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A Fuzzy Logic Based Approach for Multistage Flowshop Scheduling With Arbitrary Lags and Transportation Time

Sameer Sharma*[1], Shefali Aggarwal[2], Deepak Gupta[2] and Naveen Gulati*[3]

[1]Department of Mathematics
D.A.V. College, Jalandhar, 144008, Punjab, India
[2]Department of Mathematics & Humanities
M. M. University, Mullana, Ambala, Haryana, India
[3]Department of Mathematics
S.D. College, Ambala Cantt., Haryana, India

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Corresponding author: Department of Mathematics, D.A.V. College, Jalandhar, 144008, Punjab, India
E-mail address:
sameer31@davjalandhar.com

ABSTRACT

The aim of this paper is to introduce the concept of arbitrary lags in *n*-jobs, *m* machines flowshop scheduling problem involving the processing times and transportation times of jobs. Start lag is the minimum time which must elapse between starting of job *i* on the first machine and starting of job *i* on the last machine. The stop lag for the job *i* is the minimum time which must elapse between completing job *i* on the first machine and completing it on the last machine. The concept of fuzzy processing time to represent the uncertainty, vagueness in processing of jobs is introduced. An algorithm to find the optimal sequence so as to minimize the total elapsed time subject to some specified lag time constraint is discussed. A numerical illustration is given to demonstrate the computational efficiency of proposed algorithm as a valuable analytical tool for the researchers.

INTRODUCTION

The scheduling of jobs and control of their flow through production process is essential to modern production / manufacturing companies. Ever since the first results of modern scheduling theory appeared 50 years ago, scheduling has attracted a lot of attention from both academia and industry. In a general flowshop scheduling problem, *n* jobs are to be scheduled on *m* machines in order to optimize some measures of performance. A time lag is the minimum time delay required between the executions of two consecutive operations of the same job. Practically time lags represents: when the time needed to move a job from one machine to another is not negligible, we have to take transportation delays into account when constructing a schedule.

In the literature dealing with a flowshop scheduling problems, processing times are usually assumed to be known exactly. But, the real world is complex; complexity in the world generally arises from uncertainty. From this prospective, the concept of fuzzy environment is introduced in the theory of scheduling. The past few years have witnessed a rapid growth in the number and variety of applications of fuzzy logic. Zadeh[17] introduced the term fuzzy logic in his seminal work "Fuzzy Sets", which described the mathematics of fuzzy set theory. One of the earliest researches in arbitrary lags is due to Mitten[5] who studied sequencing of *n* jobs on two machines with arbitrary time lags. He considered Start-to-Start type combined with Finish-to-Finish lags. Kern and Nawijn[4] studied scheduling of multi-operation jobs with time lags on a single machine. MacCahon and Lee[6] discussed the job sequencing with fuzzy processing time. Ishibuchi and Lee[1] addressed the formulation of fuzzy flow shop scheduling problem with fuzzy processing time. Martin and Roberto[7] discussed the concept of fuzzy scheduling with application to real life system. Reizebos and Gaalman[8] studied the time lag size in multiple operation flowshop scheduling heuristics. Shukla and Chen[9] described the real time FMS control as a comprehensive survey. Sanuja and Song[10] discussed a new approach for two machine flowshop problems with uncertain processing times. Singh *et al*[11] studied the reformation of non-fuzzy scheduling using the concept of fuzzy processing time under blocking. Gupta *et al*[12] discussed flowshop scheduling on two machines with setup time and single transport facility under fuzzy environment. Sharma *et al*[13] studied multistage bi-criteria scheduling problems involving *n* jobs on *m* machines to minimize the rental cost of machines with minimum makespan.

Gupta, Shefali and Sharm[14] studied *n* jobs, 2 machine fuzzy flowshop scheduling problem with some time lags. The present work is an attempt to extend their study by generalizing the numbers of machines in which uncertain, vagueness in processing times are represented by triangular fuzzy numbers.

ROLE OF FUZZY LOGIC IN SCHEDULING

A fuzzy system can be thought of an attempt to understand a system for which no model exists, and it does so with the information that can be uncertain in a sense of being vague, or fuzzy, or imprecise, or altogether lacking. From this angle, fuzzy logic is a method to formalize the human capacity of imprecise reasoning. Such reasoning represents the human ability to reason approximately and judge under uncertainty. In fuzzy logic all truths are partial or approximate. In this sense the reasoning has also been termed interpolative reasoning, where the process of interpolating between the binary extremes of truth and false is represented by the ability of fuzzy logic to encapsulate partial truths.

