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A semi-analytic technique to determine the propagation constant of periodically segmented Ti:LiNbO$_3$ waveguide

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ABSTRACT

A semi-analytical technique to determine the propagation constant of periodically segmented Ti:LiNbO$_3$ waveguide (PSW) is described. The 2-D refractive index profiles of the waveguide segments are computed from its fabrication parameters, such as, Ti-layer thickness and width, and the diffusion parameters. WKB method is used to transform 2-D refractive index profile to 1-D lateral effective index profile, which is then converted to equivalent refractive index profile of the PSW. Finally transfer matrix method is applied to compute the propagation constant of the waveguide.

Key words: Ti:LiNbO$_3$, Periodically segmented waveguide, Effective-index, Equivalent refractive index, Matrix method.

INTRODUCTION

There has been an increasing amount of interest in the application of periodically segmented waveguides (PSW's) in integrated optics. To date these devices have been reported in a number of different material systems including proton-exchanged LiNbO$_3$ [1], KTP [2], InP [3], annealed proton exchanged LiNbO$_3$ [4-6] and Ti:LiNbO$_3$ [7-9]. Application of PSWs ranges from nonlinear devices, which uses quasi phase – matched second harmonic generation [2], to linear devices like a 2-D mode taper [10], asymmetric directional coupler filter [11], and wavelength demultiplexer [9]. In a PSW the increase in refractive index (Δn) is modulated periodically during fabrication, as shown in Fig.1.

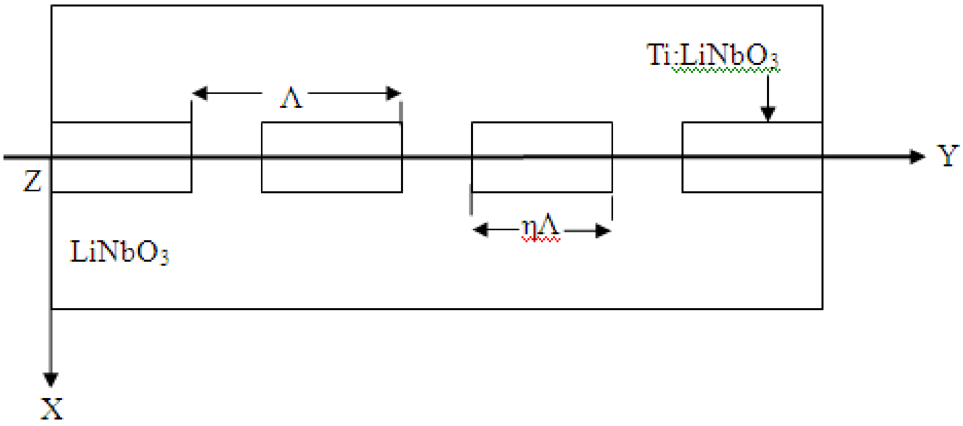

Figure 1 Periodically segmented waveguide (PSW).

As a consequence of the segmentation, the loss in the guide is increased and the effective refractive index is reduced when compared to a continuous waveguide. A PSW is characterized by its period, Λ, and duty-cycle, η (the ratio of the length of a segment and the period of the guide). It has been demonstrated that a PSW can be represented by an equivalent continuous waveguide with same depth and width, in which the average index difference (Δn_{eq}) is taken to be the weighted average of the index along the propagation direction. This is represented by equation (1), [12]

$$\Delta n_{eq} = \eta \Delta n \quad (1)$$

By choosing the duty-cycle, the average index can be spatially modified along the waveguide. The refractive index change determines the mode size, propagation constant and cut-off wavelength of the PSW.

Periodically segmented waveguides formed by Ti-indiffusion in a $LiNbO_3$ substrate will have a graded concentration variation along depth and lateral directions. In this work, the propagation constants of periodically segmented Ti:$LiNbO_3$ waveguides have been computed by applying the effective-index-based matrix method (EIMM) along with equivalent waveguide concept. In the first step, the depth and lateral refractive index profiles of each Ti:$LiNbO_3$ segment has been computed directly from its fabrication parameters, such as, Ti-layer thickness and width, and the diffusion parameters. Effective index method has been applied to transfer the 2-dimentsional index distribution to 1-dimensional lateral effective-index distribution. In the next step, the average refractive index has been computed to transfer the PSW into an equivalent continuous waveguide. Finally, the transfer matrix method is used to compute the propagation constant of the equivalent waveguide.

