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A fuzzy logic based Approach to Minimize the Rental Cost of Machines for Specially Structured Three Stages Flow Shop Scheduling

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ABSTRACT

This paper is an attempt to introduce the concept of specially structured n-jobs, three machine flow shop scheduling in which the processing time of jobs are uncertain that is not known exactly and are in fuzzy environment. Fuzzy logic poses the ability to mimic the human mind to effectively employ modes of reasoning that are approximate rather than exact. The fuzzy processing times are described by triangular fuzzy membership function. Further the average high ranking (AHR) of fuzzy processing times are not random but bear a well defined relationship to one another. The objective of the present paper is to develop a heuristic algorithm to minimize the rental cost of machines under the specified rental policy. A numerical example is provided to demonstrate the computational efficiency of proposed algorithm.

Keywords: Specially structured flowshop scheduling, Rental Policy, Processing Time, Fuzzy processing time, Average High Ranking, Utilization Time.
Mathematical Subject Classification: 90B30, 90B35.

INTRODUCTION

Scheduling problem is concerned with searching for an optimal or near optimal schedule subject to a number of constraints. A variety of approaches have been developed to solve the problem of scheduling. However, many of these approaches are often impractical in dynamic real-world environments where there are complex constraints and a variety of unexpected disruptions. In most real-world environments scheduling is an ongoing reactive process where the presence of real-time information continually forces reconsideration and revision of pre-established schedules. Over the last decades several theories such as Fuzzy-set theory, Probability theory, D-S theory and approaches based on certainty factors have been developed to account for uncertainty. Among them, fuzzy set theory is more and more frequently used in intelligent control because of its simplicity and similarity to human reasoning. Moreover, the fuzzy approach seems a natural extension of its crisp counterpart so that we need to know how the fuzziness of processing times affects the job sequence itself. In most of the literature processing time of jobs is considered to be random with a goal to minimize the makespan and mean flow time. But there are significant situations in which the processing times are not random, but follow some well defined structural conditions. One of the earliest results in flowshop scheduling theory is an algorithm given by Johnson's [1] for scheduling jobs in a two, three machine flowshop to minimize the time at which all jobs are completed. Gupta, J.N.D [2] gave an algorithm to find the optimal schedule for specially structured flowshop scheduling. MacCahon and Lee [4] discussed the job sequencing with fuzzy processing time. Ishibuchi and Lee [6] addressed the formulation of fuzzy flowshop scheduling problem with fuzzy processing time. Hong and Chuang [8] developed a new triangular Johnson algorithm. Some of the noteworthy approaches are due to Yager [3], McCahon [5] , Shukla and Chen [7], Marin and Roberto [9], Yao and Lin [10], Singh and Gupta [12], Sanuja and Song [13], Singh, Sunita and Allawalia [14].

Gupta, D., Sharma, S., and Shashi [15] studied specially structured two stage flow shop scheduling to minimize the rental cost of machines. The objective of this paper is to develop a heuristic algorithm to minimize the rental cost for three stages specially structured flow shop scheduling problem under a specified rental policy with fuzzy processing time represented by triangular fuzzy numbers.

Practical Situation

Fuzzy set theory is now applied to problems in engineering, business, medical and related health sciences, and the natural sciences. Various practical situations occur in real life when one has got the assignments but does not have one's own machine or does not have enough money or does not want to take risk of investing huge amount of money to purchase machine. Under such circumstances, the machine has to be taken on rent in order to complete the assignments. Medical science can save the patient's life but proper care leads to a faster recovery. Care giving techniques often require hi-tech, expensive medical equipment. Many of these equipments can even help in saving the life of critical patients. Most of these equipments are expensive & they are often needed for a few days or weeks thus buying them do not make much sense even if one can afford them. Many patients even lose their lives just because they can not afford to buy these products. In his starting career, we find a medical practitioner does not buy expensive machines say X-ray machine, the Ultra Sound Machine, Rotating Triple Head Single Positron Emission Computed Tomography Scanner, Patient Monitoring Equipment, and Laboratory Equipment etc., but instead takes on rent. Rental of medical equipment is an affordable and quick solution for hospitals, nursing homes, physicians, which are presently constrained by the availability of limited funds due to the recent global economic recession. Renting enables saving working capital, gives option for having the equipment, and allow up gradation to new technology.

