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A Simple model for squeezing flow of red blood cell in narrow capillary

Rekha Bali, Swati Mishra and P. N. Tandon*

Department of Mathematics, Harcourt Butler Technological Institute, Kanpur (India)

ABSTRACT

This paper describes a model for the analysis of drag on flexible red blood cell considering different assumed shapes of red blood cells in narrow capillary of diameter of the same order as that of the cells or slightly greater. We have considered Krogh cylindrical model of the capillary surrounded by tissue. The blood cell was assumed as particle suspended in plasma flowing in axisymmetric Stoke's flow. Flow in the narrow gap between the particle and the gap have been studied. The gap between the cell and the capillary wall is small so that the lubrication theory is applicable. Both prolate and oblate spheroids shape of the red cell have been considered. Results for total drag have been brought out for different shapes of parameters involved.

Keywords: Drag, prolate and oblate shape of cell, Permeability, Pressure.

INTRODUCTION

The study of flow of closely fitting incompressible elastic pellets in tubes has been developed as model of blood flows in capillaries. The blood cells were assumed as particles suspended in plasma flowing in axisymmetric Stoke□s flow [8]. When blood flows through capillaries whose diameter is less than that of a red blood cell, it is obvious that the red cell must be deformed. Observations show that cell may also be deformed in somewhat larger vessels [10,11,12,13,14].

In either case, it is to be expected that deformations dependent on the magnitude of the viscous stress and hence on the velocity or pressure gradient, will result during flow. The shape of the particle will then be velocity dependent. Since the increased pressure drop is a function of shape as discussed for the rigid particles [19]. It may be expected that the apparent viscosity of a suspension of deformable particle is a function of mean velocity or pressure gradient in a given size tube and it will vary in a nonlinear manner with tube diameter.

The qualitative ideas were first incorporated in to a quantitative theory by Lighthill using the approximations of lubrication theory and a amplified linear relation of the cell deflection to the local pressure [9]. The original formulation by Lighthill has been modified by Fitzgerald [6, 7] to make it more separate and realistic with respect to the blood flow problems the qualitative results are the same. Lighthill and Fitzgerald both include more realistic analysis. Stoke's flow in a cylindrical tube containing a line of spheroidal particles was studied by Chen and Skalak under large and very narrow gap between the particle and the capillary [3]. In their later work they used lubrication theory [5]. Canhun and Burton [4] have thoroughly studied the surface area and volume as $138.1 \pm 17.4\ \mu m^2$ and $107 \pm 16.8 \mu m^3$ respectively with 8.065 μm as diameter. The density of RBC is about 1.10 and that of plasma is usually assumed to be naturally buoyant particle in stoke□s flow [2]. The resultant force and moment on the particle due to pressure and viscous force must be zero [17] as given by the equation

$$c_0^2\left[P(-b_0)-P(b_0)\right]=\int_{-b_0}^{b_0}\frac{dP}{dx}\left(r_0^2-\frac{2r_0h+h^2}{2\ln\left\{1+\frac{h}{r_0}\right\}}\right)+\frac{2U}{\ln\left\{1+\frac{h}{r_0}\right\}}dx$$

where c_0, b_0 are major & minor axes in meridian plane section of spheroid , $P(x)$ is the pressure and U is the velocity of plasma in the gap between the cell and capillary wall. Wang and Skalak [20] find out the solution for a line of Spherical particles and Chen and Skalak [3] given the solution for a spheroid particle. Bungay and Breener [2] Solved full Navier Stoke's equation by using singular perturbation equation. In narrow capillaries, the flexibilities of red blood cells tend to make themselves centering and they do not rotate. Chen and Skalak [3] treated an idealized problem of blood flow in capillary in which blood cells represent particle whose diameter are of the order of magnitude as that of the capillary. Both prolate and oblate spheroids equally spaced have been considered. Numerical computations for total drag have been brought out for different sets of parameters involved [11, 12]. These authors have also applied lubrication theory as mentioned above for simplified analysis and compared the results the drag pressure drop and drag coefficient. In practice, It is found impractical to compute pressure and drag for large spheroids because of the slow convergence of the series solutions. For this region, lubrication theory for large spheroids is introduced at this point which may be expected to give food accuracy for large spheroids closely fitting the cylinder with very narrow gap. Earlier Tandon et al have also studied fluid exchange and or nutritional transport in between capillary and surrounding tissue. Bali et al [1] developed a model of fluid and nutritional exchange when the gap between the cell and the capillary wall is small and discussed the fluid flux at the capillary tissue interface. The effect of thickness of the porous material on the peristaltic pumping when the tube wall is provided with non-erodible porous lining [14]. A mathematical model is developed to study the steady flow of Casson fluid through an inclined tube of non-uniform cross section with multiple stenosis [18].

