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A Mathematical Model for the Prediction of Optimum Mortar Joint Thickness in brick-Mortar Couplet

Nwofor T. C. and Sule S.

Department of Civil and Environmental Engineering, University of Portharcourt, Portharcourt, Rivers State, Nigeria

ABSTRACT

In this paper, a mathematical model is formulated for the prediction of optimum mortar joint thickness in brick-mortar couplet. The brick-mortar couplet is assumed to be sandwiched between two mortar joints and loaded in compression. The predicted curves agreed favorably with those of the experimental values. It was shown that the optimum mortar thickness for both the predicted and experimental curves was 10mm. The correlation coefficient between the experimental and predicted values was high (0.945) showing the efficacy of the formulated model in the analysis and design of brick-mortar couplet under compressive loading.

Keywords: Mathematical model, mortar joint, brick-mortar couplet, sandwiched, optimum.

INTRODUCTION

The problem of shelter is as old as man. Apart from food, the most basic problem of man is shelter. It is very unfortunate that this basic need of man has not materialized for the poor masses over the years. Efforts to deliver mass housing for the rural poor and low income earners at affordable rates have not been successful due to high cost of construction materials such as cement used for production of building blocks [1]-[10].

Nigeria is a country that is naturally endowed with abundant building materials such as laterite, bricks, sand etc. That Nigeria is blessed with the above-mentioned building materials is no guarantee for Nigerian citizens to afford shelters of their own due to inflation, unemployment, high rate of population growth and poor economic resources. The project undertaken by the world labour organization to provide house for all by the year 2000 was a failure due to high cost of building material such as cement in the country. Bricks which are readily available in Nigeria in commercial quantities are resorted to as alternative construction materials [7]-[9]. A brick-mortar couplet is indeed a promising masonry unit. However, the failure which occurs in its usage is traceable to lack of predictive models for optimum mortar thickness under compressive loading.

In this paper, a mathematical model is formulated for the optimization of mortar joint thickness in masonry units. It is believed that the formulated models will enhance optimum utilization of brick-mortar couplet as construction materials.

3.0 Model Derivation

Consider a brick sandwiched between two mortar joints in opposite direction and loaded in compression as shown (Figure 1).

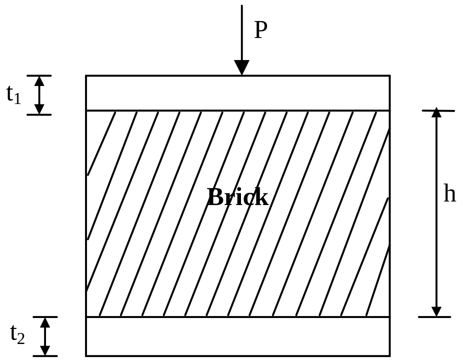

Figure 1: A brick- mortar couplet under compressive loading

Let
B represents the width of brick under a compressive loading P.
t_1, t_2 = thickness of mortar joint at top and bottom of brick
h = overall thickness of brick including the mortar joint thickness t_1 and t_2.

From Figure 1, the neutral axis distance from the centre line of bottom mortar joint is given by:

$$\bar{y} = \frac{Bht_1}{B(t_1 + t_2)} \tag{1}$$

Since $t_1 = t_2$

$$\Rightarrow = \bar{y} = \frac{h}{2} \tag{2}$$

The bending stiffness of the brick is assumed to be ignored. The moment of inertia of the entire section is given by:

$$I_{section} = \frac{Bt_1^2}{12} + \frac{Bt_2^3}{12} + \frac{Bt_2h^2t_1^2}{(t_1+t_2)^2} + \frac{Bt_1t_2^2h^2}{(t_1+t_2)^2} \tag{3}$$

Again, $t_1 = t_2$

$$\Rightarrow \quad I = \frac{Bt^3}{12} + \frac{Bt^3}{12} + \frac{Bh^2t^3}{4t^2} + \frac{Bt^3h^2}{4t^2} \tag{4}$$

Summing up the like terms in equation (4) gives:

$$I_{section} = \frac{Bt^3}{6} + \frac{Bh^2t}{2} \tag{5}$$

Equation (5) is simplified and factorized to give:

$$I_{section} = \frac{Bt(t^2 + 3h^2)}{6} \tag{6}$$

From strength of materials, section modulus,

$$Z = \frac{I_{section}}{\bar{y}} \tag{7}$$

Substituting for $I_{section}$ and $\bar{y}$ in equations (2) and (6) gives;

$$Z = Bt\left(\frac{t^2}{3h} + h\right) \tag{8}$$

The maximum stress in the brick mortar couplet is given by;

$$\sigma_{max} = \sigma_c = \frac{M_{max}}{Z} \tag{9}$$

where

$\sigma_{max} = \sigma_c$ = compressive stress induced in the brick mortar couplet due to load P.
m = m_{ax} = maximum bending moment in the brick mortar couplet.
t = mortar thickness
h = overall depth of brick including mortar joints

