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Advances in Applied Science Research, 2012, 3 (6):3491-3501

ISSN: 0976-8610
CODEN (USA): AASRFC

A numerical approach of a typical three species syn-eco-system

[*]B. Hari Prasad[1] and N.Ch. Pattabhi Ramacharyulu[2]

[1]Department of Mathematics, Chaitanya Degree, Hanamkonda A.P. India
[2]Department of Mathematics, NIT, Warangal India.

ABSTRACT

In this paper,we are studing the numerical approach of a typical three species syn eco-system. The system comprises of a commensal (S_1), two hosts S_2 and S_3 ie., S_2 and S_3 both benefit S_1, without getting themselves effected either positively or adversely. Further S_2 is a commensal of S_3 and S_3 is a host of both S_1, S_2.. Limited resources are considered for all three species in this case. The model equations of the system constitute a set of three first order non-linear ordinary differential coupled equations. In all, eight equilibrium points of the model are identified. In this paper the numerical solutions for the growth rate equations are computed using Runga-Kutta fourth order method.

AMS Classification: 92D25, 92D40
Keywords: Commensal, Equilibrium Point, Host, Runge-Kutta method.

INTRODUCTION

Ecology is the study of the inter-relationships between organisms and environment. It is natural that two or more species living in a common habitat interact in different ways. Mathematical modeling has been playing an important role for the last half a century in explaining several phenomena concerned with individuals and groups of populations in nature. Lotka[14] and Volterra [22] pioneered theoretical ecology significantly and opened new eras in the filed of life and biological sciences. The Ecological interactions can be broadly classified as Ammensalism, Competition, Commensalism, Neutralism, Mutualism, Predation and Parasitism. The general concept of modeling has been presented in the treatises of Meyer[15], Kushing[12], Kapur[10,11]. Srinivas[21] studied competitive ecosystem of two species and three species with limited and unlimited resources. Laxminarayan and Pattabhi Ramacharyulu [13] studied prey-predator ecological models with partial cover for the prey and alternate food for the predator. Archana Reddy [1] and Bhaskara Rama Sharma [2] investigated diverse problems related to two species competitive systems with time delay, employing analytical and numerical techniques. Phani Kumar [16] studied some mathematical models of ecological commensalism. Ravindra Reddy [17] discussed on the stability of two mutually interacting species with mortality rate for the second species. Further, Shiva Reddy et al [20] and Srilatha et al [18, 19] studied stability analysis of three and four species. The present authors Hari Prasad and Pattabhi Ramacharyulu [3 to 9] discussed on the stability of a three and four species syn-ecosystems.

The present investigation is on numerical approach of a typical three species (S_1, S_2, S_3) syn-eco system. The system comprises of a commensal (S_1), two hosts S_2 and S_3 ie, S_2 and S_3 both benefit S_1, without getting themselves effected either positively or adversely. Further S_2 is a commensal of S_3 and S_3 is a host of both S_1, S_2 where all the three species with limited resources. Figure 1 shows a schematic diagram of the interaction under study.

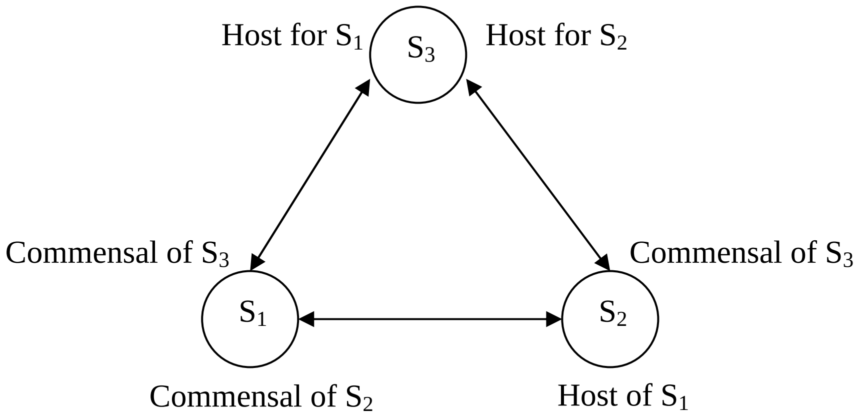

Figure 1. Schematic Sketch of the Syn Eco-System.

2. Basic Equations of the Model:

The model equations for a typical three species ecosystem is given by the following system of first order non-linear ordinary differential equations employing the following notation.

Notation Adopted

S_1	:	Commensal of S_2 and S_3
S_2	:	Host of S_1 and commensal of S_3
S_3	:	Host of S_1 and S_2
$N_i(t)$	:	The population strength of S_i at time t, i = 1,2,3.
t	:	Time instant.
a_i	:	Natural growth rate of S_i , i = 1,2, 3.
a_{ii}	:	Self inhibition coefficients of S_i, i=1, 2, 3.
a_{12}, a_{13}	:	Interaction coefficients of S_1 due to S_2 and S_1 due to S_3.
a_{23}	:	Interaction coefficient of S_2 due to S_3
$k_i = \frac{a_i}{a_{ii}}$	:	Carrying capacities of S_i, i = 1, 2, 3.
t^*	:	The dominance reversal time.