Formulation of the Problem:

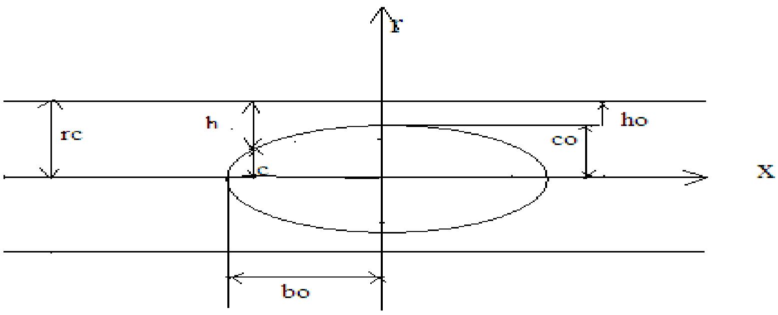

Figure 1 Geometrical diagram for the prolate spheroid

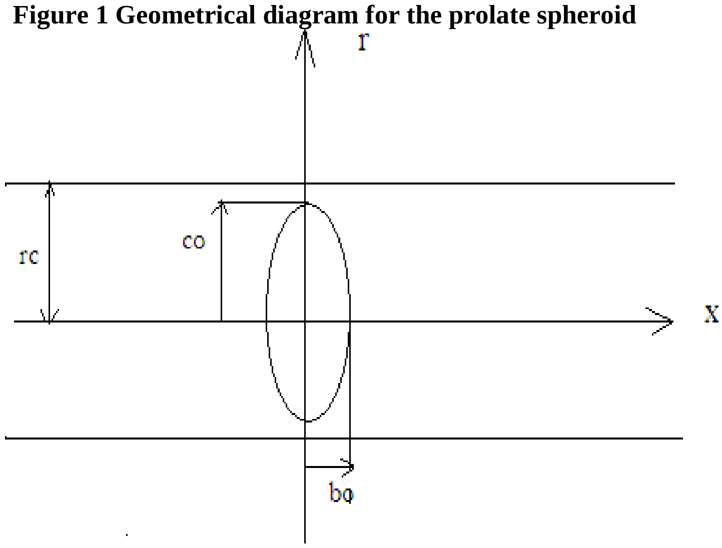

Figure 2 Geometrical diagrams for the oblate spheroid

Equation of motion in capillary region is

$$0=-\frac{\partial \mathrm{P}}{\partial x}+\frac{\mu}{r'}\left(r'\frac{\partial u'}{\partial r'}\right) \qquad (1)$$

In non dimensional form

$$r\frac{\partial \mathrm{P}}{\partial x}+\frac{1}{\mathrm{Re}}\left(r\frac{\partial u}{\partial r}\right)=0 \qquad (2)$$

$$u=\mathrm{Re}\frac{r^2}{4}\frac{\partial \mathrm{P}}{\partial x}+A\log r+B \qquad (3)$$

Boundary Condition:

$$u=0 \qquad r=r_0 \qquad (4)$$

$$u=-U-\sigma\frac{\partial u}{\partial r} \qquad r=1 \qquad (5)$$

Where

$$A=\left[\frac{U+\frac{\mathrm{Re}}{2}\frac{\partial \mathrm{P}}{\partial x}\left(\sigma+\frac{\left(1-r_0^2\right)}{2}\right)}{\log r_0-\sigma}\right]$$

$$B=-\mathrm{Re}\frac{r_0^2}{4}\frac{\partial \mathrm{P}}{\partial x}-A\log r_0$$