Fuzzy Membership Function

All information contained in a fuzzy set is described by its membership function. The triangular membership functions are used to represent fuzzy processing times in our algorithm. The membership value of the x denoted by $\mu_x, x \in R^+$, can be calculated according to the formula

$$\mu_x = \begin{cases} 0; x \le a \\ \dfrac{x-a}{b-a}; a \le x \le b \\ \dfrac{c-x}{c-b}; b < x < c \\ o; x \ge c \end{cases}$$

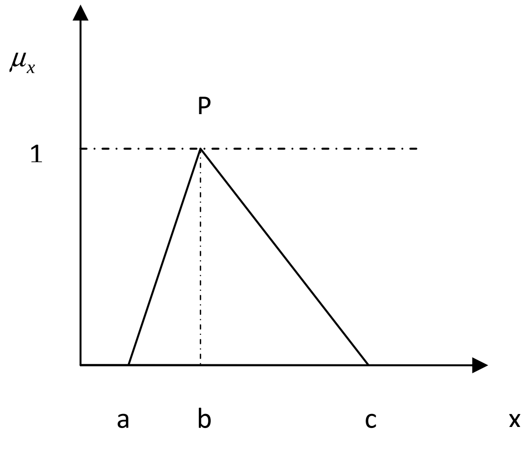

Fig. 1: Triangular membership function

Figure 1 shows the triangular membership function of a fuzzy set $\tilde{P}$, $\tilde{P}$ =(a, b, c). The membership value reaches the highest point at '*b*', while '*a*' and '*c*' denote the lower bound and upper bound of the set $\tilde{P}$ respectively.

Average High Ranking (A.H.R.)
To find the optimal sequence, the processing times of the jobs are calculated by using Yager's(1981) average high ranking formula (AHR) = $h(A) = \dfrac{3b + c - a}{3}$.

Fuzzy Arithmetic Operations
If $A_1 = (m_{A_1}, \alpha_{A_1}, \beta_{A_1})$ and $A_2 = (m_{A_2}, \alpha_{A_2}, \beta_{A_2})$ be the two triangular fuzzy numbers, then

1. $A_1 + A_2 = (m_{A_1}, \alpha_{A_1}, \beta_{A_1}) + (m_{A_2}, \alpha_{A_2}, \beta_{A_2})$ = $(m_{A_1} + m_{A_2}, \alpha_{A_1} + \alpha_{A_2}, \beta_{A_1} + \beta_{A_2})$
2. $A_1 - A_2 = (m_{A_1}, \alpha_{A_1}, \beta_{A_1}) - (m_{A_2}, \alpha_{A_2}, \beta_{A_2})$ = $(m_{A_1} - m_{A_2}, \alpha_{A_1} + \beta_{A_2}, \alpha_{A_2} + \beta_{A_1})$
3. $kA_1 = k(m_{A_1}, \alpha_{A_1}, \beta_{A_1}) = (km_{A_1}, k\alpha_{A_1}, k\beta_{A_1})$; if k>0.
4. $kA_1 = k(m_{A_1}, \alpha_{A_1}, \beta_{A_1}) = (k\beta_{A_1}, k\alpha_{A_1}, km_{A_1})$; if k<0.

1. Notations

S : Sequence of jobs 1, 2, 3,….,n
S_k : Sequence obtained by applying Johnson's procedure, k = 1, 2 , 3, -------
M_j : Machine j, j= 1,2,3
M : Minimum makespan
a_{ij} : Fuzzy processing time of i^{th} job on machine M_j i=1,2,3,....,n; j=1,2,3
A_{ij} : AHR of processing time of i^{th} job on machine M_j
$t_{ij}(S_k)$: Completion time of i^{th} job of sequence S_k on machine M_j
$I_{ij}(S_k)$: Idle time of machine M_j for job i in the sequence S_k
$U_j(S_k)$: Utilization time for which machine M_j is required
$R(S_k)$: Total rental cost for the sequence S_k of all machine
C_i : Rental cost of i^{th} machine.
$h(R(S_k))$: AHR of the rental cost for sequence S_k
$CT(S_i)$: Total completion time of the jobs for sequence S_i

Rental Policy
The machines will be taken on rent as and when they are required and are returned as and when they are no longer required i.e. the first machine will be taken on rent in the starting of the processing the jobs, 2nd machine will be taken on rent at time when 1st job is completed on 1st machine and transported to 2nd machine, 3rd machine will be taken on rent at time when 1st job is completed on the 2nd machine and transported.