Further the variables N_1, N_2, N_3 are non-negative and the model parameters a_1, a_2, a_3, a_{11}, a_{22}, a_{33}, a_{13}, a_{23} are assumed to be non-negative constants.

The model equations for the growth rates of S_1, S_2, S_3 are

$$\frac{dN_1}{dt} = a_1 N_1 - a_{11} N_1^2 + a_{12} N_1 N_2 + a_{13}\ N_1 N_3 \tag{2.1}$$

$$\frac{dN_2}{dt} = a_2 N_2 - a_{22} N_2^2 + a_{23} N_2 N_3 \tag{2.2}$$

$$\frac{dN_3}{dt} = a_3 N_3 - a_{33} N_3^2 \tag{2.3}$$

3. Equilibrium States:

The system under investigation has 8 equilibrium states given by

$$\frac{dN_i}{dt} = 0,\ i = 1,\ 2,\ 3 \tag{3.1}$$

(i) Fully washed out state

$$E_1 : \overline{N}_1 = 0,\ \overline{N}_2 = 0,\ \overline{N}_3 = 0$$

(ii) States in which two of the tree species are washed out and third is not.

$$E_2 : \overline{N}_1 = 0, \overline{N}_2 = 0, \overline{N}_3 = k_3$$
$$E_3 : \overline{N}_1 = 0, \overline{N}_2 = k_3, \overline{N}_3 = 0$$
$$E_4 : \overline{N}_1 = k_1, \overline{N}_2 = 0, \overline{N}_3 = 0$$

(iii) Only one of the three species is washed out while the other two are not.

$$E_5 : \overline{N}_1 = 0, \overline{N}_2 = k_2 + \frac{a_{23}k_3}{a_{22}}, \overline{N}_3 = k_3$$

$$E_6 : \overline{N}_1 = k_1 + \frac{a_{13}k_3}{a_{11}}, \overline{N}_2 = 0, \overline{N}_3 = k_3$$

$$E_7 : \overline{N}_1 = k_1 + \frac{a_{12}k_2}{a_{11}}, \overline{N}_2 = k_2, \overline{N}_3 = 0$$

(iv) The co-existent state or normal steady state.

$$E_8 : \overline{N}_1 = k_1 + \frac{a_{12}}{a_{11}}\left(k_2 + \frac{a_{23}k_3}{a_{22}} \right) + \frac{a_{13}k_3}{a_{11}}, \overline{N}_2 = k_2 + \frac{a_{23}k_3}{a_{11}}, \overline{N}_3 = k_3$$

4. A Numerical solution of the Growth Rate Equations:

The numerical solutions of the growth rate basic equations (2.1), (2.2) and (2.3) have been computed employing the fourth order Runge-Kutta method. Some specific typically chosen values of system parameters characterizing in ecological model under investigation and properly chosen initial conditions. Making use of Mat Lab facility. What follows are the results of numerical computation and these are illustrated and some observations made here under.

Case (i): If $N_{i0} < \frac{K_i}{2}$, i = 1,2,3.

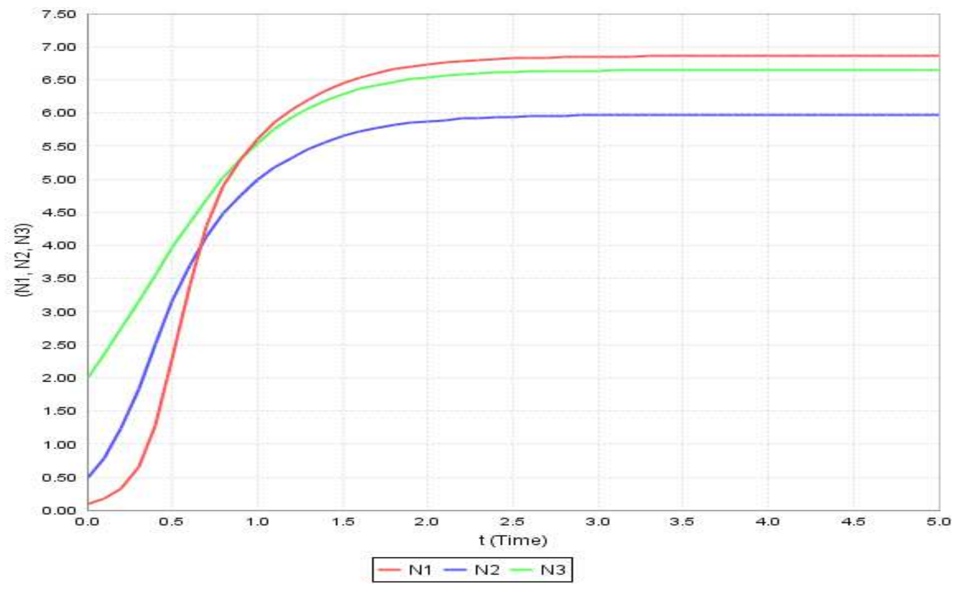

Figure 2. Variation of N_1, N_2 and N_3 against time (t) for a_1=1, a_2=2.54, a_3=2.46, K_1=0.38, K_2=1.13, K_3 =6.65, a_{12}=0.46, a_{13}=2.15, a_{23}=1.63, N_{10}=0.1, N_{20}=0.5, N_{30}=2.