Theoretical Approach

Determination of effective refractive index of Ti:$LiNbO_3$ PSW

In the case of Ti:$LiNbO_3$ waveguides the Ti-concentration profile can be represented by [13]

$$C(x,z) = \frac{1}{4} C_o \left[erf\left(\frac{\tau + z}{d_z}\right) + erf\left(\frac{\tau - z}{d_z}\right) \right]$$

$$\times\left[erf\left(\frac{W+x}{d_x}\right)+erf\left(\frac{W-x}{d_x}\right)\right] \quad (2)$$

where C_o is the solid solubility of Ti into $LiNbO_3$, 2W and τ are the deposited Ti strip width and thickness, and d_x and d_z are the diffusion lengths along x and z axes of the crystal

The refractive indices of ordinary and extraordinary ray, n_o and n_e, of the congruently grown $LiNbO_3$ crystal, at room temperature (25°C), can be obtained using the modified Sellmeier equations [14]

$$n_o^2 = 4.9048 - \frac{0.11768}{(0.04750-\lambda^2)} - 0.027169\lambda^2 \quad (3)$$

$$n_e^2 = 4.5820 - \frac{0.099169}{(0.04432-\lambda^2)} - 0.021950\lambda^2 \quad (4)$$

where λ is the wavelength in μm. The refractive index change induced by Ti-indiffusion for both ordinary and extraordinary rays are related to the titanium concentration for $LiNbO_3$ as [13]

$$\Delta n_{o,e}(x,z,\lambda) = A_{o,e}(\lambda)[C(x,z)]^{\alpha_{o,e}} \quad (5)$$

where $A_{o,e}$ are dependent on λ, and $\alpha_{o,e}$ are 0.5 and 0.85 for ordinary and extraordinary rays. Details of the λ dependence of $A_{o,e}$ are given in ref.[13], which are valid within the wavelength range 0.6 ≤ λ (μm) ≤ 1.6. So by using equations (2-5) one can determine the 2-dimensional refractive index distribution of the segment of a Ti:$LiNbO_3$ PSW directly from its fabrication parameters.

Now a waveguide mode exists only if the total transverse phase shift (along Z) for one round-trip across the guide equals an integral multiple of 2π, i.e., [15]

$$\left(\frac{4\pi}{\lambda}\right)\int_0^{Z_b}\left[n^2(z,x)-n_{eff}^2(x)\right]^{1/2} dz + \phi_t + \phi_b = 2m\pi$$

$$m = 0,1,2,\ldots \quad (6)$$

This refers to the wave confinement over the YZ plane at a particular lateral position (x). The first term is the total phase change of the wave as it travels between the crystal surface z = 0 and the return point z = Z_b in the bulk and back. ϕ_t and ϕ_b are Goos-Hanchen phase shifts corresponding to total internal reflection at z = 0 and z = Z_b, respectively. For the $LiNbO_3$-air interface, ϕ_t is about -0.93π and -0.98π for TE and TM modes, respectively, and ϕ_b approaches (-π/2) for both polarizations [15]. Equation (6) is known as the WKB quantization condition. This leads to a set of discrete angles of propagation corresponding to different guided modes. So by solving equation (6) numerically for the fundamental mode (m = 0) one can determine the effective refractive index, n_{eff} (x), of the single-mode Ti:$LiNbO_3$ waveguide. For the computation of refractive index of equivalent waveguide for a PSD with duty-cycle η one has to use equation (1), i.e.,

$$n_{eq}(x) = \eta\, n_{eff}(x) \quad (7)$$

In the next step, this n_{eg} (x) is approximated by a staircase-type step-index profile and matrix method is applied to that layered structure.