Scheduling is an enduring process where the existence of real time information frequently forces the review and modification of pre-established schedules. The real world is complex; complexity in the world generally arises from uncertainty. From this prospective, the concept of fuzzy environment is introduced in the theory of scheduling.

Fuzzy Membership Function

All information contained in a fuzzy set is described by its membership function. The triangular membership functions are used to represent fuzzy processing times in our algorithm. The membership value of the x denoted by $\mu_x, x \in R^+$, can be calculated according to the formula

$$\mu_x = \begin{cases} 0; x \le a \\ \dfrac{x-a}{b-a}; a \le x \le b \\ \dfrac{c-x}{c-b}; b < x < c \\ o; x \ge c \end{cases}$$

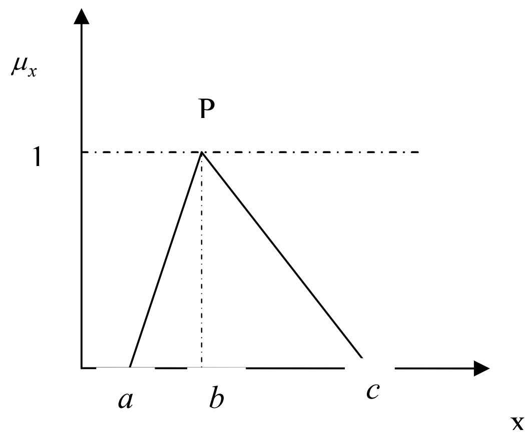

Fig. 1: Triangular membership function

The above figure shows the triangular membership function of a fuzzy set $\tilde{P}$, $\tilde{P}$=(a, b, c). The membership value reaches the highest point at '*b*', while '*a*' and '*c*' denote the lower bound and upper bound of the set $\tilde{P}$ respectively.

AVERAGE HIGH RANKING (A.H.R.)

The system characteristics are described by membership function; it preserves the fuzziness of input information. However, the designer would prefer one crisp value for one of the system characteristics rather than fuzzy set. In order to overcome this problem, we defuzzify the fuzzy values of system characteristic by using the Yager's[10] approximation formula

$$\text{crisp}(A) = h(A) = \frac{3a_2 + a_3 - a_1}{3}.$$

FUZZY ARITHMETIC OPERATIONS

The following are the four operations that can be performed on triangular fuzzy numbers:

Let $A = (a_1, a_2, a_3)$ and $B = (b_1, b_2, b_3)$ be the two triangular fuzzy numbers then

Addition: $A + B = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$

Subtraction: $A - B = (a_1 - b_1, a_2 - b_2, a_3 - b_3)$.

This subtraction operation exist only if the following condition is satisfied DP(A) ≥ DP(B), where and $DP(B)=(b_3-b_1)/2,\ DP(A)=(a_3-a_1)/2$; where DP denote difference point of a triangular fuzzy number

else; $A-B=(a_1-b_3, a_2-b_2, a_3-b_1)$

Multiplication: $A\times B=(\min(a_1b_1, a_1b_3, a_3b_1, a_3b_3), \max(a_1b_1, a_1b_3, a_3b_1, a_3b_3))$

Division: $A/B=(\min(a_1/b_1, a_1/b_3, a_3/b_1, a_3/b_3), \max(a_1/b_1, a_1/b_3, a_3/b_1, a_3/b_3))$.

NOTATIONS & DEFINITIONS

The following notations have been used in the progress of the paper:

S	: Sequence of jobs 1,2,3,….,n
S_k	: Sequence obtained by applying Johnson's procedure, k = 1, 2, 3, -----
M_j	: Machine j, j= 1, 2
M	: Minimum makespan
a_{ij}	: Fuzzy Processing time of i^{th} job on machine M_j
D_i	: Start lag for job i
E	: Stop lag for job i
A_{ij}	: AHR of processing time of i^{th} job on machine M_j
U_{iX}	: Starting time of any job i on machine X
T_{iX}	: Completion time of any job i on machine X
$CT(S_i)$	: Total completion time of jobs for the sequence S_i
$T_{i,j\to k}$	: Transportation time of i^{th} job from j^{th} machine to k^{th} machine
$T'_{i,j\to k}$	: Effective transportation time of i^{th} job from j^{th} machine to k^{th} machine.

