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A nonlinear variation approach to the study of compressible fluid flow

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ABSTRACT

In this paper, an attempt has been made to provide an approximation method for the analysis of fluid flow. The steady flow of a noniviscous compressible fluids were examined and it is discovered that uniqueness of solution requires that the – ρ (pressure) be convex for subsonic flow. For there to be flow, mass and momentum must be conserved.

Key words: fluid flow, compressible fluids, subsonic flow, streamlines flow, mass and momentum.

INTRODUCTION

The equations of hydrodynamics are highly nonlinear and difficult to solve. It is therefore necessary to study an associated extremum principles which, if they exist, can by used to provide an approximation method for the analysis of fluid flows.

The unsteady pseudoplastic flow near a moving wall was investigated by Bird 1959, and this has resulted in much interest on non-Newtonian flows. Hassanien et al, 1998 studied fluid flow and heat transfer over a non-isothermal stretching sheet. They showed that friction and heat transfer rate exhibit strong dependence on the fluid parameters.

In this paper, the steady flow of a noniviscous compressible fluid was examined. The problem of identification of the basic quantity $W(U, \varphi)$ related to the generalized Hamiltonian, $H(x, u\ \varphi,)$ is a little less obvious that in some of the eelier application, and so we shall develop the canonical equations from the familiar equations of hydrodynamics. We considered the extreme property of $T(\varphi)$, the trial function and examine the variations of velocity hence we see that uniqueness of solution requires that - ρ be convex for subsonic flow equation. Hence, for there to be flud flow, mass and momentum must be conserved.

Three-D laminar flow problems using a control volume approach was solved by Sharma and Patankar 2005. The key feature of this approach is that the computational overhead to solve for the motion of rigid particle is very small. The formulation is convenient for handing irregular geometries, Glowinski et al, 2001 discussed a methodology that allows the direct numerical simulation of incompressible viscous fluid flow past moving rigid bodies. The method is particularly well suited to the direct numerical simulation of particulate flow, such as the flow of mixtures of rigid solid particulates and incompressible viscous fluids, possibly non – Newtonian. They presented the results of various numerical experiments, including the simulation of store separation for rigid airfoils and of sedimentation and fluidization phenomena in two and three dimensions.

Decheng Wan and Stefan Turek,2007 investigated Numerical Simulation of particulate flows using a new moving mesh method combined with the multigrid fictitious boundary method. The fictitious boundary method for the implicit treatment of Dirichlet boundary conditions with applications to incompressible fluid was highly treated.

Blasco Jordi et al,2009 developed a fictitious Domain, parallel numerical method for direct numerical simulation of the flow of rigid particles in an incompressible viscous Newtonian fluid. A simultaneous direction implicit algorithm is employed which gives the model a high level of parallelization. The projection of fluid velocity onto rigid motion on the particles is based on a fast computational of linear and angular momenta,

Feng Zhi and Efstathios ,2009 developed a method for solving the heat transfer equations for the computation of thermal convection in particulate flows. The numerical method makes use of a finite difference method in combination with the Immersed Boundary (IB) method for treating the particulate phase.

Mathematical Formulation

In the study of flow of a noniviscous compressible fluid in the absence of external forces, the basic equations of conservation of mass and momentum are

$$div(\rho V) = 0 \quad (1)$$

$$(V.grad)=-1/\rho\ grad\ P \quad (2)$$

where ρ = density, V = velocity and ρ is the pressure. Pressure, ρ is a function of ρ and entropy Thus

$$p=g(\rho,S) \quad (3)$$

conservation of entropy is expressed by

$$V.\ grad\ S = 0 \quad (4)$$

If entropy, S is constant everywhere in space, it is homentropic hence p is a function of density ρ only. Flows could be irrotational meaning

$$Curl\ V=0 \quad (5)$$

Equation (2) can be written as

$$\frac{1}{2}\ grad\ V^2 + \frac{1}{\rho}\ grad\ P = 0\ or \frac{1}{2}\ V^2 + \int \frac{dp}{\rho} = constant\ for\ a\ streamiline \qquad (6)$$

Since p is a function of ρ, equation (6) could be written as

$$f^{'}(\rho) - \frac{1}{2}V^2 = 0 \qquad (7)$$

But $\frac{dp}{\rho} = -f^{''}(\rho) = C^2$ (8)

where C = speed of sound in the fluid. Integrating equation (8) yields

$$p = f(\rho) - \rho f^{'}(\rho) - \frac{1}{2}\rho V^2 \qquad (9)$$

Bateman (1959) considered the case of homentropic, irrotational flow in two dimensions. In two dimensions $V = (V_{x,}\ \ V_{y,})$ and equation (5) becomes

$$\frac{\partial Vx}{\partial y} - \frac{\partial Vx}{\partial x} = 0 \qquad (10)$$

