{4\xi^2}\sum_{n=1}^{\infty}\frac{r^2S_0(k_1,k_2,\mu_n r)}{C_n(1-c^2\lambda_n^2)}\left\{\sum_{m=1}^{\infty}(-1)^{m+1}m[\lambda_m\cos\lambda_m(z-\xi)+c\lambda_m^2\sin\lambda_m(z-\xi)]\right\}$$

$$\times \int_0^t \left[-[\mathrm{PI}]_{z=0} - c \left[\frac{d\mathrm{PI}}{dz}\right]_{z=0} \right] e^{-k({\mu_n}^2+{\lambda_m}^2)(t-t')} dt'$$
$$+ \frac{(1+\nu)}{2(1-\nu)} a_t \sum_{n=1}^{\infty} \frac{r^2 S_0(k_1,k_2,\mu_n r)}{C_n} \int_0^t [\ L^{-1}\{\mathrm{PI}\}] e^{-k({\mu_n}^2+{\lambda_m}^2)(t-t')} dt' \qquad (4.3)$$

5. DETERMINATION OF STRESS FUNCTIONS

Using (4.2 and (4.3) in (2.16),(2.17),(2.18),(2.19 the stress functions are obtained as

$$\sigma_{rr} = (\lambda + 2G) \frac{(1+\nu)}{(1-\nu)} a_t \frac{k\pi}{2\xi^2} \sum_{n=1}^{\infty} \frac{[2S_0 + 4r\,\mu_n S'_0 + r^2 S''_0]}{C_n(1-c^2 n^2)} \times \left\{ \sum_{m=1}^{\infty} (-1)^{m+1} m[\sin(\lambda_m z) - c\lambda_m \cos(\lambda_m z)] \right\}$$
$$\times \int_0^t \left[\bar{f}^*(\mu_n, t) - [\mathrm{PI}]_{z=\xi} - c \left[\frac{d\mathrm{PI}}{dz}\right]_{z=\xi} \right] e^{-k({\mu_n}^2+{\lambda_m}^2)(t-t')} dt'$$
$$- \frac{(1+\nu)}{(1-\nu)} a_t \frac{k\pi}{2\xi^2} \sum_{n=1}^{\infty} \frac{[2S_0 + 4r\,\mu_n S'_0 + r^2 S''_0]}{C_n(1-c^2 {\lambda_n}^2)} \left\{ \sum_{m=1}^{\infty} (-1)^{m+1} m[\sin\lambda_m(z-\xi) - c\lambda_m \cos\lambda_m(z-\xi)] \right\} \times$$
$$\int_0^t \left[-[\mathrm{PI}]_{z=0} - c \left[\frac{d\mathrm{PI}}{dz}\right]_{z=0} \right] e^{-k({\mu_n}^2+{\lambda_m}^2)(t-t')} dt'$$
$$+ \frac{(1+\nu)}{2(1-\nu)} a_t \sum_{n=1}^{\infty} \frac{[2S_0 + 4r\,\mu_n S'_0 + r^2 S''_0]}{C_n} \int_0^t [\ L^{-1}\{\mathrm{PI}\}] e^{-k({\mu_n}^2+{\lambda_m}^2)(t-t')} dt'$$
$$+\lambda \frac{(1+\nu)}{(1-\nu)} a_t \frac{k\pi}{4\xi^2} \sum_{n=1}^{\infty} \frac{[2S_0 + r\mu_n S'_0]}{C_n(1-c^2 {\lambda_n}^2)} \left\{ \sum_{m=1}^{\infty} (-1)^{m+1} m[\sin(\lambda_m z) - c\lambda_m \cos(\lambda_m z)] \right\}$$
$$\times \int_0^t \left[\bar{f}^*(\mu_n, t) - [\mathrm{PI}]_{z=\xi} - c \left[\frac{d\mathrm{PI}}{dz}\right]_{z=\xi} \right] e^{-k({\mu_n}^2+{\lambda_m}^2)(t-t')} dt'$$
$$- \frac{\lambda(1+\nu)}{(1-\nu)} a_t \frac{k\pi}{4\xi^2} \sum_{n=1}^{\infty} \frac{[2S_0 + r\mu_n S'_0]}{C_n(1-c^2 {\lambda_n}^2)} \left\{ \sum_{m=1}^{\infty} (-1)^{m+1} m[\sin\lambda_m(z-\xi) - c\lambda_m \cos\lambda_m(z-\xi)] \right\} \times$$
$$\int_0^t \left[-[\mathrm{PI}]_{z=0} - c \left[\frac{d\mathrm{PI}}{dz}\right]_{z=0} \right] e^{-k({\mu_n}^2+{\lambda_m}^2)(t-t')} dt'$$
$$+ \frac{\lambda(1+\nu)}{4(1-\nu)} a_t \sum_{n=1}^{\infty} \frac{[2S_0 + r\mu_n S'_0]}{C_n} \int_0^t [\ L^{-1}\{\mathrm{PI}\}] e^{-k({\mu_n}^2+{\lambda_m}^2)(t-t')} dt'$$
$$+ \frac{\lambda(1+\nu)}{(1-\nu)} a_t \frac{k\pi}{4\xi^2} \sum_{n=1}^{\infty} \frac{r^2 S_0(k_1,k_2,\mu_n r)}{C_n(1-c^2 {\lambda_n}^2)} \left\{ \sum_{m=1}^{\infty} (-1)^{m+1} m\left[-{\lambda_m}^2 \sin(\lambda_m z) + c{\lambda_m}^3 \cos(\lambda_m z)\ \right] \right\} \times$$
$$\int_0^t \left[\bar{f}^*(\mu_n, t) - [\mathrm{PI}]_{z=\xi} - c \left[\frac{d\mathrm{PI}}{dz}\right]_{z=\xi} \right] e^{-k({\mu_n}^2+{\lambda_m}^2)(t-t')} dt'$$
$$- \frac{\lambda(1+\nu)}{(1-\nu)} a_t \frac{k\pi}{4\xi^2} \sum_{n=1}^{\infty} \frac{r^2 S_0(k_1,k_2,\mu_n r)}{C_n(1-c^2 {\lambda_n}^2)} \left\{ \sum_{m=1}^{\infty} (-1)^{m+1} m\left[-{\lambda_m}^2 \sin\lambda_m(z-\xi) + c{\lambda_m}^3 \cos\lambda_m(z-\xi)\right] \right\}$$
$$\times \int_0^t \left[-[\mathrm{PI}]_{z=0} - c \left[\frac{d\mathrm{PI}}{dz}\right]_{z=0} \right] e^{-k({\mu_n}^2+{\lambda_m}^2)(t-t')} dt'$$
$$+ \frac{\lambda(1+\nu)}{2(1-\nu)} a_t \sum_{n=1}^{\infty} \frac{r^2 S_0(k_1,k_2,\mu_n r)}{C_n} \int_0^t [\ L^{-1}\{\mathrm{PI}\}] e^{-k({\mu_n}^2+{\lambda_m}^2)(t-t')} dt' \qquad (5.1)$$