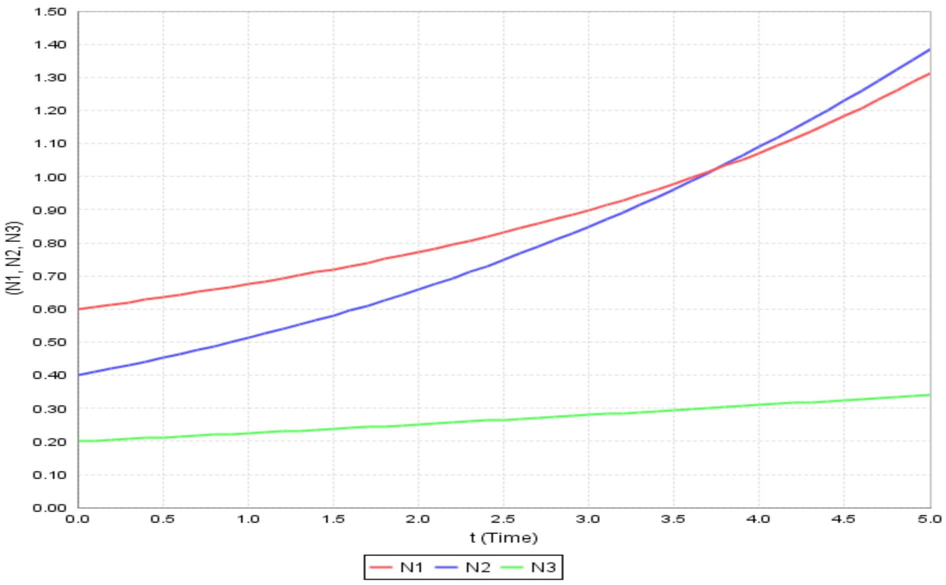

Figure 3. Variation of N_1, N_2 and N_3 against time (t) for a_1=0.07,a_2=0.13,a_3=0.16,K_1=1.4, K_2=1, K_3=0.8, a_{12}=0.13, a_{13}=0.1, a_{23}=0.83, N_{10}=0.6, N_{20}=0.4, N_{30}=0.2.

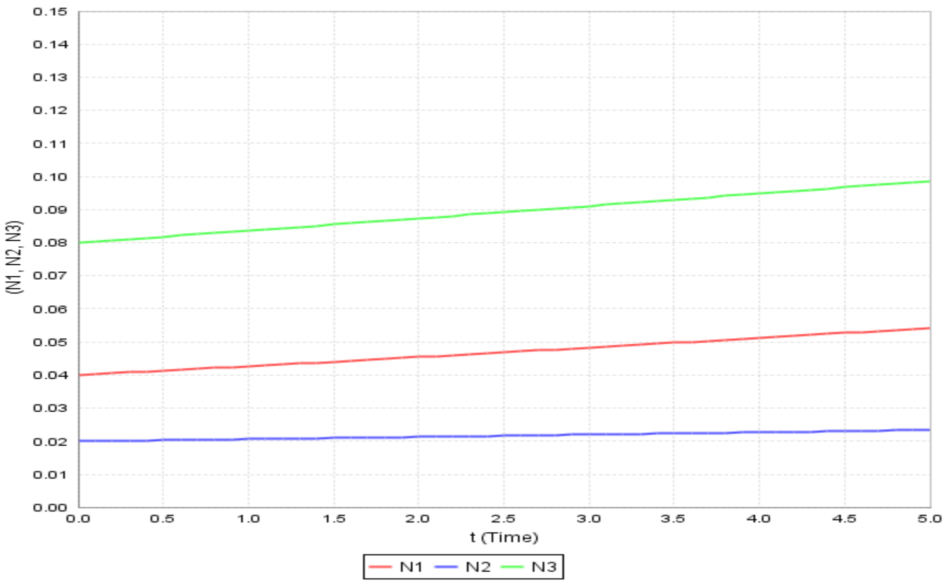

Figure 4. Variation of N_1, N_2 and N_3 against time (t) for a_1=0.088,a_2=0.042,a_3=0.074,K_1=0.119, K_2=0.056, K_3=0.206, a_{12}=0.148, a_{13}=0.049, a_{23}=0.063, N_{10}=0.04, N_{20}=0.02, N_{30}=0.08.