Determination of propagation constant

The propagation constants of the Ti:$LiNbO_3$ PSW's are here determined by matrix method. If the medium is considered to be made of a number of layers (each of thickness d_i and refractive index n_i), then for an incident plane wave in the first layer, the electric field associated with each layer can be represented as [15]

$$E_i = u_i^+ E_i^+ \exp(j\Delta_i)\exp\left[j\{\omega t-(K_i\cos\theta_i)x-\beta y\}\right] + u_i^- E_i^- \exp(-j\Delta_i)\exp\left[j\{\omega t+(K_i\cos\theta_i)x-\beta y\}\right] \quad (8)$$

where

$$\Delta_1=\Delta_2=0;\quad \Delta_3=K_3 d_2\cos\theta_3;$$

$$\Delta_4=K_4(d_2+d_3)\cos\theta_4;$$

$$\Delta_i=K_i(d_2+d_3+\cdots+d_{i-1})\cos\theta_i;\qquad K_i=K_o n_i=\frac{2\pi}{\lambda}n_i;$$

$$\beta_i=K_i\sin\theta_i. \quad (9)$$

E_i^+ and E_i^- are the magnitude of electric field vectors associated with the downward –and upward-propagating waves, respectively, and u_i^+ and u_i^- are their corresponding unit vectors, and β_i is the propagation constant along the layers and is the same for all the layers, which is consistent with the law of refraction.

Applying the boundary conditions at the interfaces, one obtains

$$\begin{pmatrix}E_1^+\\E_1^-\end{pmatrix}=S_1\begin{pmatrix}E_2^+\\E_2^-\end{pmatrix}=\cdots=S_1S_2\cdots S_{N-1}\begin{pmatrix}E_N^+\\E_N^-\end{pmatrix} \quad (10)$$

where

$$S_{i-1}=\frac{1}{t_i}\begin{pmatrix}\exp(j\delta_i) & r_i\exp(j\delta_i)\\ r_i\exp(-j\delta_i) & \exp(-j\delta_i)\end{pmatrix} \quad (11)$$

$$\delta_i=\left(\frac{2\pi}{\lambda_o}\right)n_i d_i\cos\theta_i,\qquad i=2,3,4,\ldots\ldots,N.$$

S_{i-1} is the transmission matrix from the (i-1)-th and i-th layer; r_i and t_i represent the amplitude reflection and transmission coefficients, respectively, from the (i-1)-th layer to the i-th layer. Using equations (8-11) one can determine the field amplitude in any layer in terms of the incident field amplitude E_1^+.

The above method when applied to equivalent refractive index profile of equation (7) can be used to determine the propagation constant of Ti:$LiNbO_3$ periodically segmented waveguides. Prism coupling approach has been applied for the purpose. Initially a higher refractive index layer is considered as layer 1 and excitation efficiency of the guiding layer is computed by matrix method for different incident angles. Out of the different incident angles only a single will excite the guided mode in the single-mode waveguide refractive index structure. Excitation efficiency versus incident angle or propagation constant characteristics will show only one peak for the guided mode, from which propagation constant of the PSW can be determined. The details of the matrix method are given in ref. [16].

Computational Procedure and Results

Computational Procedure

The computer program written for calculating propagation constant of Ti:$LiNbO_3$ PSW requires the following parameters as inputs:

1. Ti film parameters: thickness, strip width, gap between the segments, and length of segments,
2. Ti diffusion parameters: diffusion temperature, time, and ambient,
3. Beam parameters: wavelength and polarization (TE / TM),
4. Computational parameters: sampling area (x range, z range) and the layer thicknesses of the discretised $n_{eq}(x)$ profile.

The first three sets of parameters are taken as the fabrication parameters of Ti:$LiNbO_3$ PSW, while the x range and z range are taken as -15 to 15 μm and 0 to 15 μm, respectively, for most computations. A uniform layer thickness of 0.01 μm has been used. However, in order to reduce the computation time without loss of accuracy, nonuniform partitioning of $n_{eq}(x)$ involving a fewer number of layers may also be considered.

The computational steps to determine the propagation constants of Ti:$LiNbO_3$ PSWs are as follows:

1. Computation of two-dimensional refractive index profiles, n(x, z), of Ti:$LiNbO_3$ waveguide from its fabrication parameters.
2. Computation of one-dimensional lateral effective index profile, $n_{eff}(x)$, from the two-dimensional refractive index distribution by WKB method.
3. Determination of equivalent refractive index profile, $n_{eq}(x)$, for the PSW from $n_{eff}(x)$.
4. Application of matrix method onto $n_{eq}(x)$ to determine the propagation constant of Ti:$LiNbO_3$ PSW.