The effective transportation time of job i denoted by $T'_{i,s\to s+1}$ is defined as

$$T'_{i,s\to s+1}=\max(D_i-G_i, E_i-H_i, T_{i,s\to s+1}); s=1,2,3,....(m-1)$$

Where; $G_i = A_{i1}+A_{i2}+A_{i3}+....+A_{i(m-1)}; H_i = A_{i2}+A_{i3}+A_{i4}+....+A_{im}\ \forall i$

.

THEOREMS

The following theorems have been established to find the optimal sequence optimizing the multistage fuzzy flowshop scheduling problems with time lag contraints.

a. Let n jobs J_1, J_2, J_3… J_n are processed through m machines M_j (j = 1, 2,…, m) in order M_1-M_2-M_3-……..-M_m with no passing allowed. Let t_{ij} represents the processing time of i^{th} job (i =1, 2, …, n) on j^{th} machine (j = 1, 2, …., m) such that $\min t_{is} \geq \max t_{i(s+1)}$; s = 1, 2, …, (m-2), then the optimal schedule minimizing the total elapsed time is given by the following decision rule: job J_k proceeds job J_{k+1} if $\min\{G_k, H_{k+1}\} < \min\{G_{k+1}, H_k\}$; where $G_i = t_{i1}+t_{i2}+---+t_{i(m-1)}$ and $H_i = t_{i2}+t_{i3}+---+t_{im}$.

Proof: Let S be a sequence of jobs defined as $S=\{J_1-J_2-J_3---J_{k-1}-J_k-J_{k+1}-J_{k+2}----J_n\}$. Let S' be another sequence of jobs processing in which jobs J_k and J_{k+1} are switched, .i.e.

$S' = \{J_1 - J_2 - J_3 - - - J_{k-1} - J_{k+1} - J_k - J_{k+2} - - - - J_n\}$. Let $A_{p,j}$ and $C_{p,j}$ denote the processing time and completion time of p^{th} job on machine M_j in job schedule *S*. Let $A'_{p,j}$ and $C'_{p,j}$ denote the processing time and completion time of p^{th} job on machine M_j in job schedule S'. First, we shall prove the following lemma:

Lemma: The completion time of i^{th} job J_i on $(m\text{-}1)^{th}$ machine M_{m-1} is given by $C_{i,(m-1)} = C_{i,1} + A_{i,2} + A_{i,3} + - - - + A_{i,(m-1)}$; m=2,3,4,...,m .

We shall prove the result with the help of mathematical induction. Let *P*(*i*) denote the statement

$P(i)$: $C_{i,(m-1)} = C_{i,1} + A_{i,2} + A_{i,3} + - - - + A_{i,(m-1)}$; for any natural number *i* . --- (1)

For *i* = 1, we have $C_{1,(m-1)} = A_{1,1} + A_{1,2} + A_{1,3} + - - - + A_{1,(m-1)} = C_{1,1} + A_{1,2} + A_{1,3} + - - - + A_{1,(m-1)} \; (\because C_{1,1} = A_{1,1})$

Therefore, statement P (1) is true.

Let us assume that the result (1) is true for any arbitrary numbers say *k*, .i.e. P(*k*) is true. Therefore, we have

$P(k)$: $C_{k,(m-1)} = C_{k,1} + A_{k,2} + A_{k,3} + - - - + A_{k,(m-1)}$ --- (2)

Let a new statement a new statement $P'(S)$ as

$P'(S) : C_{(r+1),s} \geq C_{r,(s+1)}$; $s = 1, 2, 3, ..., (m-2)$, m being a natural number.

Now, first we validate this statement with the help of induction.

Since $C_{(r+1),1} = C_{r,1} + A_{(r+1),1}$ --- (3)

and $C_{r,2} = C_{r,1} + A_{r,2}$ (using (2)) --- (4)

From the structural relationship it is obvious that $A_{(r+1),1} \geq A_{r,2}$ --- (5)

On combining results (3), (4) and (5), we get

$C_{(r+1),1} \geq C_{r,2}$, hence $P'(1)$ is true.

Let us assume that the statement $P'(S)$ is true for any arbitrary value say *q*.