Case (ii): If $\frac{K_i}{2} < N_{i0} < K_i$ **, i = 1,2,3.**

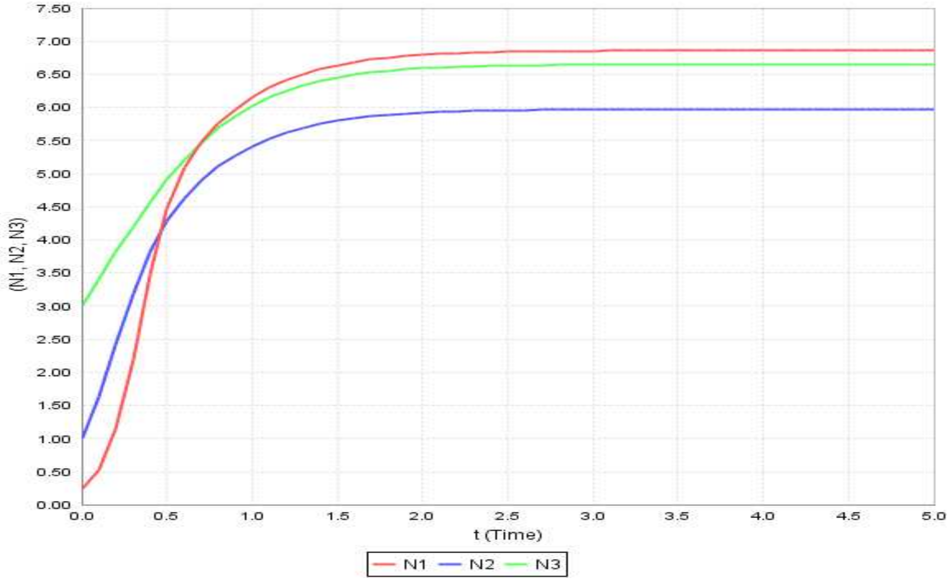

Figure 5. Variation of N_1, N_2 and N_3 against time (t) for a_1=1,a_2=2.54,a_3=2.46,K_1=0.38, K_2=1.13, K_3=6.65, a_{12}=0.46, a_{13}=2.15, a_{23}=1.63, N_{10}=0.25, N_{20}=1, N_{30}=3.

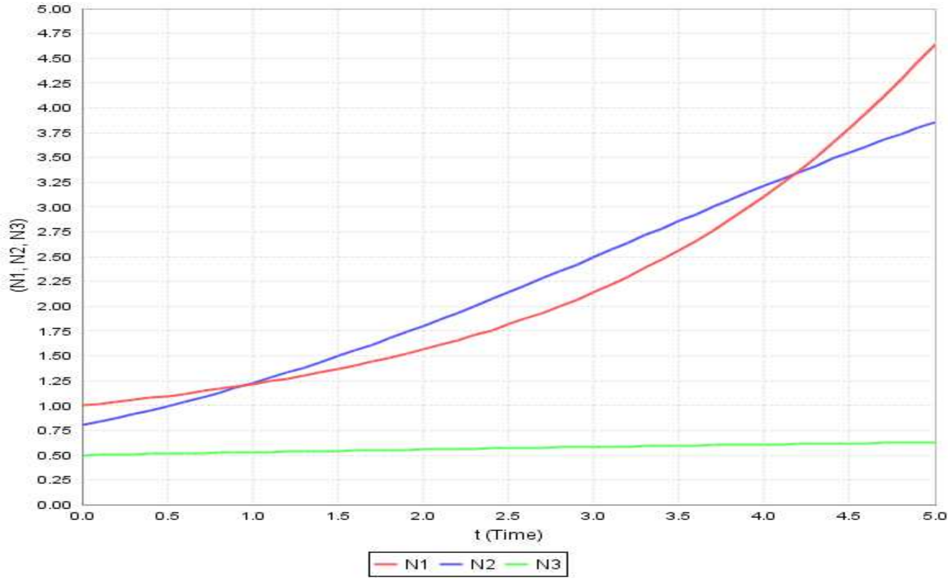

Figure 6. Variation of N_1, N_2 and N_3 against time (t) for a_1=0.07,a_2=0.13,a_3=0.16,K_1=1.4, K_2=1, K_3=0.8, a_{12}=0.13, a_{13}=0.1, a_{23}=0.83, N_{10}=1, N_{20}=0.8, N_{30}=0.5.