$$\sigma_{zz} = (\lambda + 2G) \frac{(1+\nu)}{(1-\nu)} a_t \frac{k\pi}{4\xi^2} \sum_{n=1}^{\infty} \frac{r^2 S_0(k_1,k_2,\mu_n r)}{C_n(1-c^2 {\lambda_n}^2)} \times \quad \sum_{m=1}^{\infty} (-1)^{m+1} m\left[-{\lambda_m}^2 \sin(\lambda_m z) + c{\lambda_m}^3 \cos(\lambda_m z)\ \right]$$
$$\times \int_0^t \left[\bar{f}^*(\mu_n, t) - [\mathrm{PI}]_{z=\xi} - c \left[\frac{d\mathrm{PI}}{dz}\right]_{z=\xi} \right] e^{-k({\mu_n}^2+{\lambda_m}^2)(t-t')} dt' \ -$$
$$\frac{(1+\nu)}{(1-\nu)} a_t \frac{k\pi}{4\xi^2} \sum_{n=1}^{\infty} \frac{r^2 S_0(k_1,k_2,\mu_n r)}{C_n(1-c^2 {\lambda_n}^2)} \left\{ \sum_{m=1}^{\infty} (-1)^{m+1} m\left[-{\lambda_m}^2 \sin\lambda_m(z-\xi) + c{\lambda_m}^3 \cos\lambda_m(z-\xi)\right] \right\}$$
$$\times \int_0^t \left[-[\mathrm{PI}]_{z=0} - c \left[\frac{d\mathrm{PI}}{dz}\right]_{z=0} \right] e^{-k({\mu_n}^2+{\lambda_m}^2)(t-t')} dt'$$
$$+ \frac{(1+\nu)}{2(1-\nu)} a_t \sum_{n=1}^{\infty} \frac{r^2 S_0(k_1,k_2,\mu_n r)}{C_n} \int_0^t [\ L^{-1}\{\mathrm{PI}\}] e^{-k({\mu_n}^2+{\lambda_m}^2)(t-t')} dt'$$

$$+\lambda\frac{(1+\nu)}{(1-\nu)}a_t\frac{k\pi}{2\xi^2}\sum_{n=1}^{\infty}\frac{[2S_0+4r\,\mu_n S'_0+r^2S"_0]}{C_n(1-c^2\lambda_n{}^2)}\left\{\sum_{m=1}^{\infty}(-1)^{m+1}m[\sin(\lambda_m z)-c\lambda_m\cos(\lambda_m z)]\right\}$$
$$\times\int_0^t\left[\bar{f}^*(\mu_n,t)-[\mathrm{PI}]_{z=\xi}-c\left[\frac{\mathrm{dPI}}{\mathrm{dz}}\right]_{z=\xi}\right]e^{-k(\mu_n{}^2+\lambda_m{}^2)(t-t')}dt'$$
$$-\frac{\lambda(1+\nu)}{(1-\nu)}a_t\frac{k\pi}{2\xi^2}\sum_{n=1}^{\infty}\frac{[2S_0+4r\,\mu_n S'_0+r^2S"_0]}{C_n(1-c^2\lambda_n{}^2)}\left\{\sum_{m=1}^{\infty}(-1)^{m+1}m[\sin\lambda_m(z-\xi)-c\lambda_m\cos\lambda_m(z-\xi)]\right\}$$
$$\times\int_0^t\left[-[\mathrm{PI}]_{z=0}-c\left[\frac{\mathrm{dPI}}{\mathrm{dz}}\right]_{z=0}\right]e^{-k(\mu_n{}^2+\lambda_m{}^2)(t-t')}dt'$$
$$+\frac{\lambda(1+\nu)}{2(1-\nu)}a_t\sum_{n=1}^{\infty}\frac{[2S_0+4r\,\mu_n S'_0+r^2S"_0]}{C_n}\int_0^t[\,\mathrm{L}^{-1}\{\mathrm{PI}\}]e^{-k(\mu_n{}^2+\lambda_m{}^2)(t-t')}dt'$$
$$+\lambda\frac{(1+\nu)}{(1-\nu)}a_t\frac{k\pi}{4\xi^2}\sum_{n=1}^{\infty}\frac{[2S_0+r\mu_n S'_0]}{C_n(1-c^2\lambda_n{}^2)}\left\{\sum_{m=1}^{\infty}(-1)^{m+1}m[\sin(\lambda_m z)-c\lambda_m\cos(\lambda_m z)]\right\}$$
$$\times\int_0^t\left[\bar{f}^*(\mu_n,t)-[\mathrm{PI}]_{z=\xi}-c\left[\frac{\mathrm{dPI}}{\mathrm{dz}}\right]_{z=\xi}\right]e^{-k(\mu_n{}^2+\lambda_m{}^2)(t-t')}dt'$$
$$-\frac{\lambda(1+\nu)}{(1-\nu)}a_t\frac{k\pi}{4\xi^2}\sum_{n=1}^{\infty}\frac{[2S_0+r\mu_n S'_0]}{C_n(1-c^2\lambda_n{}^2)}\left\{\sum_{m=1}^{\infty}(-1)^{m+1}m[\sin\lambda_m(z-\xi)-c\lambda_m\cos\lambda_m(z-\xi)]\right\}$$
$$\times\int_0^t\left[-[\mathrm{PI}]_{z=0}-c\left[\frac{\mathrm{dPI}}{\mathrm{dz}}\right]_{z=0}\right]e^{-k(\mu_n{}^2+\lambda_m{}^2)(t-t')}dt'$$
$$+\frac{\lambda(1+\nu)}{4(1-\nu)}a_t\sum_{n=1}^{\infty}\frac{[2S_0+r\mu_n S'_0]}{C_n}\int_0^t[\,\mathrm{L}^{-1}\{\mathrm{PI}\}]e^{-k(\mu_n{}^2+\lambda_m{}^2)(t-t')}dt' \tag{5.2}$$