Equation of motion in tissue region is

$$\frac{\partial^2\overline{\mathrm{P}}}{\partial x^2}+\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \overline{\mathrm{P}}}{\partial r}\right)=0 \qquad (6)$$

$$\overline{\mathrm{P}}=\sum E_n\cos\left(\alpha_n x\right)\left\{I_0\left(\alpha_n r\right)+\frac{I_1\left\{\alpha_n\left(1+H\right)\right\}}{K_1\left\{\alpha_n\left(1+H\right)\right\}}K_0\left(\alpha_n r\right)\right\} \qquad (7)$$

Boundary Condition:

$$\overline{\mathrm{P}}=0 \qquad x=b_0 \qquad (8)$$

$$\frac{\partial \overline{\mathrm{P}}}{\partial x}=0 \qquad x=0 \qquad (9)$$

$$\frac{\partial \bar{P}}{\partial r}=0 \qquad r=1+H \tag{10}$$

Pressure in capillary region is obtained by solving equation of continuity:

$$\mathrm{P}=\frac{4K}{g_{16}}\sum E_nF(\alpha_n r)\frac{1}{\alpha_n}\left[-\left(1-\mathrm{P}_0\frac{g_{17}}{g_{16}}\right)\frac{\cos(\alpha_n x)}{\alpha_n}+\frac{g_{18}}{g_{16}}\left\{-\frac{x^2\cos(\alpha_n x)}{\alpha_n}+\frac{4x\sin(\alpha_n x)}{\alpha_n{}^2}+\frac{2\cos(\alpha_n x)}{\alpha_n{}^3}\right\}\right]$$
$$+\mathrm{P}_0-\frac{4K}{g_{16}}\sum E_nF(\alpha_n r)\frac{1}{\alpha_n}\left[-\left(1-\mathrm{P}_0\frac{g_{17}}{g_{16}}\right)\frac{\cos(\alpha_n)}{\alpha_n}+\frac{g_{18}}{g_{16}}\left\{-\frac{\cos(\alpha_n)}{\alpha_n}+\frac{4\sin(\alpha_n)}{\alpha_n{}^2}+\frac{2\cos(\alpha_n)}{\alpha_n{}^3}\right\}\right] \tag{11}$$

Following Chen and Skalak [3], the total drag is given by

$$D=4\pi r_0{}^2\left[\tau_{rx}\right]_{r=r_0}+2\pi r_0{}^3\frac{\partial \mathrm{P}}{\partial x} \tag{12}$$

Boundary Condition

$$\tau_{rx}=\frac{\partial u}{\partial r} \qquad r=r_0 \tag{13}$$

$$D=2\pi r_0\left[(\mathrm{Re}+1)r_0{}^2+\frac{\left\{\sigma+\frac{(1-r_0{}^2)}{2}\right\}}{(\log r_0-\sigma)}\right]_{r=r_0}$$
$$+\frac{4K}{g_{16}}\sum E_nF(\alpha_n r)\frac{1}{\alpha_n}\left[-\left(1-\mathrm{P}_0\frac{g_{17}}{g_{16}}\right)\frac{\cos(\alpha_n x)}{\alpha_n}+\frac{g_{18}}{g_{16}}\left\{-\frac{x^2\cos(\alpha_n x)}{\alpha_n}+\frac{4x\sin(\alpha_n x)}{\alpha_n{}^2}+\frac{2\cos(\alpha_n x)}{\alpha_n{}^3}\right\}\right] \tag{15}$$

Pressure drop is given by

$$a^2\left[\mathrm{P}(-b_0)-\mathrm{P}(b_0)\right]=\int_{-b_0}^{b_0}\frac{\partial \mathrm{P}}{\partial x}\left[r_0{}^2-\frac{2r_0(1-r_0)+(1-r_0)^2}{2\ln\left\{1+\frac{(1-r_0)}{r_0}\right\}}\right]dx \tag{16}$$