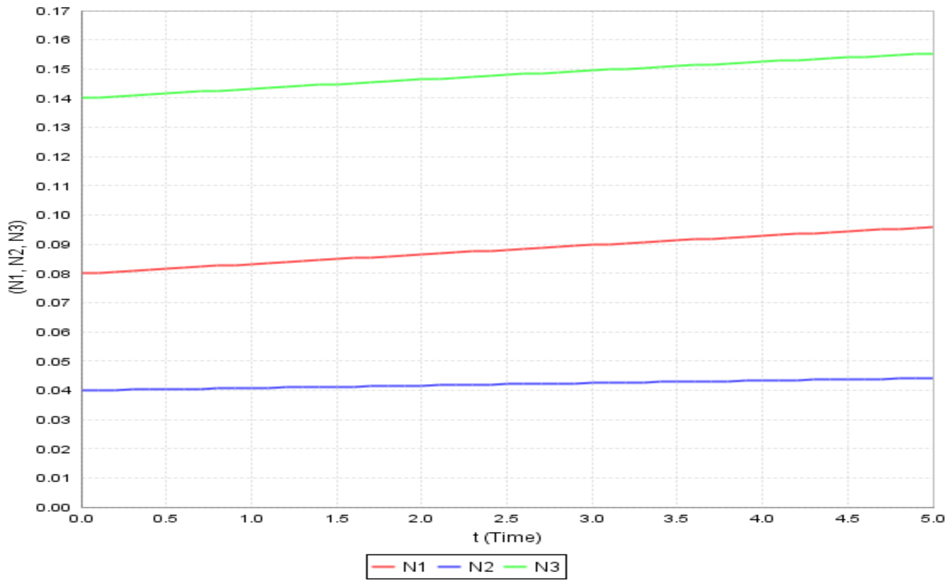

Figure 7. Variation of N_1, N_2 and N_3 against time (t) for a_1=0.088,a_2=0.042,a_3=0.074,K_1=0.119, K_2=0.056, K_3=0.206, a_{12}=0.148, a_{13}=0.049, a_{23}=0.063, N_{10}=0.08, N_{20}=0.04, N_{30}=0.14.

Case (iii): If $N_{i0} > K_i$ **, i = 1,2,3.**

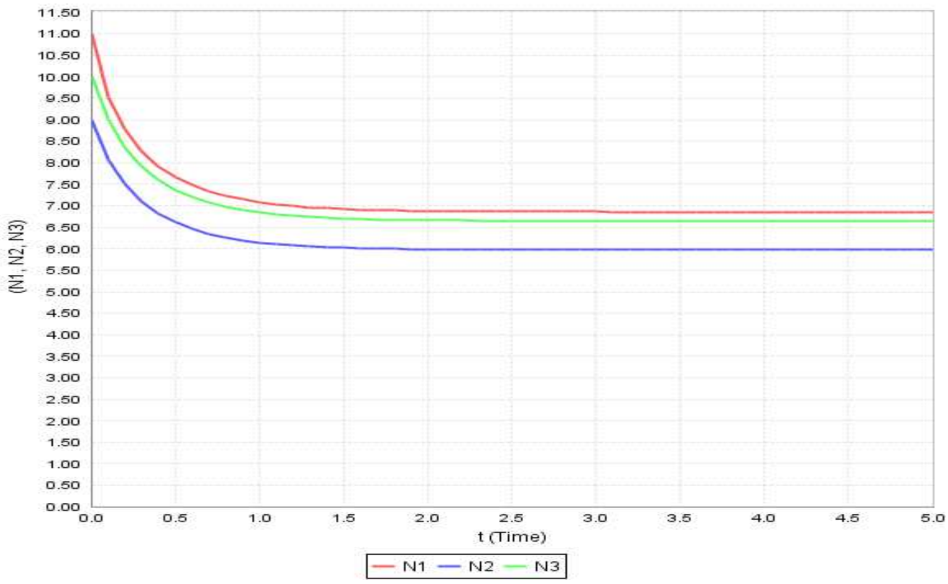

Figure 8. Variation of N_1, N_2 and N_3 against time (t) for a_1=1,a_2=2.54,a_3=2.46,K_1=0.38, K_2=1.13, K_3=6.65, a_{12}=0.46, a_{13}=2.15, a_{23}=1.63, N_{10}=11, N_{20}=9, N_{30}=10.