Substituting for section modulus, Z transforms equation (9) to:

$$\sigma_c = \frac{M_{max}}{Bt\left(\frac{t^2}{3h} + h\right)} \tag{10}$$

The brick mortar couplet is assumed to be subjected to centre-point loading and simply supported. Therefore,

$$M_{max} = \frac{PL}{4} \tag{11}$$

Substituting for M_{max} in equation (10) gives:

$$\sigma_c = \frac{PL}{4Bt\left(\frac{t^2}{3h} + h\right)} \tag{12}$$

The effect of mortar curing factor α, air content factor β and plastic mortar age factor T, transform equation (12) to:

$$\sigma_c = \frac{\alpha\beta TPL}{4Bt\left(\frac{t^2}{3h} + h\right)} \tag{13}$$

According to Dayaratnam [10],

$\alpha = 0.49, \qquad \beta = 0.32 \qquad and \qquad T = 0.55$

Therefore,

$$\sigma_c = \frac{0.08624PL}{4Bt\left(\frac{t^2}{3h} + h\right)} \tag{14}$$

Again, according to Dayaratnam [10], the failure of brick-mortar couplet in compression reaches its maximum when:

$$\sigma_c = \alpha_f \sigma_{cu} \quad (15)$$

where;

$$\alpha_f = 0.95 \quad and$$

σ_{cu} = compressive strength of brick mortar.

Therefore,

$$\sigma_c = \frac{0.083PL}{4Bt\left(\frac{t^2}{3h} + h\right)} \quad (16)$$

Equation (16) gives the compressive strength of brick-mortar couplet.

Hence the compressive strength of brick- mortar couplets obtained are tabulated below for easy comparison

Table 1: Compressive strength of brick-mortar couplet at 7 days for 7mm, 10mm, 13mm, 15mm and 17.5m mortar thickness

Thickness (mm)	Compressive strength (N/mm^2)	
	Expt (N/mm^2)	Predicted
7	6.81	7.45
10	11.24	11.87
13	6.58	7.19
15	4.98	5.13
17.5	3.14	3.40

Table 2: Compressive strength of brick-mortar couplet at 21 days for 7mm, 10mm, 13mm, 15mm and 17.5mm mortar thickness

Thickness (mm)	Compressive strength (N/mm^2)	
	Expt (N/mm^2)	Predicted
7	12.78	13.65
10	19.76	20.43
13	12.69	13.16
15	6.94	7.08
17.5	4.83	5.07

Table 3: Compressive strength of brick-mortar couplet at 28 days for 7mm, 10mm, 13mm, 15mm and 17.5mm mortar thickness

Thickness (mm)	Compressive strength (N/mm^2)	
	Expt (N/mm^2)	Predicted
7	14.4	14.90
10	20.99	21.74
13	7.83	8.22
15	7.91	7.84
17.5	5.41	5.58

RESULTS AND DISCUSSION

The predicted results of mathematical models formulated using the concept of sandwiched beam has been presented, it can be seen that both the experimental and predicted results agreed favorably and statistics show that they have close variance. From the experimental and predicted results it can be seen that the compressive strength increases with age with the maximum compressive strength at 28 days. This may be due to the fact that concrete strength increases with continued hydration with full strength at 28 days. The volume of the pores in the hardened brick-mortar couplet reduces with age and therefore, the more the gel/space ratio which is an important factor that influences concrete strength. From table 1-3, it can be seen that the compressive strength increases with increased value of mortar thickness and maximized at 10mm mortar thickness after which it started decreasing continually. This trend may be due to the fact that as the mortar thickness increased the gel/space ratio decreased with a consequent reduction in strength [10]. The slight difference between the experimental and predicted results may be attributed to error in the model assumption which was used in the model formulation and also error in the experimental data.

CONCLUSION

From the study the following conclusions are drawn:
(i) The formulated models gave results that were almost identical with those of the experimental data showing the high predictive ability of the formulated model.
(ii) The rate of gain of brick-mortar couplet strength increases with age with full compressive strength valves at 28 days.
(iii) The optimum mortar thickness that corresponds to maximum compressive strength of brick-mortar couplet is 10mm.

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