$$\sigma_{\theta\theta}=(\lambda+2G)\frac{(1+\nu)}{(1-\nu)}a_t\frac{k\pi}{4\xi^2}\sum_{n=1}^{\infty}\frac{[2S_0+r\mu_n S'_0]}{C_n(1-c^2\lambda_m{}^2)}\left\{\sum_{m=1}^{\infty}(-1)^{m+1}m[\sin(\lambda_m z)-c\lambda_m\cos(\lambda_m z)]\right\}$$
$$\times\int_0^t\left[\bar{f}^*(\mu_n,t)-[\mathrm{PI}]_{z=\xi}-c\left[\frac{\mathrm{dPI}}{\mathrm{dz}}\right]_{z=\xi}\right]e^{-k(\mu_n{}^2+\lambda_m{}^2)(t-t')}dt'$$
$$-\frac{(1+\nu)}{(1-\nu)}a_t\frac{k\pi}{4\xi^2}\sum_{n=1}^{\infty}\frac{[2S_0+r\mu_n S'_0]}{C_n(1-c^2\lambda_n{}^2)}\left\{\sum_{m=1}^{\infty}(-1)^{m+1}m[\sin\lambda_m(z-\xi)-c\lambda_m\cos\lambda_m(z-\xi)]\right\}\times$$
$$\int_0^t\left[-[\mathrm{PI}]_{z=0}-c\left[\frac{\mathrm{dPI}}{\mathrm{dz}}\right]_{z=0}\right]e^{-k(\mu_n{}^2+\lambda_m{}^2)(t-t')}dt'$$
$$+\frac{(1+\nu)}{4(1-\nu)}a_t\sum_{n=1}^{\infty}\frac{[2S_0+r\mu_n S'_0]}{C_n}\int_0^t[\,\mathrm{L}^{-1}\{\mathrm{PI}\}]e^{-k(\mu_n{}^2+\lambda_m{}^2)(t-t')}dt'$$
$$\lambda\frac{(1+\nu)}{(1-\nu)}a_t\frac{k\pi}{2\xi^2}\sum_{n=1}^{\infty}\frac{[2S_0+4r\,\mu_n S'_0+r^2S"_0]}{C_n(1-c^2\lambda_n{}^2)}\left\{\sum_{m=1}^{\infty}(-1)^{m+1}m[\sin(\lambda_m z)-c\lambda_m\cos(\lambda_m z)]\right\}$$
$$\times\int_0^t\left[\bar{f}^*(\mu_n,t)-[\mathrm{PI}]_{z=\xi}-c\left[\frac{\mathrm{dPI}}{\mathrm{dz}}\right]_{z=\xi}\right]e^{-k(\mu_n{}^2+\lambda_m{}^2)(t-t')}dt'$$
$$-\frac{\lambda(1+\nu)}{(1-\nu)}a_t\frac{k\pi}{2\xi^2}\sum_{n=1}^{\infty}\frac{[2S_0+4r\,\mu_n S'_0+r^2S"_0]}{C_n(1-c^2\lambda_n{}^2)}\left\{\sum_{m=1}^{\infty}(-1)^{m+1}m[\sin\lambda_m(z-\xi)-c\lambda_m\cos\lambda_m(z-\xi)]\right\}$$
$$\times\int_0^t\left[-[\mathrm{PI}]_{z=0}-c\left[\frac{\mathrm{dPI}}{\mathrm{dz}}\right]_{z=0}\right]e^{-k(\mu_n{}^2+\lambda_m{}^2)(t-t')}dt'$$
$$+\frac{\lambda(1+\nu)}{2(1-\nu)}a_t\sum_{n=1}^{\infty}\frac{[2S_0+4r\,\mu_n S'_0+r^2S"_0]}{C_n}\int_0^t[\,\mathrm{L}^{-1}\{\mathrm{PI}\}]e^{-k(\mu_n{}^2+\lambda_m{}^2)(t-t')}dt'$$
$$+\frac{\lambda(1+\nu)}{(1-\nu)}a_t\frac{k\pi}{4\xi^2}\sum_{n=1}^{\infty}\frac{r^2S_0(\mathrm{k}_1,\mathrm{k}_2,\mu_n r)}{C_n(1-c^2\lambda_n{}^2)}\left\{\sum_{m=1}^{\infty}(-1)^{m+1}m[-\lambda_m{}^2\sin(\lambda_m z)+c\lambda_m{}^3\cos(\lambda_m z)\,]\right\}$$