Where

$$g_{16}=\left\{-\frac{1}{4}+(c_0-\alpha\mathrm{P}_0)^2-\frac{\ln(c_0-\alpha\mathrm{P}_0)}{2}\right\}+\left\{1+\frac{\left(\sigma+\frac{1}{2}-\frac{1}{2}(c_0-\alpha\mathrm{P}_0)\right)}{\ln(c_0-\alpha\mathrm{P}_0)-\sigma}\right\}$$

$$g_{17} = \frac{\alpha}{(c_0 - \alpha P_0)}\left\{2(c_0 - \alpha P_0)^2 - \frac{1}{2}\right\} + \left\{1 + \frac{\left(\sigma + \frac{1}{2} - \frac{1}{2}(c_0 - \alpha P_0)\right)}{\ln(c_0 - \alpha P_0) - \sigma}\right\}\left\{\frac{\alpha}{(c_0 - \alpha P_0)}\left\{2(c_0 - \alpha P_0)^2 - \frac{1}{2}\right\}\right\}$$

$$+\left\{\frac{\alpha}{\sigma + \frac{1}{2} - \frac{1}{2}(c_0 - \alpha P_0)} - \frac{\alpha}{(c_0 - \alpha P_0)}\right\}\ln(c_0 - \alpha P_0) + \left\{-\frac{1}{4} + (c_0 - \alpha P_0)^2 - \frac{\ln(c_0 - \alpha P_0)}{2}\right\}$$

$$g_{18} = \frac{\alpha}{2b_0^2(c_0 - \alpha P_0)}\left\{2(c_0 - \alpha P_0)^2 - \frac{1}{2}\right\} + \left\{\frac{\left(\sigma + \frac{1}{2} - \frac{1}{2}(c_0 - \alpha P_0)\right)}{\ln(c_0 - \alpha P_0) - \sigma}\right\}\left\{\frac{\alpha}{2b_0^2(c_0 - \alpha P_0)}\left\{2(c_0 - \alpha P_0)^2 - \frac{1}{2}\right\}\right\}$$

$$+\left[\frac{\alpha}{2b_0^2(c_0 - \alpha P_0)}\left\{2(c_0 - \alpha P_0)^2 - \frac{1}{2}\right\}\right]\left\{\frac{c_0}{2b_0^2\left(\sigma + \frac{1}{2} - \frac{1}{2}(c_0 - \alpha P_0)\right)} - \frac{c_0}{2b_0^2(c_0 - \alpha P_0)\ln(c_0 - \alpha P_0)}\right\}$$

$$+\left\{-\frac{1}{4} + (c_0 - \alpha P_0)^2 - \frac{\ln(c_0 - \alpha P_0)}{2}\right\}$$

RESULTS AND DISCUSSION

The governing equations have been solved using analytical methods with proper boundary and interfacial conditions. The results of analysis have been obtained and discussed through graph 3 to 7.

Figure 3 depicts the variation of drag with axial distance for different values of permeability. As permeability increases more fluid goes inside the tissue therefore more pressure developed in between the gap (i.e. between cell and capillary wall). Due to increased pressure cell deformed and squeezed out easily. i.e. drag decreases.

Figures 4 to 5 show the variation of drag with axial distance for different values of cell velocity and slip parameter which is similar to the effect of permeability because as permeability increases slip parameter increases. It is clear from the graph that cell velocity increases drag decreases. This is obvious from the definition of drag that drag is inversely proportional to the velocity.

Figure 6 shows the effect of deformation parameter on drag. As deformation parameter increases effect of outside pressure is large. Therefore red cell deformation will be large so that red cell can squeeze out easily i.e. drag decreases.

Figure 7 depicts the variation of Pressure with axial distance for different values of cell velocity this show that as red blood cell velocity increases Pressure decreases. Which supports the result of Tandon & Bhardwaj [19].