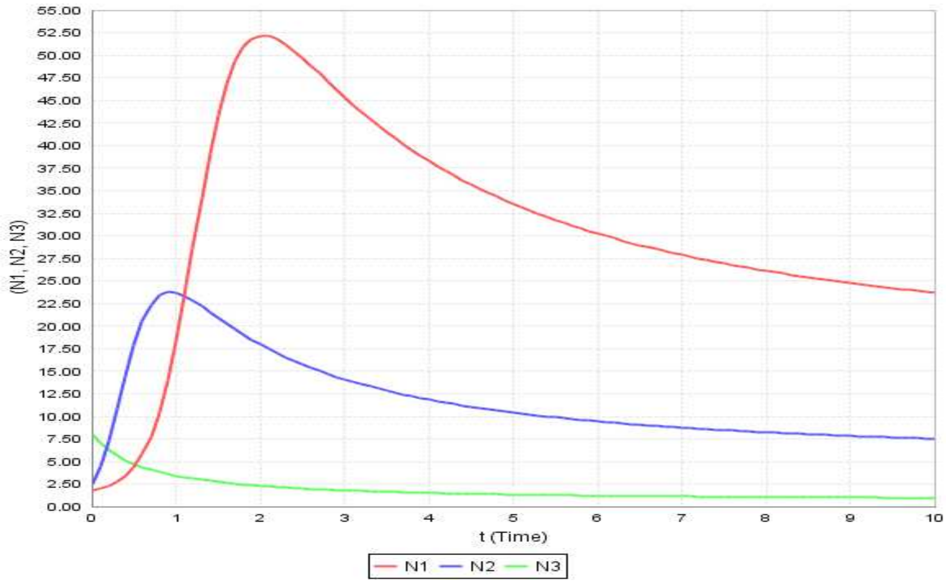

Figure 9. Variation of N_1, N_2 and N_3 against time (t) for a_1=0.07,a_2=0.13,a_3=0.16,K_1=1.4, K_2=1, K_3=0.8, a_{12}=0.13, a_{13}=0.1, a_{23}=0.83, N_{10}=1.8, N_{20}=2.5, N_{30}=8.

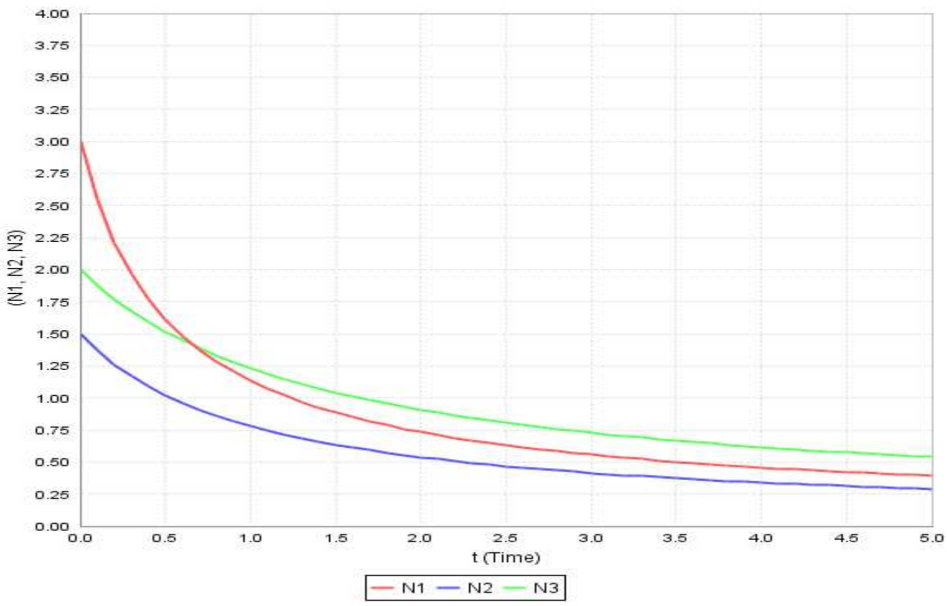

Figure 10. Variation of N_1, N_2 and N_3 against time (t) for a_1=0.088,a_2=0.042,a_3=0.074,K_1=0.119, K_2=0.056, K_3=0.206, a_{12}=0.148, a_{13}=0.049, a_{23}=0.063, N_{10}=3, N_{20}=1.5, N_{30}=2.

Case (iv): If $N_{i0} = K_i$ **, i = 1,2,3.**

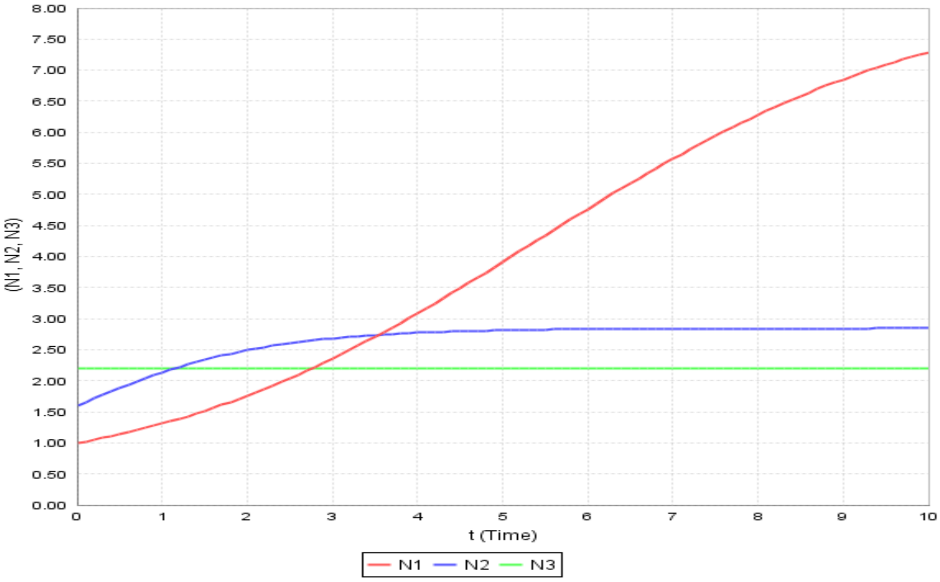

Figure 11. Variation of N_1, N_2 and N_3 against time (t) for a_1=0.05,a_2=0.48,a_3=0.44,K_1=1, K_2=1.6, K_3=2.2, a_{12}=0.09, a_{13}=0.05, a_{23}=0.17, N_{10}=1, N_{20}=1.6, N_{30}=2.2.