$$\times \int_0^t \left[\bar{f}^*(\mu_n, t) - [\mathrm{PI}]_{z=\xi} - c \left[\frac{dPI}{dz}\right]_{z=\xi} \right] e^{-k(\mu_n{}^2+\lambda_m{}^2)(t-t')} dt'$$
$$- \frac{\lambda(1+\nu)}{(1-\nu)} a_t \frac{k\pi}{4\xi^2} \sum_{n=1}^{\infty} \frac{r^2 S_0(k_1, k_2, \mu_n r)}{C_n(1-c^2\lambda_n{}^2)} \left\{ \sum_{m=1}^{\infty} (-1)^{m+1} m \left[-\lambda_m{}^2 \sin\lambda_m(z-\xi) + c\lambda_m{}^3 \cos\lambda_m(z-\xi) \right] \right\}$$
$$\times \int_0^t \left[-[\mathrm{PI}]_{z=0} - c \left[\frac{dPI}{dz}\right]_{z=0} \right] e^{-k(\mu_n{}^2+\lambda_m{}^2)(t-t')} dt'$$
$$+ \frac{\lambda(1+\nu)}{2(1-\nu)} a_t \sum_{n=1}^{\infty} \frac{r^2 S_0(k_1, k_2, \mu_n r)}{C_n} \int_0^t [\, L^{-1}\{\mathrm{PI}\}] e^{-k(\mu_n{}^2+\lambda_m{}^2)(t-t')} dt' \tag{5.3}$$

$$\sigma_{rz} = G \frac{(1+\nu)}{(1-\nu)} a_t \frac{k\pi}{2\xi^2} \sum_{n=1}^{\infty} \frac{[2rS_0 + r^2\mu_n S'_0]}{C_n(1-c^2\lambda_n{}^2)} \left\{ \sum_{m=1}^{\infty} (-1)^{m+1} m \left[\lambda_m \cos(\lambda_m z) + c\lambda_m{}^2 \sin(\lambda_m z) \right] \right\}$$
$$\times \int_0^t \left[\bar{f}^*(\mu_n, t) - [\mathrm{PI}]_{z=\xi} - c \left[\frac{dPI}{dz}\right]_{z=\xi} \right] e^{-k(\mu_n{}^2+\lambda_m{}^2)(t-t')} dt'$$
$$- \frac{G(1+\nu)}{(1-\nu)} a_t \frac{k\pi}{2\xi^2} \sum_{n=1}^{\infty} \frac{[2rS_0 + r^2\mu_n S_0]}{C_n(1-c^2\lambda_n{}^2)} \left\{ \sum_{m=1}^{\infty} (-1)^{m+1} m \left[\lambda_m \cos\lambda_m(z-\xi) + c\lambda_m{}^2 \sin\lambda_m(z-\xi) \right] \right\}$$
$$\times \int_0^t \left[-[\mathrm{PI}]_{z=0} - c \left[\frac{dPI}{dz}\right]_{z=0} \right] e^{-k(\mu_n{}^2+\lambda_m{}^2)(t-t')} dt'$$
$$+ G \frac{(1+\nu)}{(1-\nu)} a_t \sum_{n=1}^{\infty} \frac{[2rS_0 + r^2\mu_n S'_0]}{C_n} \int_0^t [\, L^{-1}\{\mathrm{PI}\}] e^{-k(\mu_n{}^2+\lambda_m{}^2)(t-t')} dt' \tag{5.4}$$

6. SPECIAL CASE AND NUMERICAL RESULTS

Set $f(r,t) = (1 - e^{-t}) \frac{\delta(r-r_0)}{2\pi r} \xi$ (6.1)

Where δ Dirac-delta function

Applying the finite Marchi-Zgrablich integral transform to (6.1) one obtains

$$\bar{f}(\mu_n, t) = \int_a^b r(1-e^{-t}) \frac{\delta(r-r_0)}{r} \xi S_0(k_1, k_2, \mu_n r) dr$$

$\Psi(r, z, t) = \frac{\delta(r)}{r} \delta(z-h)$, $Q = 0$ where $\lambda_m = \frac{\pi m}{\xi}$

, a=1 m ,b=2m h=1m, $\xi = 1.5m$, $k_1 = k_2 = 1$,

k=0.86 (for copper metal), t=1 sec. , μ_n is the root of transcendental equation.

$[\mathrm{PI}]_{z=\xi} = constant$, $\left[\frac{dPI}{dz}\right]_{z=\xi} = constnat$