Following the matrix method outlined in previous section, the excitation efficiency is computed for various values of incidence angle θ_1 from first high refractive index medium (added) for $\theta_c \leq \theta_1 \leq 90^o$ to find the values of θ_1 and hence β [using equation (9)] for which excitation efficiency shows sharp resonance peaks. The gap between the added medium and the boundary of the waveguide is increased until the limiting values of β are obtained. The flow chart of the entire computational procedure is given in Fig.2.

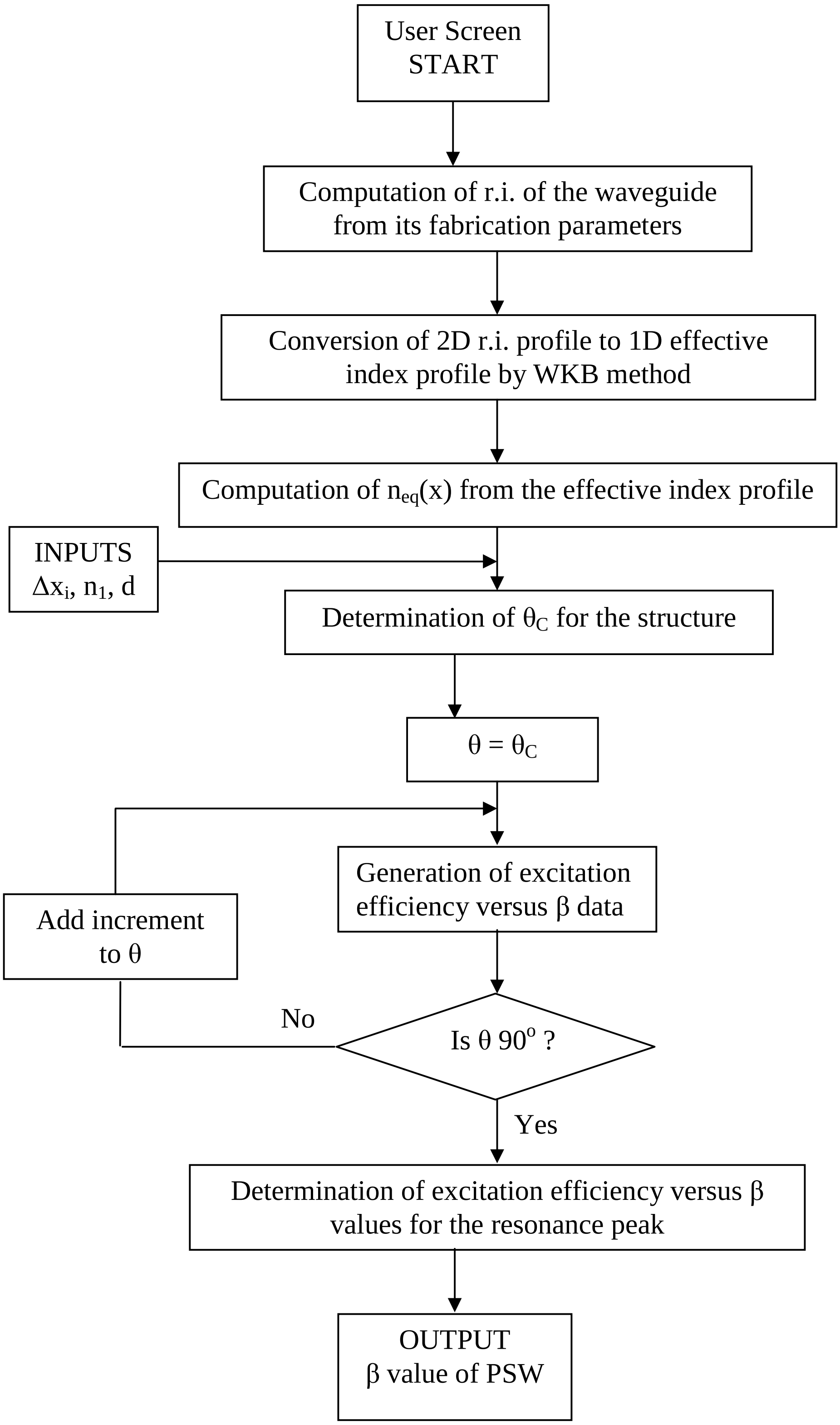

Figure 2 Flow-chart for computation of the propagation constant of Ti:$LiNbO_3$ PSW.

