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A Study of Ureteral Peristalsis in Cylindrical Tube through Porous Medium

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ABSTRACT

This paper deals with the study of ureteral peristalsis in cylindrical tube through porous medium. In reality, there are solid particles present in the flow, but it is necessary to understand the behaviour of the fluid flow. The flow is analyzed for a single traveling wave in an axisymmetric tube with an incompressible, Newtonian fluid. The wavelength of the traveling wave assumed to be large. Analytical expressions for the stream function, axial velocity, and pressure gradient have been obtained. The effect of various parameters on the flow is discussed with the help of graphs.

Keywords: Peristalsis, Porous medium, cylindrical tube.

INTRODUCTION

The transport of fluid through axisymmetric tube by peristaltic motion is a fundamental physiological process has found great importance in many engineering and biological system including study of humans. The phenomenon created by peristaltic is an interesting problem because of its applications in understanding many physiological transport processes through vessels under peristaltic motion. The description of the phenomenon associated to this transport has been studied by many authors. In order to understand peristalsis in various situations, many theoretical and experimental investigations have been made since the first attempt of Latham [9]. Many investigators Jaffrin and Shapiro [6], Yin and Fung [18], Barton and Raynor [2], Manton [10], Srvastava and Srivastava [14] have analyzed the peristaltic flow of viscous fluid. Recently, Rathod and Asha [12] have worked on effect of couple stress fluid and an endoscope on peristaltic motion. Rathod et. al. [11] has studied the peristaltic transport of a couple stress fluid in uniform and non-uniform annulus. Particle motion in unsteady two-dimensional peristaltic flow with application to the ureter has been studied by Joel Jiménez-Lozano et. al. [8]

In resent years, peristaltic transport through porous medium has been of considerable interest among geophysical and fluid dynamists. Peristaltic transport through porous medium has been investigated by El- Sheshawey et. al .[4]. Varshney [16] has studied the fluctuating flow of a viscous fluid through porous medium bounded by porous and horizontal space.

By applying Darcy's law flow through a porous medium has been discussed by Raptis and Perdikis [13]. A mathematical model of peristalsis in tubes through a porous medium has been studied by Hayat et al [6]. Sobh and Mady [15] have investigated the peristaltic flow through a porous medium in a non-uniform channel. Ahmadi and Manvi [1] have studied the equation of motion for viscous flow through a rigid porous medium. Casson fluid flow in a pipe filled with a homogeneous porous medium is discussed by Dash et.al [3]**.** Effect of porous boundaries on peristaltic transport through a porous medium was studied by Elshehaway et al. [5]. Peristaltic pumping of a variable viscosity fluid in a non-uniform tube with permeable wall is presented by Vijayaraj et al. [17].

The aim of the present paper is to study the peristaltic transport of a viscous incompressible fluid through a porous medium and geometrical form of the ureter is considered as a single wave axisymmetric. Graphical results for streamline, velocity profile and the pressure gradient are presented to illustrate the nature of the analytical results.

Formulation of the problem
Consider the flow of an incompressible Newtonian fluid through an axisymmetric tube with a traveling sinusoidal wave along the wall. In cylindrical co-ordinate system $(\bar{r},\bar{z})$ the dimensional equation for the tube radius for an in finite wave train is

$$\bar{R} = \bar{\eta}(\bar{z},\bar{t}) = R_b + a\mathrm{Sin}\left(\frac{2\pi}{\lambda}(\bar{Z} - c\bar{t})\right) \qquad (1)$$

where t is time, R_b is average radius of the tube, a is the amplitude of the wave, λ is wavelength and c is wave speed. In the laboratory frame (Z, R) the flow is unsteady. It becomes steady in moving coordinate (z, r) traveling at the speed of the wave is used. The system of coordinate in the laboratory frame and wave frame are related through

$$\bar{z} = \bar{Z} - c\bar{t}, \quad \bar{r} = \bar{R} \qquad 1(a)$$

and the velocity components are related by

$$\bar{u}_r(\bar{z},\bar{r}) = \bar{U}_r(\bar{Z} - c\bar{t}, R) \ ; \bar{u}_z(\bar{z},\bar{r}) = \bar{U}_z(\bar{Z} - c\bar{t}, R) - c \qquad 1(b)$$

where $\bar{u}_z$ and $\bar{u}_r$ are velocity component in frame.