$[\mathrm{PI}]_{z=0} = 0$, $\left[\frac{dPI}{dz}\right]_{z=0} = 0$

$L^{-1}[PI] = constant$

$$\mathrm{T}(r,z,t) = \frac{2k\pi}{\xi^2} \sum_{n=1}^{\infty} \frac{S_0(k_1, k_2, \mu_n r)}{C_n(1-c^2\lambda_n{}^2)} \left\{ \sum_{m=1}^{\infty} (-1)^{m+1} m [\sin(\lambda_m z) - c\lambda_m \cos(\lambda_m z)] \right\}$$
$$\int_0^t \left[\bar{f}^*(\mu_n, t) - [\mathrm{PI}]_{z=\xi} - c \left[\frac{dPI}{dz}\right]_{z=\xi} \right] e^{-k(\mu_n{}^2+\lambda_m{}^2)(t-t')} dt'$$
$$+ \sum_{n=1}^{\infty} \frac{S_0(k_1, k_2, \mu_n r)}{C_n} \int_0^t [\, L^{-1}\{\mathrm{PI}\}] e^{-k(\mu_n{}^2+\lambda_m{}^2)(t-t')} dt' \tag{6.3}$$

where $\lambda_m = \frac{\pi m}{\xi}$

To find unknown function put z=h in 6.2

$$g(r,t) = \frac{2k\pi}{\xi^2} \sum_{n=1}^{\infty} \frac{S_0(k_1, k_2, \mu_n r)}{C_n(1-c^2\lambda_n{}^2)} \left\{ \sum_{m=1}^{\infty} (-1)^{m+1} m [\sin(\lambda_m h) - c\lambda_m \cos(\lambda_m h)] \right\}$$
$$\int_0^t \left[\bar{f}^*(\mu_n, t) - [\mathrm{PI}]_{z=\xi} - c \left[\frac{dPI}{dz}\right]_{z=\xi} \right] e^{-k(\mu_n{}^2+\lambda_m{}^2)(t-t')} dt'$$
$$+ \sum_{n=1}^{\infty} \frac{S_0(k_1, k_2, \mu_n r)}{C_n} \int_0^t [\, L^{-1}\{\mathrm{PI}\}] e^{-k(\mu_n{}^2+\lambda_m{}^2)(t-t')} dt' \tag{6.4}$$

DETERMINATION OF THERMO ELASTIC DISPLACEMENT

$$\phi(r,z,t) = \frac{(1+\nu)}{(1-\nu)} a_t \frac{k\pi}{2\xi^2} \sum_{n=1}^{\infty} \frac{r^2 S_0(k_1,k_2,\mu_n r)}{C_n(1-c^2\lambda_n^2)} \left\{\sum_{m=1}^{\infty} (-1)^{m+1} m[\sin(\lambda_m z) - c\lambda_m \cos(\lambda_m z)]\right\}$$
$$\times \int_0^t \left[\bar{f}^*(\mu_n,t) - [\mathrm{PI}]_{z=\xi} - c\left[\frac{d\mathrm{PI}}{dz}\right]_{z=\xi}\right] e^{-k(\mu_n^2+\lambda_m^2)(t-t')} dt'$$
$$+ \frac{(1+\nu)}{2(1-\nu)} a_t \sum_{n=1}^{\infty} \frac{r^2 S_0(k_1,k_2,\mu_n r)}{C_n} \int_0^t [\,L^{-1}\{\mathrm{PI}\}] e^{-k(\mu_n^2+\lambda_m^2)(t-t')} dt' \tag{6.5}$$

Using (4.1) in (2.12) and (2.13) one obtain the radial and axial displacement U and W

$$U = \frac{(1+\nu)}{(1-\nu)} a_t \frac{k\pi}{4\xi^2} \sum_{n=1}^{\infty} \frac{[2rS_0 + r^2\mu_n S'_0]}{C_n(1-c^2\lambda_n^2)} \left\{\sum_{m=1}^{\infty} (-1)^{m+1} m[\sin(\lambda_m z) - c\lambda_m \cos(\lambda_m z)]\right\}$$
$$\times \int_0^t \left[\bar{f}^*(\mu_n,t) - [\mathrm{PI}]_{z=\xi} - c\left[\frac{d\mathrm{PI}}{dz}\right]_{z=\xi}\right] e^{-k(\mu_n^2+\lambda_m^2)(t-t')} dt'$$
$$+ \frac{(1+\nu)}{4(1-\nu)} a_t \sum_{n=1}^{\infty} \frac{[2rS_0 + r^2\mu_n S'_0]}{C_n} \int_0^t [\,L^{-1}\{\mathrm{PI}\}] e^{-k(\mu_n^2+\lambda_m^2)(t-t')} dt' \tag{6.6}$$

$$W = \frac{(1+\nu)}{(1-\nu)} a_t \frac{k\pi}{4\xi^2} \sum_{n=1}^{\infty} \frac{r^2 S_0(k_1,k_2,\mu_n r)}{C_n(1-c^2\lambda_n^2)} \left\{\sum_{m=1}^{\infty} (-1)^{m+1} m\left[\lambda_m \cos(\lambda_m z) + c\lambda_m^2 \sin(\lambda_m z)\right]\right\}$$
$$\times \int_0^t \left[\bar{f}^*(\mu_n,t) - [\mathrm{PI}]_{z=\xi} - c\left[\frac{d\mathrm{PI}}{dz}\right]_{z=\xi}\right] e^{-k(\mu_n^2+\lambda_m^2)(t-t')} dt'$$
$$+ \frac{(1+\nu)}{2(1-\nu)} a_t \sum_{n=1}^{\infty} \frac{r^2 S_0(k_1,k_2,\mu_n r)}{C_n} \int_0^t [\,L^{-1}\{\mathrm{PI}\}] e^{-k(\mu_n^2+\lambda_m^2)(t-t')} dt' \tag{6.7}$$