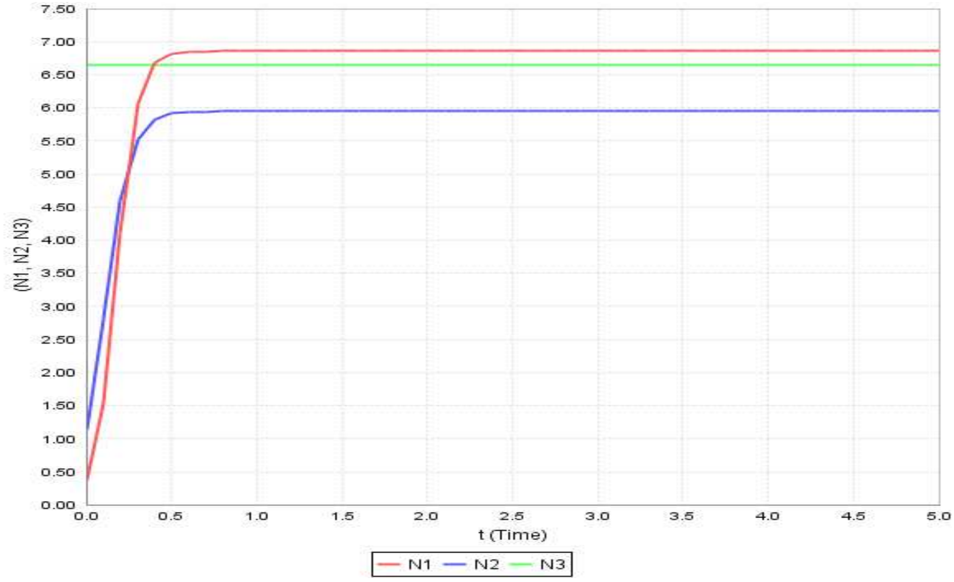

Figure 12. Variation of N_1, N_2 and N_3 against time (t) for a_1=1, a_2=2.54, a_3=2.46, K_1=0.38, K_2=1.13, K_3 =6.65, a_{12}=0.46, a_{13}=2.15, a_{23}=1.63, N_{10}=0.38, N_{20}=1.13, N_{30}=6.65.

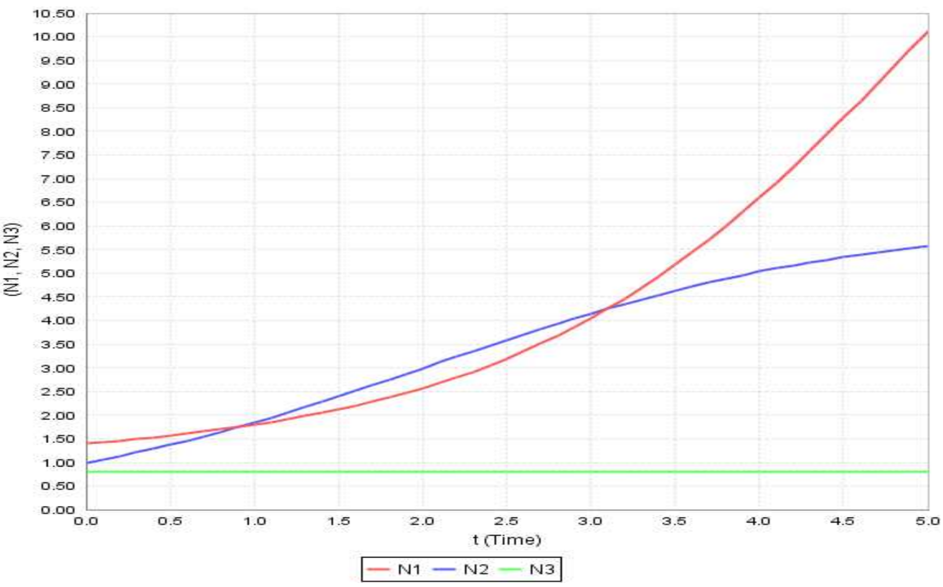

Figure 13. Variation of N_1, N_2 and N_3 against time (t) for a_1=0.07, a_2=0.13, a_3=0.16, K_1=1.4, K_2=1, K_3 =0.8, a_{12}=0.13, a_{13}=0.1, a_{23}=0.83, N_{10}=1.4, N_{20}=1, N_{30}=0.8.