The governing equations are the continuity equation and the Navier- Stokes equations for incompressible fluid with steady two-dimensional asymmetric flow. These equations are:

$$\frac{1}{\bar{r}}\frac{\partial}{\partial \bar{r}}(\bar{r}\,\bar{u}_r) + \frac{\partial \bar{u}_z}{\partial \bar{z}} = 0 \qquad (2)$$

$$\bar{u}_r\frac{\partial \bar{u}_r}{\partial \bar{r}} + \bar{u}_z\frac{\partial \bar{u}_r}{\partial \bar{z}} = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial \bar{r}} + \nu\left\{\frac{\partial}{\partial \bar{r}}\left(\frac{1}{\bar{r}}\frac{\partial}{\partial \bar{r}}(\bar{r}\,\bar{u}_r)\right) + \frac{\partial^2 \bar{u}_r}{\partial \bar{z}^2}\right\} - \frac{\nu}{K}u_r \qquad (3)$$

$$\bar{u}_r\frac{\partial \bar{u}_z}{\partial \bar{r}} + \bar{u}_z\frac{\partial \bar{u}_z}{\partial \bar{z}} = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial \bar{z}} + \nu\left\{\frac{1}{\bar{r}}\frac{\partial}{\partial \bar{r}}\left(\bar{r}\frac{\partial \bar{u}_z}{\partial \bar{r}}\right) + \frac{\partial^2 \bar{u}_z}{\partial \bar{z}^2}\right\} - \frac{\nu}{K}u_z \qquad (4)$$

In order to simplify the solution of this non-linear system, it is necessary to introduce non-dimensional parameters. For simplification purposes, the Reynolds number is introduced as well:

$$\bar{r}=\frac{r}{R_b},\ \bar{z}=\frac{z}{\lambda},\ \varepsilon=\frac{R_b}{\lambda},\quad \bar{\eta}=\frac{\eta}{R_b},\quad \bar{\psi}=\frac{\psi}{cR_b^{\,2}},$$
$$\bar{u}_r=\frac{u_r}{\varepsilon c},\ \bar{u}_z=\frac{u_z}{c},\ \bar{p}=\frac{p}{\frac{\mu c}{\varepsilon R_b}},\ R_e=\frac{R_b c}{\nu},\ \bar{K}=\frac{k}{R_b^{\,2}}$$

The governing equations, Eqs. (2) - (4) can be rewritten using the dimensionless forms

$$\frac{1}{r}\frac{\partial}{\partial r}(r\,u_r)+\frac{\partial u_z}{\partial z}=0 \tag{5}$$

$$\varepsilon^3 R_e\left[u_r\frac{\partial u_r}{\partial \bar{r}}+u_z\frac{\partial u_r}{\partial z}\right]=-\frac{\partial p}{\partial r}+\varepsilon^2\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r\,u_r)\right)+\varepsilon^4\frac{\partial^2 u_r}{\partial z^2}-\varepsilon^2\frac{1}{K}u_r \tag{6}$$

$$\varepsilon R_e\left[u_r\frac{\partial u_z}{\partial r}+u_z\frac{\partial u_z}{\partial z}\right]=-\frac{\partial p}{\partial z}+\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_z}{\partial r})\right)+\varepsilon^2\frac{\partial^2 u_z}{\partial z^2}-\frac{1}{K}u_z \tag{7}$$

The stream function is related to the velocity components by:

$$u(r,z)=-\frac{1}{r}\frac{\partial \psi}{\partial z};\ \ u_z(r,z)=\frac{1}{r}\frac{\partial \psi}{\partial r} \tag{8}$$