DETERMINATION OF STRESS FUNCTIONS

$$\sigma_{rr} = (\lambda + 2G) \frac{(1+\nu)}{(1-\nu)} a_t \frac{k\pi}{2\xi^2} \sum_{n=1}^{\infty} \frac{[2S_0 + 4r\,\mu_n S'_0 + r^2 S''_0]}{C_n(1-c^2 n^2)} \times \left\{\sum_{m=1}^{\infty} (-1)^{m+1} m[\sin(\lambda_m z) - c\lambda_m \cos(\lambda_m z)]\right\}$$
$$\times \int_0^t \left[\bar{f}^*(\mu_n,t) - [\mathrm{PI}]_{z=\xi} - c\left[\frac{d\mathrm{PI}}{dz}\right]_{z=\xi}\right] e^{-k(\mu_n^2+\lambda_m^2)(t-t')} dt'$$
$$+ \frac{(1+\nu)}{2(1-\nu)} a_t \sum_{n=1}^{\infty} \frac{[2S_0 + 4r\,\mu_n S'_0 + r^2 S''_0]}{C_n} \int_0^t [\,L^{-1}\{\mathrm{PI}\}] e^{-k(\mu_n^2+\lambda_m^2)(t-t')} dt'$$
$$+\lambda \frac{(1+\nu)}{(1-\nu)} a_t \frac{k\pi}{4\xi^2} \sum_{n=1}^{\infty} \frac{[2S_0 + r\mu_n S'_0]}{C_n(1-c^2\lambda_n^2)} \left\{\sum_{m=1}^{\infty} (-1)^{m+1} m[\sin(\lambda_m z) - c\lambda_m \cos(\lambda_m z)]\right\}$$
$$\times \int_0^t \left[\bar{f}^*(\mu_n,t) - [\mathrm{PI}]_{z=\xi} - c\left[\frac{d\mathrm{PI}}{dz}\right]_{z=\xi}\right] e^{-k(\mu_n^2+\lambda_m^2)(t-t')} dt'$$
$$+ \frac{\lambda(1+\nu)}{4(1-\nu)} a_t \sum_{n=1}^{\infty} \frac{[2S_0 + r\mu_n S'_0]}{C_n} \int_0^t [\,L^{-1}\{\mathrm{PI}\}] e^{-k(\mu_n^2+\lambda_m^2)(t-t')} dt'$$
$$+ \frac{\lambda(1+\nu)}{(1-\nu)} a_t \frac{k\pi}{4\xi^2} \sum_{n=1}^{\infty} \frac{r^2 S_0(k_1,k_2,\mu_n r)}{C_n(1-c^2\lambda_n^2)} \left\{\sum_{m=1}^{\infty} (-1)^{m+1} m\left[-\lambda_m^2 \sin(\lambda_m z) + c\lambda_m^3 \cos(\lambda_m z)\ \right]\right\} \times$$
$$\int_0^t \left[\bar{f}^*(\mu_n,t) - [\mathrm{PI}]_{z=\xi} - c\left[\frac{d\mathrm{PI}}{dz}\right]_{z=\xi}\right] e^{-k(\mu_n^2+\lambda_m^2)(t-t')} dt'$$
$$+ \frac{\lambda(1+\nu)}{2(1-\nu)} a_t \sum_{n=1}^{\infty} \frac{r^2 S_0(k_1,k_2,\mu_n r)}{C_n} \int_0^t [\,L^{-1}\{\mathrm{PI}\}] e^{-k(\mu_n^2+\lambda_m^2)(t-t')} dt' \tag{6.8}$$

$$\sigma_{zz} = (\lambda + 2G) \frac{(1+\nu)}{(1-\nu)} a_t \frac{k\pi}{4\xi^2} \sum_{n=1}^{\infty} \frac{r^2 S_0(k_1,k_2,\mu_n r)}{C_n(1-c^2\lambda_n^2)} \times \quad \sum_{m=1}^{\infty} (-1)^{m+1} m\left[-\lambda_m^2 \sin(\lambda_m z) + c\lambda_m^3 \cos(\lambda_m z)\ \right]$$

$$\times \int_0^t \left[\bar{f}^*(\mu_n, t) - [PI]_{z=\xi} - c \left[\frac{dPI}{dz}\right]_{z=\xi} \right] e^{-k(\mu_n{}^2+\lambda_m{}^2)(t-t')} dt'$$

$$+ \frac{(1+\nu)}{2(1-\nu)} a_t \sum_{n=1}^{\infty} \frac{r^2 S_0(k_1, k_2, \mu_n r)}{C_n} \int_0^t [\ L^{-1}\{PI\}] e^{-k(\mu_n{}^2+\lambda_m{}^2)(t-t')} dt'$$

$$+\lambda \frac{(1+\nu)}{(1-\nu)} a_t \frac{k\pi}{2\xi^2} \sum_{n=1}^{\infty} \frac{[2S_0 + 4r\,\mu_n S'_0 + r^2 S''_0]}{C_n(1 - c^2\lambda_n{}^2)} \left\{ \sum_{m=1}^{\infty} (-1)^{m+1} m[\sin(\lambda_m z) - c\lambda_m \cos(\lambda_m z)] \right\}$$

$$\times \int_0^t \left[\bar{f}^*(\mu_n, t) - [PI]_{z=\xi} - c \left[\frac{dPI}{dz}\right]_{z=\xi} \right] e^{-k(\mu_n{}^2+\lambda_m{}^2)(t-t')} dt'$$

$$+ \frac{\lambda(1+\nu)}{2(1-\nu)} a_t \sum_{n=1}^{\infty} \frac{[2S_0 + 4r\,\mu_n S'_0 + r^2 S''_0]}{C_n} \int_0^t [\ L^{-1}\{PI\}] e^{-k(\mu_n{}^2+\lambda_m{}^2)(t-t')} dt'$$

$$+\lambda \ \frac{(1+\nu)}{(1-\nu)} a_t \frac{k\pi}{4\xi^2} \sum_{n=1}^{\infty} \frac{[2S_0 + r\mu_n S'_0]}{C_n(1 - c^2\lambda_n{}^2)} \left\{ \sum_{m=1}^{\infty} (-1)^{m+1} m[\sin(\lambda_m z) - c\lambda_m \cos(\lambda_m z)] \right\}$$