RESULTS AND DISCUSSION

The computed propagation constant versus gap between the segments of Ti:$LiNbO_3$ PSW is shown in Fig.3 for TM mode at 1.31 μm transmitting wavelength. The

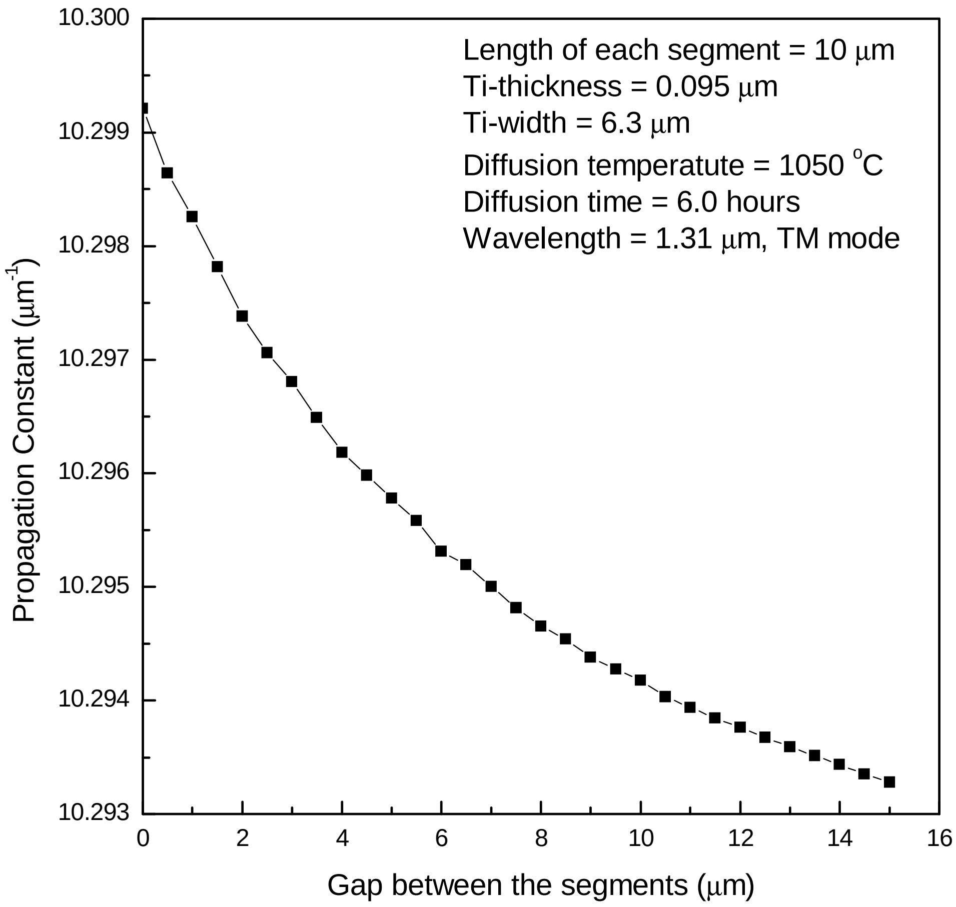

Figure 3 Computed propagation constant versus gap between the segments of a Ti:$LiNbO_3$ PSW.