Problem Formulation
Let some job *i* (*i* =1, 2, 3, ..., n) is to be processed on three machines M_j(j=1,2,3) under the specified rental policy P. Let a_{ij} be the processing time of i^{th} job on j^{th} machine described by triangular fuzzy numbers. Let A_{ij} ; *i*=1,2,3,.....n; *j*=1,2,3 be the average high ranking (AHR) of the processing times on three machines M_1, M_2 and M_3 such that either $A_{j2} \le A_{i1}$ or $A_{j2} \le A_{i3}$ for all values of *i*, *j*. Our aim is to find the sequence {S_k} of the jobs which minimize the rental cost of the machines.

Table 1: The mathematical model of the problem in matrix form is

Jobs	Machine M1	Machine M2
i	ai1	ai2
1	a11	a12
2	a21	a22
3	a31	a32
4	a41	a42
-	-	-
n	an1	an2

Mathematically, the problem can be stated as follows:

Minimize $R(S_k) = \sum_{i=1}^{n} a_{i1} \times C_1 + U_2(S_k) \times C_2 + U_3(S_k) \times C_3$

Subject to Constraint: Rental Policy P.

Our objective is to minimize the rental cost of machines while minimizing their utilization time.

Algorithm
The following algorithm provides a heuristic approach to minimize the utilization time and hence rental cost of three stages specially structured flow shop scheduling with processing time in fuzzy environment without taking into consideration the makespan.

Step 1: Find the average high ranking (AHR) A_{ij} ; $i=1,2,3,...,n$; $j=1,2,3$ of the processing times for all the jobs on three machines M_1, M_2 and M_3.

Step 2: Check the condition: either $A_{j2} \le A_{i1}$ or $A_{j2} \le A_{i3} \forall i, j$

.i.e. either Max $\{A_{i1}\} \ge$ Min $\{A_{j2}\}$ or Max $\{A_{i3}\} \ge$ Min $\{A_{j2}\}$ $\forall i, j$.

If the conditions are satisfied then go to Step 3, else the data is not in the standard form.

Step 3: Introduce the two fictitious machines G and H with processing times G_i and H_i as

$$G_i = A_{i1} + A_{i2}, \; H_i = A_{i2} + A_{i3} \; \forall \; i.$$

Step 4: Obtain the sequence S_1 (say) by applying Johnson's [1] algorithm on machines G & H.

Step 5: Obtain other sequences by putting 2nd , 3rd ,…. , nth jobs of the sequence S_1 in the 1st position and all other jobs of S_1 in the same order. Let these sequences be S_2, S_3, S_4, --------, S_{n-1}.

Step 6: Compute $\sum_{i=1}^{n} a_{i1}, U_2(S_k), U_3(S_k)$ and $R(S_k) = \sum_{i=1}^{n} a_{i1} \times C_1 + U_2(S_k) \times C_2 + U_3(S_k) \times C_3$ for all the possible sequences S_k (k=1, 2, 3,……, n).

Step 7: Find $\min h\{R(S_k)\}; k = 1, 2, 3, ..., n$. Let it be minimum for the sequence S_p, then the sequence S_p will be the optimal sequence with rental cost $R(S_p)$.