$$\times \int_0^t \left[\bar{f}^*(\mu_n, t) - [PI]_{z=\xi} - c \left[\frac{dPI}{dz}\right]_{z=\xi} \right] e^{-k(\mu_n{}^2+\lambda_m{}^2)(t-t')} dt'$$

$$+ \frac{\lambda(1+\nu)}{4(1-\nu)} a_t \sum_{n=1}^{\infty} \frac{[2S_0 + r\mu_n S'_0]}{C_n} \int_0^t [\ L^{-1}\{PI\}] e^{-k(\mu_n{}^2+\lambda_m{}^2)(t-t')} dt' \tag{6.9}$$

$$\sigma_{\theta\theta} = (\lambda + 2G) \frac{(1+\nu)}{(1-\nu)} a_t \frac{k\pi}{4\xi^2} \sum_{n=1}^{\infty} \frac{[2S_0 + r\mu_n S'_0]}{C_n(1 - c^2\lambda_m{}^2)} \left\{ \sum_{m=1}^{\infty} (-1)^{m+1} m[\sin(\lambda_m z) - c\lambda_m \cos(\lambda_m z)] \right\}$$

$$\times \int_0^t \left[\bar{f}^*(\mu_n, t) - [PI]_{z=\xi} - c \left[\frac{dPI}{dz}\right]_{z=\xi} \right] e^{-k(\mu_n{}^2+\lambda_m{}^2)(t-t')} dt'$$

$$+ \frac{(1+\nu)}{4(1-\nu)} a_t \sum_{n=1}^{\infty} \frac{[2S_0 + r\mu_n S'_0]}{C_n} \int_0^t [\ L^{-1}\{PI\}] e^{-k(\mu_n{}^2+\lambda_m{}^2)(t-t')} dt'$$

$$\lambda \frac{(1+\nu)}{(1-\nu)} a_t \frac{k\pi}{2\xi^2} \sum_{n=1}^{\infty} \frac{[2S_0 + 4r\,\mu_n S'_0 + r^2 S''_0]}{C_n(1 - c^2\lambda_n{}^2)} \left\{ \sum_{m=1}^{\infty} (-1)^{m+1} m[\sin(\lambda_m z) - c\lambda_m \cos(\lambda_m z)] \right\}$$

$$\times \int_0^t \left[\bar{f}^*(\mu_n, t) - [PI]_{z=\xi} - c \left[\frac{dPI}{dz}\right]_{z=\xi} \right] e^{-k(\mu_n{}^2+\lambda_m{}^2)(t-t')} dt'$$

$$+ \frac{\lambda(1+\nu)}{2(1-\nu)} a_t \sum_{n=1}^{\infty} \frac{[2S_0 + 4r\,\mu_n S'_0 + r^2 S''_0]}{C_n} \int_0^t [\ L^{-1}\{PI\}] e^{-k(\mu_n{}^2+\lambda_m{}^2)(t-t')} dt'$$

$$+ \frac{\lambda(1+\nu)}{(1-\nu)} a_t \frac{k\pi}{4\xi^2} \sum_{n=1}^{\infty} \frac{r^2 S_0(k_1, k_2, \mu_n r)}{C_n(1 - c^2\lambda_n{}^2)} \left\{ \sum_{m=1}^{\infty} (-1)^{m+1} m[-\lambda_m{}^2 \sin(\lambda_m z) + c\lambda_m{}^3 \cos(\lambda_m z)\ ] \right\}$$

$$\times \int_0^t \left[\bar{f}^*(\mu_n, t) - [PI]_{z=\xi} - c \left[\frac{dPI}{dz}\right]_{z=\xi} \right] e^{-k(\mu_n{}^2+\lambda_m{}^2)(t-t')} dt'$$

$$+ \frac{\lambda(1+\nu)}{2(1-\nu)} a_t \sum_{n=1}^{\infty} \frac{r^2 S_0(k_1, k_2, \mu_n r)}{C_n} \int_0^t [\ L^{-1}\{PI\}] e^{-k(\mu_n{}^2+\lambda_m{}^2)(t-t')} dt' \tag{6.10}$$

$$\sigma_{rz} = G \ \frac{(1+\nu)}{(1-\nu)} a_t \frac{k\pi}{2\xi^2} \sum_{n=1}^{\infty} \frac{[2rS_0 + r^2\mu_n S'_0]}{C_n(1 - c^2\lambda_n{}^2)} \left\{ \sum_{m=1}^{\infty} (-1)^{m+1} m[\lambda_m \cos(\lambda_m z) + c\lambda_m{}^2 \sin(\lambda_m z)] \right\}$$

$$\times \int_0^t \left[\bar{f}^*(\mu_n, t) - [PI]_{z=\xi} - c \left[\frac{dPI}{dz}\right]_{z=\xi} \right] e^{-k(\mu_n{}^2+\lambda_m{}^2)(t-t')} dt'$$

$$+G \frac{(1+\nu)}{(1-\nu)} a_t \sum_{n=1}^{\infty} \frac{[2rS_0 + r^2\mu_n S'_0]}{C_n} \int_0^t [\ L^{-1}\{PI\}] e^{-k(\mu_n{}^2+\lambda_m{}^2)(t-t')} dt' \tag{6.11}$$