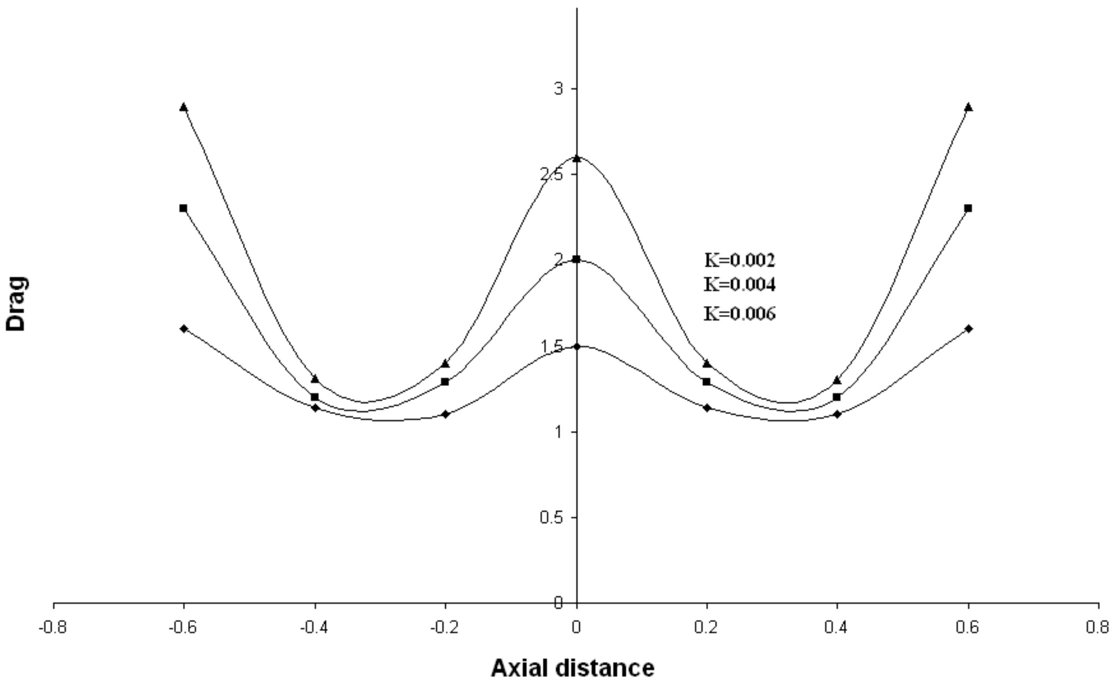

Figure 3 Variation of total drag with axial distance for different values of permeability for σ = 1, U=1.5

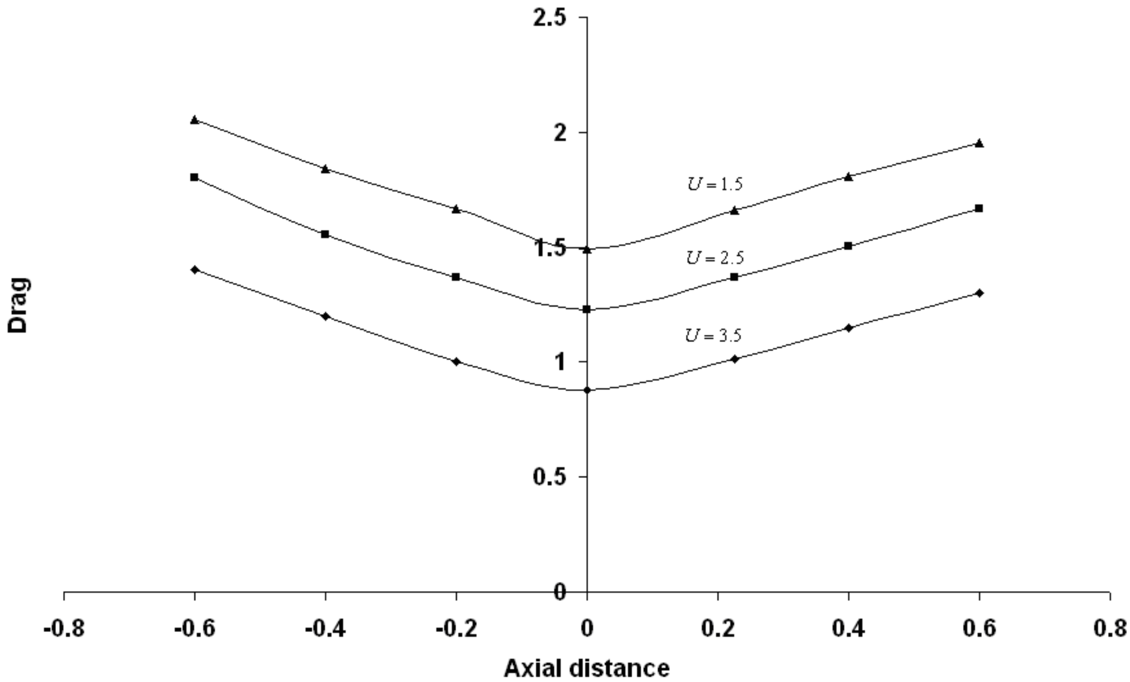

Figure 4 Variation of total drag with axial distance for different values of cell velocity for k = 0.002, σ = 1.5