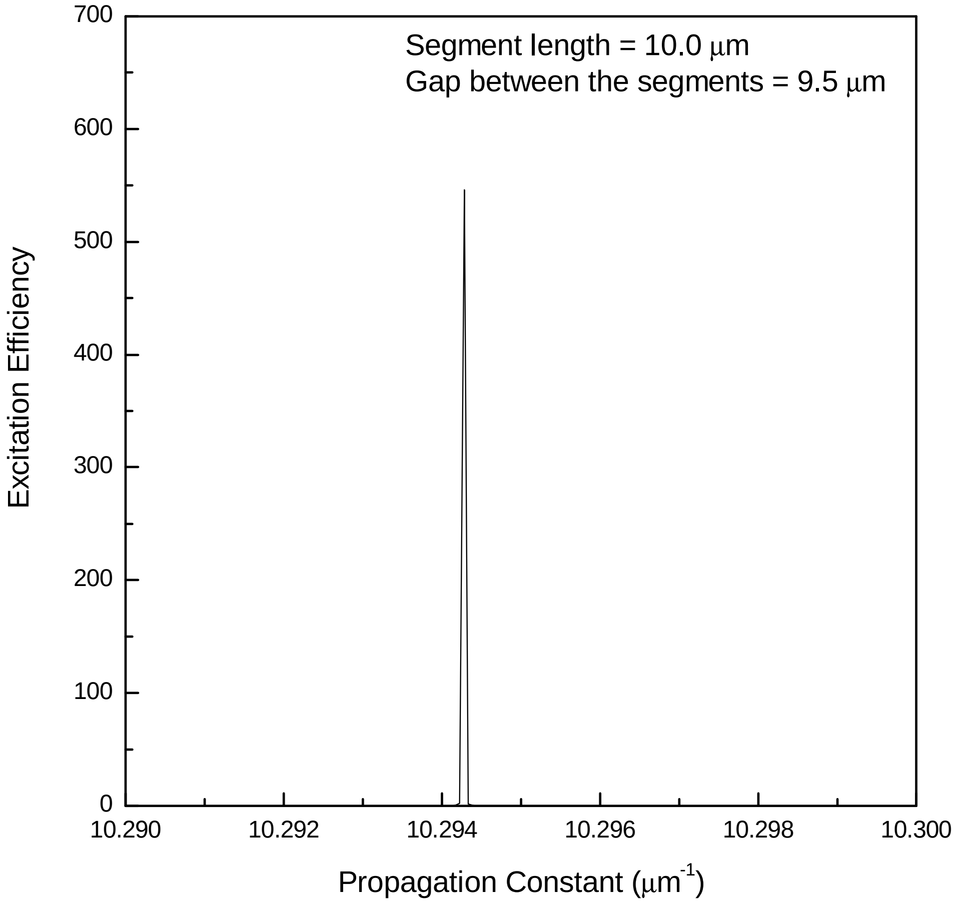

Figure 4 Typical resonance peak of excitation efficiency versus propagation constant plot of Ti:LiNbO$_3$ PSW.

length of the segments is taken as 10 μm. It may be noted from the figure that as the gap between the segments increases the propagation constant decreases. This nature of the characteristic is obvious from the fact that as gap between the segments increases, the n_{eq} of PSW decreases and hence propagation constant of the waveguide decreases. Typical excitation efficiency versus propagation constant plot for gap 9.5 μm is shown in Fig.4, which shows a sharp peak indicating the guided mode propagation constant of the PSW.

It may be noted that the present analytical model is usable for both TE and TM polarized lights for different sets of PSW fabrication parameters within a wavelength range from 0.6 to 1.6 μm. The method involves only straightforward multiplication of 2x2 transfer matrices of the layers. It does not require the solution of any transcendental or differential equation. Iterations are used only once to evaluate the effective refractive index of the waveguide. This makes the present model very fast and simple to use with a PC. The computed propagation constant will be an essential input for design consideration for several PSW components of optical integrated circuits, such as asymmetric directional couplers and Y-junctions.

CONCLUSION

Propagation constants of Ti:$LiNbO_3$ periodically segmented waveguides (PSW's) are determined from its fabrication parameters by effective index based matrix method (EIMM) along with equivalent continuous waveguide model. The 2-D refractive index profiles of the waveguide segments are computed from its fabrication parameters. WKB method is used to transform 2-D refractive index profile to 1-D lateral effective index profile, which is then converted to equivalent refractive index profile of the PSW. Finally transfer matrix method is applied to compute the propagation constant of the waveguide. This semi-analytical technique is computationally fast and simple to use with a PC.

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