By applying the definition of stream function, the continuity equation is satisfied identically and the equation of motion becomes

$$\varepsilon^3 R_e\left[\frac{1}{r}\frac{\partial \psi}{\partial z}\frac{\partial}{\partial r}-\frac{1}{r}\frac{\partial \psi}{\partial r}\frac{\partial}{\partial z}\right]\frac{1}{r}\frac{\partial \psi}{\partial z}=-\frac{\partial p}{\partial r}-\varepsilon^2\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{\partial \psi}{\partial z}\right)\right)-\varepsilon^4\frac{\partial^2}{\partial z^2}\left(\frac{1}{r}\frac{\partial \psi}{\partial z}\right)+\varepsilon^2\frac{1}{K}\left(\frac{1}{r}\left(\frac{\partial \psi}{\partial z}\right)\right) \tag{9}$$

$$-\varepsilon R_e\left[\frac{1}{r}\frac{\partial \psi}{\partial z}\frac{\partial}{\partial r}-\frac{1}{r}\frac{\partial \psi}{\partial r}\frac{\partial}{\partial z}\right]\frac{1}{r}\frac{\partial \psi}{\partial r}=-\frac{\partial p}{\partial z}+\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial \psi}{\partial r}\right)\right)+\varepsilon^2\frac{\partial^2}{\partial z^2}\left(\frac{1}{r}\frac{\partial \psi}{\partial r}\right)-\frac{1}{K}\left(\frac{1}{r}\left(\frac{\partial \psi}{\partial r}\right)\right) \tag{10}$$

The dimensionless wall equation of the tube is

$$\eta(z)=1+\phi\,\mathrm{Sin}z \tag{11}$$

where $\phi=\frac{a}{R_b}$, for $0<\phi<1$.

Boundary Conditions

The boundary conditions are determined based on the flow rate relation of the stream function in moving frame at two different locations. The first location is at the centerline (r = 0), and the

second is at the wall (r = η). At the centerline, the velocity in the axial direction is constant and the volume flux is zero. At the wall, the axial velocity is -1 and the volume flux is F.
In conclusion, the dimensionless boundary condition is expressed as

$$\psi = 0, \quad \frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial \psi}{\partial r}\right) = 0 \quad for \quad r = 0 \tag{12}$$

$$\psi = F, \quad \left(\frac{1}{r}\frac{\partial \psi}{\partial r}\right) = -1 \quad for \quad r = \eta(z) \tag{13}$$

Equations For Large Wavelength

By neglecting the terms containing (ε), and R_e using long wavelength and low Reynolds number approximation. Eq. (9) and (10) tend to the following system.

$$\frac{\partial p}{\partial r} = 0 \tag{14}$$

$$\frac{\partial p}{\partial z} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial \psi}{\partial r}\right)\right) - M^2\left(\frac{1}{r}\left(\frac{\partial \psi}{\partial r}\right)\right) \tag{15}$$

where $M = \sqrt{\frac{1}{k}}$

With dimensionless boundary conditions

$$\psi = 0, \quad \frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial \psi}{\partial r}\right) = 0 \quad for \quad r = 0 \tag{16}$$

$$\psi = F, \quad \left(\frac{1}{r}\frac{\partial \psi}{\partial r}\right) = -1 \quad for \quad r = \eta(z) \tag{17}$$

We note from Eq. (9) that p = p(z) and , Eq.(9) and (10) can be cross differentiated and substracted to eliminate the dependence to the pressure term. The resulting expression, one finds that

$$\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial \psi}{\partial r}\right)\right)\right) - M^2\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial \psi}{\partial r}\right) = 0 \tag{18}$$

Integrating of Eq. (18) and the use of boundary conditions (16) yields.

$$\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial \psi}{\partial r}\right) - \frac{M^2}{r}\psi = \frac{r}{2}C_1 \tag{19}$$

in which C_1 is an arbitrary function of z. Equation (19) can also be written as

$$r^2\frac{\partial \phi_1}{\partial r} + r\frac{\partial \phi_1}{\partial r} - (M^2 r^2 + 1)\phi_1 = 0 \tag{20}$$

where

$$\phi_1 = \frac{\psi}{r} + \frac{r}{2M^2}C_1 \qquad (21)$$

The solution of equation (20) is

$$\phi_1 = C_2 I_1(Mr) + C_3 K_1(Mr) \qquad (22)$$

and thus from Eq. (21)

$$\psi = r\left[C_2 I_1(Mr) + C_3 K_1(Mr) - \frac{C_1 r}{2M^2} \right] \qquad (23)$$

in which $I_1(Mr)$ and $K_1(Mr)$ are the modified Bessel's functions of first order, first and second kinds, respectively, and C_2 and C_3 are arbitrary functions of z. With the help of boundary condition (16) and (17) expression (23) becomes.

$$\psi = \frac{r[(2F+\eta^2)I_1(Mr) - MrFI_0(M\eta) - \eta r I_1(M\eta)]}{M\eta^2 I_2(M\eta)} \qquad (24)$$

In the above equation I_0 and I_2 are the modified Bessel's functions of order zero and two respectively.