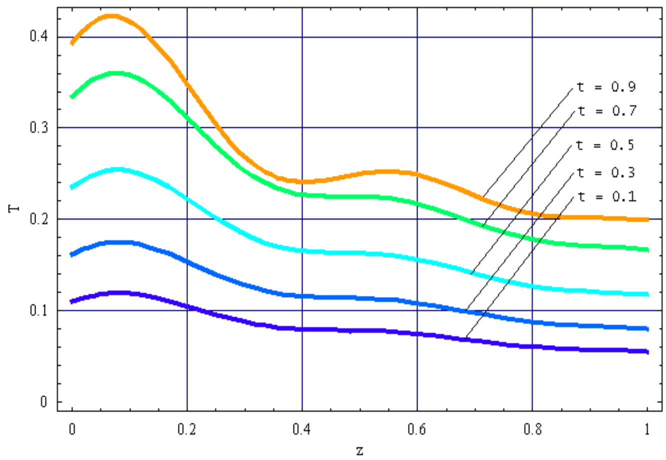

Fig.2 Temperature vs z with different time t

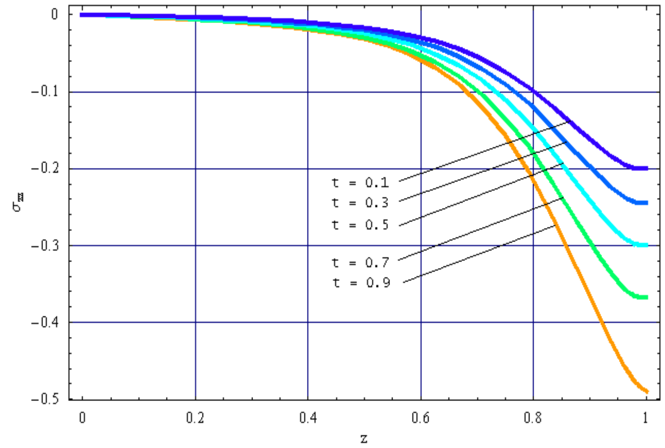

Fig.3 Axial stress vs z with different time t

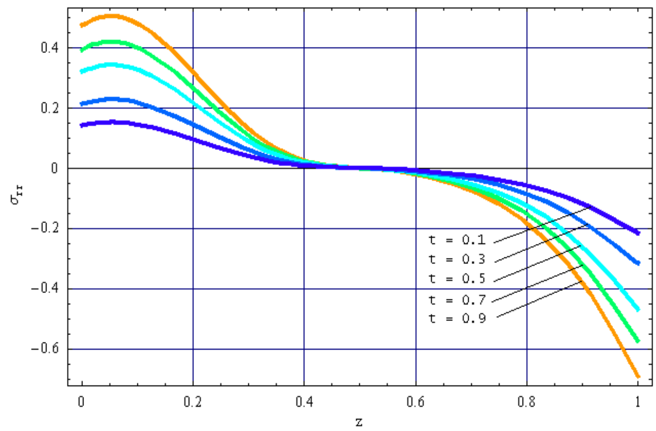

Fig.4 Radial stress vs z with different time t

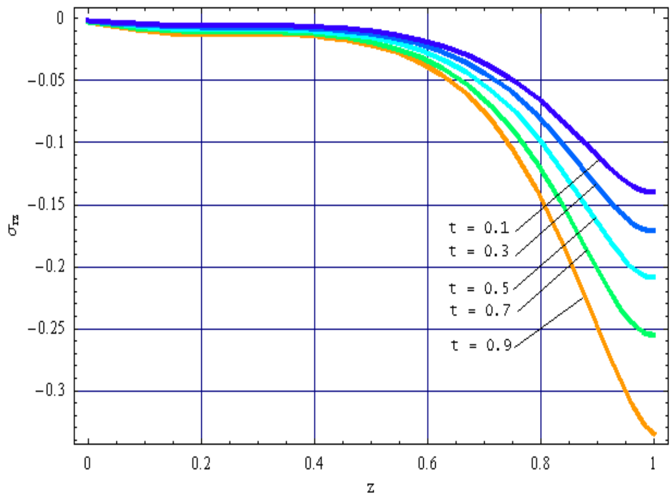

Fig.5 shear stress vs z with different time t

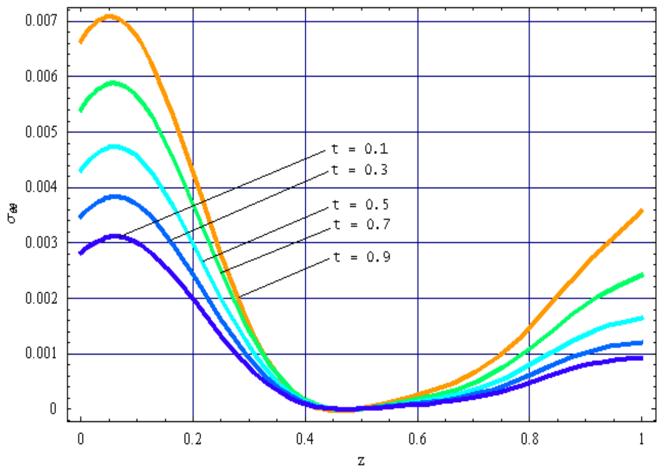

Fig.6 Tangential stress vs z with different time t

CONCLUSION

In this paper, we discussed completely the inverse unsteady state thermoelastic problem of finite length thick hollow cylinder with internal heat sources applied for upper plane surface where the heat is dissipated by convention from the boundary surfaces at r=a and r=b in to surrounding varies position and time on curved surfaces and at lower plane surface heat is dissipated to surrounding at zero temperature. The finite Marchi-Zgrablich and Laplace transforms are used to obtain the numerical results. The series solution convergent since the length of hollow cylinder is very small the temperature, Displacement and thermal stresses that are obtained can be applied to the design of useful structure or machines in engineering application. Any particular case of special interest can be derived by assigning suitable value of the parameters and function in the expression.

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