To satisfy equation (10), the velocity potential ∂ could be introduced such that

$$\frac{\partial\varphi}{\partial x} = V_{x,} \frac{\partial\varphi}{\partial y} = V_{y,} \qquad (11)$$

Or grad $\varphi = V$ (12)

Equation (1), (7) and (12) could be rewritten as

$$\text{grad}\ \varphi = \frac{1}{\rho}u = \frac{\partial H}{\partial u} \qquad (13)$$

$$\text{-divU} = 0 = \frac{\partial H}{\partial u} \qquad (14)$$

$$\rho^2(\rho) - \frac{1}{2}U^2 = 0 \qquad (15)$$

where $U = \rho^V$ (16)

Density ρ is a function of $(U\frac{2}{x} + U\frac{2}{y})\ {}^1/_2 = U$ only and from equation (13) and (14), it follows that the generalized Hamitonian H is a function of U only
Introducing

$$H\,(U\{\rho\}) = \Omega(\rho) \qquad (17)$$

then we can show by using equations (13) and (15)

$$\Omega(\rho) + \rho f'(\rho) = f(\rho) = \frac{1}{2}\rho V^2 \tag{18}$$

This is Hamiltonian which appears in the potential *I(U,φ)* of the variational problem. If we assume homogeneous boundry conditions, we find that

$$I(U,\varphi) = \int \left\{U.grad\ \varphi - f\ (\rho) - \frac{1}{2}\rho V^2\right\} dA \tag{19}$$

$$= \int \left\{(-divU)\varphi - f - \frac{1}{2}\ \rho V^2\right\} dA \tag{20}$$

where A is some sufficiently simple region with boundary ∂A

The functional $T(\varphi_t), w\square ere\ \varphi_t\ denotes\ a\ trial\ function, is\ given\ by$

$$T(\varphi_t) = I\{U(\varphi_t), \varphi_t\} \tag{21}$$

where $U(\varphi_t) = \rho_t\ grad\ \varphi_t = \rho_t V_t$ (22)

This is the solution of equation (13). Hence, from equation (9)

$$T(\varphi_t + = \int \left\{\rho_t\ (grad\ \varphi_t)\ {}^1/_2 - f(\rho_t) - \frac{2}{2}\ \rho_t V_t\right\} dA$$
$$= \int \{\frac{1}{2}\rho_t V_t^2 - f\ (\rho_t\} dA = -\int p(\rho_t) dA \tag{23}$$

Density ρ_t is a function of the trail function φ_t

The function $M(U_t)$, where $\mathrm{U_t}$ denotes a trail function given by

$$M(U_t), = I\ \{U_{t'}\varphi(U_t)\} \tag{24}$$

where $\varphi(U_t)$ is the solution of equation (14), which actually imposes the constraint.

$$div\ U_t = 0 \tag{25}$$

From equation (20)

$$M(U_t), = -\int \left\{f(\rho_t) + \frac{1}{2}\rho_t V_t^2\right\} dA = -\int \{p(\rho_t) + \rho_t V_t^2\} dA \tag{26}$$

It implies that ρ_t and $\mathrm{V_t}$ are function of the trail function $\mathrm{U_t}$ subject to the constraint *div* $\mathrm{U_t} = 0$ in *A*.

The variations of V_t and *V* could be expanded by examining the extremum property of $T(\varphi_t)$

$$T(\varphi_t) = T\varphi) - \frac{1}{2} \in^2 \int \sum_{i,j} \delta V_i \frac{\partial^2 p}{\partial V_i \partial V_j} \delta V_j dA + \cdots \quad (27)$$

where $\in \delta V_i = V_{ii} - V_i$

CONCLUSION

Considering the discussion of Sewell (1963), we discovered that uniqueness of solution requires that $-p$ be convex. This is ensured if

$$\sum_{i,j} \partial V_i \frac{\partial^2 p}{\partial V_i \partial V_j} \partial V_j \leq 0 \quad (28)$$

From equation (9)

$$\frac{\partial^2 p}{\partial V_i \partial V_j} = \rho(\frac{V_i V_j}{C^2} - \partial_{ij}) \quad (29)$$

Hence, equation (28) is satisfied by subsonic flow. In this case, we have the minimum principle

$$T(\varphi) \leq T(\varphi_t) \quad (30)$$

for φ_t sufficiently close to the exact solution φ

For subsonic flow, the complementary maximum principle is given by

$$M(U_t) \leq M(U) = T(\varphi) \quad (31)$$

For U_t sufficiently close to the exact solution *U.*

We have considered the extremum principles of $T(\varphi)$ and the variations of velocity V_t and *V*. The associated complementary variational principles with regard to the equations of compressible fluid flow was highly considered.

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