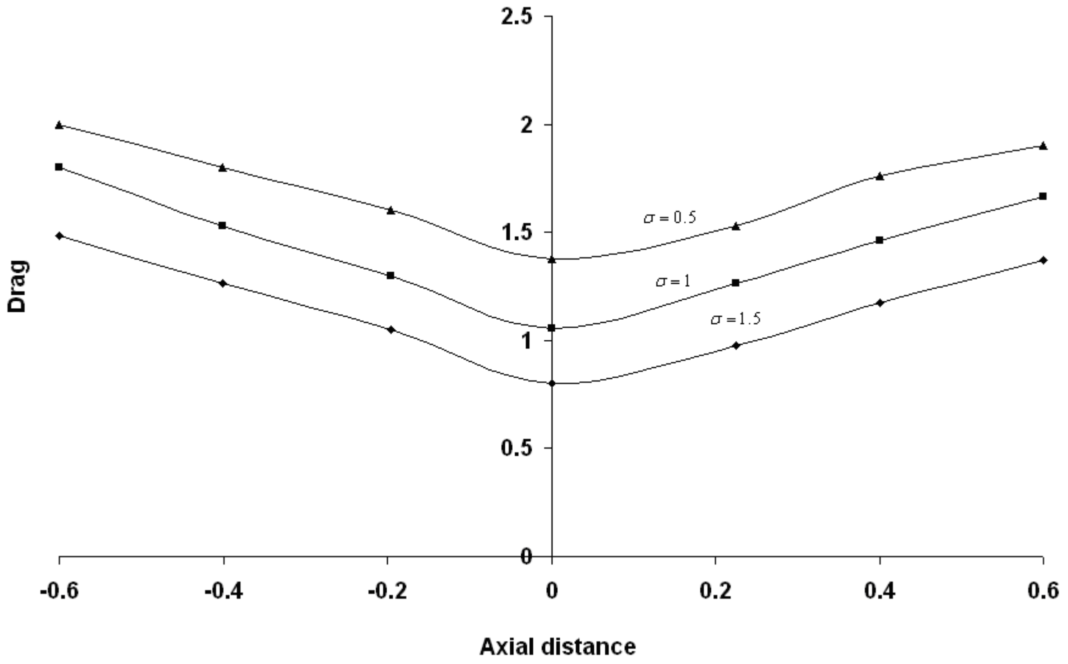

Figure.5 Variation of total drag with axial distance for different values of slip parameter for k=0.002, U = 1