Numerical Illustration
Consider 5 jobs, 3 machine flow shop problem with processing time described by triangular fuzzy numbers as given in table. The rental cost per unit time for machines M_1, M_2 and M_3 are 4 units, 2 units and 3 units respectively, under the rental policy P. Our objective is to obtain an optimal schedule to minimize the total rental cost of the machines

Table 2: The machines with fuzzy processing times

Jobs	Machine M1	Machine M2	Machine M3
i	*ai1*	*ai2*	*ai3*
1	(7,8,9)	(6,7,8)	(3,4,5)
2	(12,13,14)	(5,6,7)	(4,5,6)
3	(8,10,12)	(4,5,6)	(6,7,8)
4	(10,11,12)	(5,6,7)	(11,12,13)
5	(9,10,11)	(5,6,8)	(8,9,10)

Solution: As per step 1: The A.H.R of processing time of jobs is as follows:

Table 3: Machines with AHR processing times

Jobs	Machine M1	Machine M2	Machine M3
i	Ai1	Ai2	Ai3
1	26/3	23/3	14/3
2	41/3	20/3	17/3
3	34/3	17/3	23/3
4	35/3	20/3	38/3
5	32/3	21/3	29/3

As per step 3: The processing times for two fictitious machines G and H are

Table 4: The two fictitious machines with AHR processing time

Jobs	G_i	H_i
1	49/3	37/3
2	61/3	37/3
3	51/3	40/3
4	55/3	58/3
5	53/3	50/3

Using Johnson's [1] procedure, the sequence with minimum makespan is

S_1: 4 – 5 – 3 – 2 – 1.

Other feasible sequences which may correspond to minimum rental cost are

S_2: 5 – 4 – 3 – 2 – 1, S_3: 3 – 4 – 5– 2 – 1, S_4: 2 – 4 – 5 – 3 – 1, S_5: 1 – 4 – 5 – 3 – 2.

The In – Out table for sequence S_1 = 4 – 5 – 3 – 2 – 1is as follows

Table 5: The In-Out flow table for sequence S_1

Jobs	Machine M_1	Machine M_2	Machine M_3
i	In - Out	In - Out	In - Out
4	(10,11,12)	(15,17,19)	(26,29,32)
5	(19,21,23)	(24,27,31)	(34,38,42)
3	(27,31,35)	(31,36,41)	(40,45,50)
2	(39,44,49)	(44,50,56)	(48,55,62)
1	(46,52,58)	(52,59,66)	(55,63,71)

For sequence S_1: 4 – 5 – 3 – 2 – 1, we have

$CT(S_1)$ = (55,63,71), $U_2(S_1)$ = (37,78,83), $U_3(S_1)$ = (29,95,100), $R(S_1)$ = (345,649,698), and $h\{R(S_1)\}$= 766.666 units

Similarly, from In – Out tables for other feasible sequences, we can have

For S_2: 5 – 4 – 3 – 2 – 1

$CT(S_2)$ = (55,63,71), $U_2(S_2)$ = (38,78,82), $U_3(S_2)$ = (33,92,96), $R(S_2)$ = (359,640,684), and $h\{R(S_2)\}$= 736.333 units

For S_3: 3 – 4 – 5– 2 – 1

$CT(S_3)$ = (55,63,71), $U_2(S_3)$ = (40,77,81), $U_3(S_3)$ = (37,89,93), $R(S_3)$ = (375,629,673)), and $h\{R(S_3)\}$= 728.333 units

For S_4: 2 – 4 – 5 – 3 – 1

$CT(S_4)$ = (55,63,71), $U_2(S_4)$ = (35,80,85), $U_3(S_4)$ = (34,90,95), $R(S_4)$ = (356,638,687)), and $h\{R(S_4)\}$= 748.333 units

For S_5: 1 – 4 – 5 – 3 – 2

$CT(S_5)$ = (55,63,71), $U_2(S_5)$ = (38,75,80), $U_3(S_5)$ = (39,85,90), $R(S_5)$ = (377,613,662)), and $h\{R(S_5)\}$= 708 units

Therefore, $\min h\{R(S_k)\}$ = 708, and is for sequence S_5. Hence, R (S_5) = (377,613,662) is the minimum rental cost and the sequence S_5 : 1 – 4 – 5 – 3 – 2 is the optimal sequence with minimum rental cost irrespective of total elapsed time.

CONCLUSION

A heuristic algorithm to minimize the rental cost of the machines for a specially structured three stage flow shop scheduling is discussed irrespective of their total elapsed time The processing times of machine are uncertain and are represented by triangular fuzzy membership functions. The study may further be extended using trapezoidal fuzzy membership functions and various other constraints of flow shop scheduling problems.

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