Now, from Eqs. (14), (15) and (24), the expressions for velocity and pressure gradient are

$$u_z = \frac{[(2F+\eta^2)M\, I_0(Mr) - 2MFI_0(M\eta) - 2\eta I_1(M\eta)}{M\eta^2 I_2(M\eta)} \qquad (25)$$

$$\frac{dp}{dz} = \frac{-2[(2F+\eta^2)M^2 I_0(Mr) - M^2 FI_0(M\eta) - \eta M I_1(M\eta)]}{\eta^2 I_2(M\eta)} \qquad (26)$$

The dimensionless pressure rise Δp and friction force F_λ are defined by

$$\Delta P = \int_0^{2\pi} \frac{-2[(2F+\eta^2)M^2 I_0(Mr) - M^2 FI_0(M\eta) - \eta M I_1(M\eta)]}{\eta^2 I_2(M\eta)} dz \qquad (27)$$

$$F_\lambda = \eta^2 \int_0^{2\pi} - \frac{-2[(2F+\eta^2)M^2 I_0(Mr) - M^2 FI_0(M\eta) - \eta M I_1(M\eta)]}{\eta^2 I_2(M\eta)} dz \qquad (28)$$

RESULTS AND DISCUSSION

There are analytical solutions for the stream-function, axial velocity, pressure gradient and frictional force, it is possible to plot the behavior of the model at several location of interest. This paper presents a theoretical study of peristaltic flow through porous medium in the uereter that considers solid particles in the medium. Although the real peristaltic motion presented in the problems of physiology is very difficult, as it is in the ureter, this analysis can serve as a model, which may help in understanding the mechanics of a ureteral peristalsis.

The effect of variation of permeability K on the stream function ψ is depicted in fig.1. This figure shows that with an increase in the permeability K, the deviation in stream function becomes smaller.
The effect of variation of total flux F on the stream function ψ is exhibited in

fig.2. From this figure it can be noted that the stream function strongly depend on the choice of the total flux F.

It is possible to plot the profiles of axial velocity with respect to radial location for various values of permeability K and for the fixed value of total flux F = - 2 and radius of tube $\eta = 1$. From fig.3, it is clear that the axial velocity increases when K is increased. fig 4, shows that for F = - 0.5, a constant velocity distribution across the cross-section is formed; in this case, no pressure gradient exist near the cross-section of tube when η = 1.

When F > -0.5, the flow velocity is curved towards the positive z–direction (not shown in fig. 4) due to an exerted pressure gradient in negative z-direction, whilst for F< -0.5 a pressure gradient in the positive z–direction is required to maintain such a flux value, hence flow velocity is curved towards the negative z- direction. Also from fig. 5, it is clear that the pressure gradient decreases with increase of permeability K.

The distribution of the pressure gradient dp/dz within a wavelength $z \in [0, 2\pi]$ are shown in Fig. 6and 7. Fig. 6 is plotted for various values of the wave amplitude ϕ and fig. 7 is prepared to show the effect of occlusion F on the pressure gradient for fixed values of η, ϕ, and K. It can be clearly seen from the Figs. 6 and 7 that, on the one hand, in the wider part of the tube $z \in [0, 3.2]$ *and* $[6.2, 8]$ the pressure gradient is relatively small, that is, the flow can easily pass without imposition of large pressure gradient. On the other hand, in a narrow part of the tube, a much larger pressure gradient is required to maintain the same flux to pass it, especially for the narrowest position near $z = 3\pi/2$ when the wave amplitude ϕ (Fig.6) or the absolute value of the flux F (Fig.7) is larger. This mathematical model can be of some help in understanding the medical, engineering and industrial problems.

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