4.1 Observations of the above graphs:

Situation 1: In this situation the natural birth rates of S_1, S_2, S_3 are increasing order. It is noticed that initially the S_2 is dominated S_1 up to time instant $t^* = 0.7$ and the S_3 up to $t^* = 0.9$ after these dominate times we find reversal of the dominance as shown in Figure 2.

Situation 2: In this situation the first species dominates over the second species up to the time instant $t^* = 3.75$ after which the dominance is reversed. Further the third species with low natural birth rate. (Figure 3).

Situation 3: In this situation all the three species have almost equal birth rates and the second species is dominated by the first which itself dominated by the third. Further we notice that all the three species have low growth rates. (Figure 4).

Situation 4: In this situation initially the S_1 dominates by the S_2 up to the time $t^* = 0.48$ and the S_3 up to the time $t^* = 0.7$ and the dominances are reversed. Further the host(S_3) of both S_1 and S_2 is dominated by the commensal(S_1) after the time $t^* = 0.7$ higher value of the commensal coefficient a_{13}=2.15. (Figure 5).

Situation 5: In this situation the host (S_3) is always dominated by the commensals S_1 and S_2. In spite of higher natural growth rate, this may be attributed to lowest carrying capacity of S_3 compare with S_1. Since the carrying capacity of S_1 is greater than that of S_2. The S_1 dominates over the S_2 initially up to the time $t^* = 1$ and also often the time $t^* = 4.2$ in between these two species S_1 is dominated S_2. This is a case of a weak host the strong commensal. (Figure 6).

Situation 6: In this situation the host (S_3) dominates over the S_1 and S_2, the host (S_2) dominated by the commensal(S_1). Further we notice that all the three species have low growth rates. (Figure 7)

Situation 7: In this situation the host (S_3) is dominate over the commensal (S_2) and dominated by the common commensal(S_1). This is a case in which commensal coefficient a_{13} is highest. That is the S_1 exact higher advantages from the host (S_3) resulting the dominance of S_1 over S_2. (Figure 8).

Situation 8: This is a situation at the carrying capacity of the host (S_3) is lowest. In spite of highest initial value, the common host (S_3) monotonically decreases as other two S_1 and S_2 are benefited by host commensal coefficients a_{13} and $_{23}$. We notes this steep rise of S_1 and steeper rise greater fall common S_1 and S_2. This may be attributed to higher carrying capacity of S_1 and higher host commensal coefficient a_{12} between of S_1 and S_2, compare to a_{13}. (Figure 9).

Situation 9: In this situation the initial value of the common commensal (S_1) is highest. Further it is evident that all the three species asymptotically converge to the equilibrium point. (Figure 10).

Situation 10: This is a situation at the carrying capacity of the host (S_3) is highest. Initially the second species dominates over the first species up to the time instant $t^* = 3.5$ after which the dominance is reversed. Also the third species dominates over the second and first till the time instant $t^* = 1.2$ and $t^* = 2.7$ respectively and thereafter the dominance is reversed. Further we notice that the first species has the least natural birth rate. (Figure 11).

Situation 11: In this situation the natural birth rates of S_2 and S_3 are almost equal. It is noticed that initially the S_2 is dominated S_1 up to time instant $t^* = 0.25$ and the S_3 up to $t^* = 0.38$ after these dominate times we find reversal of the dominance. (Fig 12).

Situation 12: In this situation the first species has the least natural birth rate. Initially the first species is dominant over the second species for a short span and from the instant $t^* = 0.9$ to $t^* = 3.1$ the first species is dominant. Further the third species is a weak competitor with no appreciable growth even from the start. (Figure 13).

CONCLUSION

Investigate some relation-chains between the species such as Prey-Predation, Neutralism, Commensalism, Mutualism, Competition and Ammensalism between three species (S_1, S_2, S_3) with the population relations.

The present paper deals with an investigation on numerical approach of a typical three species syn eco-system. The system comprises of a commensal (S_1), two hosts S_2 and S_3 ie., S_2 and S_3 both benefit S_1, without getting themselves effected either positively or adversely. Further S_2 is a commensal of S_3 and S_3 is a host of both S_1, S_2. It is observed that, the numarical solutions for the growth rate equations are computed using Runge-Kutta foutrh order method in four cases.

(i): The initial values of the three species are less than half the respective their carrying capacities.
(ii): The initial values of the three species are lie between half their respective carrying capacities and its carrying capacities.
(iii): The initial values of the three species are greater than their respective carrying capacities.
(iv): The initial values of the three species are equal their respective carrying capacities.

Acknowledgment
We thank to Prof..M.A.Singara Chary,Head, Dept.of Microbiology, Kakatiya University, Warangal, (A.P), India and Prof. C. Janaiah Dept. of Zoology, Kakatiya University, Warangal (A.P), India for their valuable suggestions and encouragement. And also we acknowledge to Mr.K.Ravindranath Gupta for neat typing of this research paper.

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