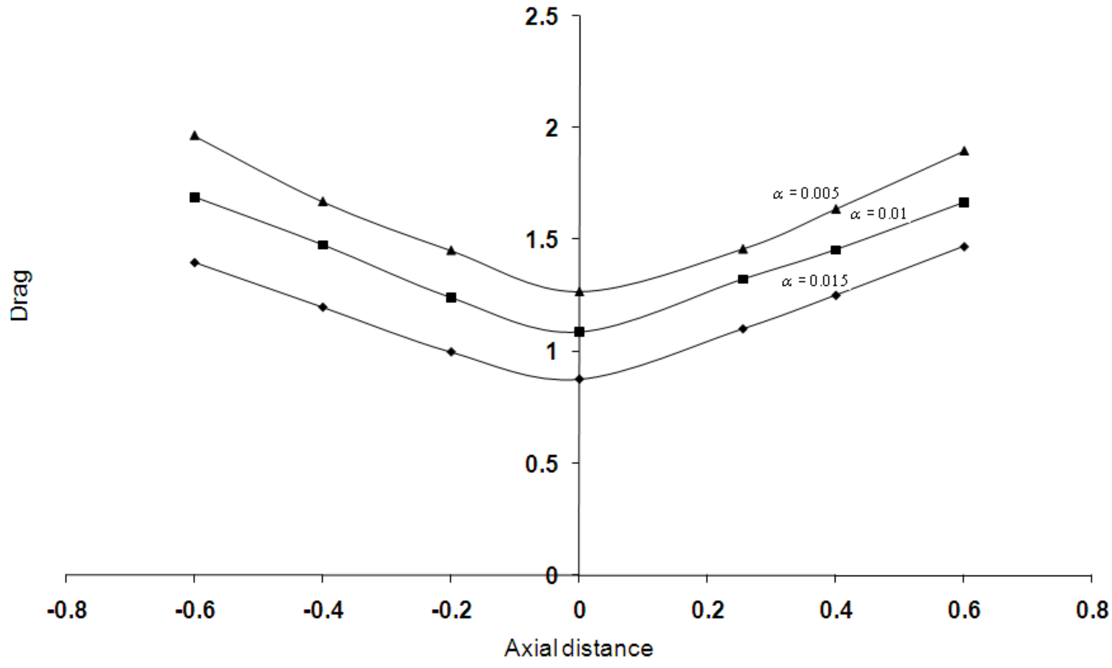

Figure 6 Variation of drag with axial distance for different values of cell shapes k=0.002, σ=0.5

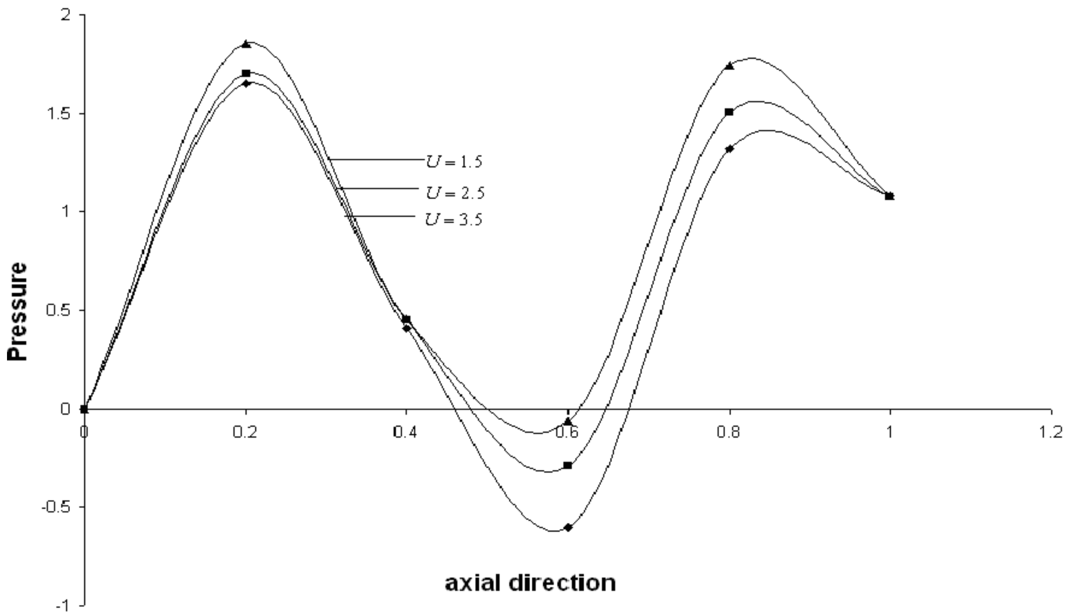

Figure 7 Variation of Pressure with axial distance for different values of cell velocity for U = 1.5, σ = 1.

CONCLUSION

Red blood cells squeeze through tiny capillaries to deliver oxygen and pickup carbon di oxide. To understand several types of disorders the study of deformation of red blood cell is very important. Mobility is the key factor in diseases such as malaria, genetic disorder sickle cell anemia and spherocytosis both render cells unable to flow through narrow capillaries and prevent them to flow which causes to red blood cells to squeeze. Therefore in this paper an attempt has been made to study the squeezing flow of red blood cell in narrow capillary which may be helpful in diagnostic / study various diseases like sickle cell anemia and spherocytosis.

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