.i.e. we have $C_{(r+1),q} \geq C_{r,(q+1)}$ --- (6)

Now, $C_{(r+1),(q+1)} = \max\left\{C_{(r+1),q}, C_{r,(q+1)}\right\} + A_{(r+1),(q+1)} = C_{(r+1),q} + A_{(r+1),(q+1)}$ --- (7)

From (1), we have

$C_{r,(q+1)} = C_{r,1} + A_{r,2} + A_{r,3} + ----+ A_{r,(q+1)}$

$C_{r,(q+2)} = C_{r,1} + A_{r,2} + A_{r,3} + ----+ A_{r,(q+1)} + A_{r,(q+2)} = C_{r,(q+1)} + A_{r,(q+2)}$ --- (8)

From structural relationship, it is obvious that $A_{(r+1),(q+1)} \geq A_{r,(q+2)}$ --- (9)

On combining results (6), (7), (8) and (9), we get

$C_{(r+1),(q+1)} \geq C_{r,(q+2)}$.

Therefore, the statement $P'(s)$ is true for s=q+1, .i.e. $P'(q+1)$ is true.

Hence, by principle of induction $P'(s)$ is true, .i.e. $C_{(r+1),s} \geq C_{r,(s+1)}$; $s = 1,2,3,---,(m-2)$ --- (10)

Let us define a new statement $P''(l) : C_{(r+1),(l+1)} = C_{(r+1),1} + A_{(r+1),2} + A_{(r+1),3} + ----+ A_{(r+1),(l+1)}$ --- (11)

Again, we have to test the consistency of result (11), by mathematical induction.

For $l = 1$, $C_{(r+1),2} = \max\left\{C_{(r+1),1}, C_{r,2}\right\} + A_{(r+1),2} = C_{(r+1),1} + A_{(r+1),2}$ $(\because \text{ of result (10)})$

Hence, $P''(1)$ is true.

Let the statement $P''(l)$ is true for any arbitrary number (say) x, .i.e.

$P''(l) : C_{(r+1),(x+1)} = C_{(r+1),1} + A_{(r+1),2} + A_{(r+1),3} + ----+ A_{(r+1),(x+1)}$ --- (12)

Now, $C_{(r+1),(x+2)} = \max\left\{C_{(r+1),(x+1)}, C_{r,(x+2)}\right\} + A_{(r+1),(x+2)} = C_{(r+1),(x+1)} + A_{(r+1),(x+2)}$ (Using (10))

$= C_{(r+1),1} + A_{(r+1),2} + A_{(r+1),3} + --- + A_{(r+1),(x+1)} + A_{(r+1),(x+2)}$

Therefore, the statement $P''(l)$ is true for $l = x + 1$.

Hence, by the mathematical induction $P''(l)$.

On taking $l = m - 2$ in result (11), we have

$C_{(r+1),(m-1)} = C_{(r+1),1} + A_{(r+1),2} + A_{(r+1),3} + --- + A_{(r+1),(m-1)}$

Therefore, statement P (i) (result (1)) is true for $i = r+1$. Hence, by mathematical induction P (i) is true, .i.e.

$C_{i,(m-1)} = C_{i,1} + A_{i,2} + A_{i,3} + - - - + A_{i,(m-1)}$; for any natural number i .

Hence, lemma is proved.

Now, we proceed to proof of main theorem. By definition, we have

$$C_{p,m} = \max\left\{C_{p,(m-1)}, C_{(p-1),m}\right\} + A_{p,m} = \max\left\{C_{p,1} + A_{p,2} + - - - - - + A_{p,(m-1)}, C_{(p-1),m}\right\} + A_{p,m} \quad \text{--- (13)}$$

Now, schedule S is preferable to S' if $C_{n,m} < C'_{n,m}$ --- (14)

.i.e. $\max\left\{C_{n,1} + A_{n,2} + - - - - - + A_{n,(m-1)}, C_{(n-1),m}\right\} + A_{n,m} < \max\left\{C'_{n,1} + A'_{n,2} + - - - - - + A'_{(n-1),m}, C'_{(n-1),m}\right\} + A'_{n,m}$

Now, we have $C_{n,1} = C'_{n,1} = \sum_{i=1}^{n} t_{i,1}$; Also $A_{n,j} = A'_{n,j}$ $(j = 1,2,3,...,m)$.

The result (14) is true, if $C_{(n-1),m} < C'_{(n-1),m}$.

Continuing in this manner, we get $C_{(k+1),m} < C'_{(k+1),m}$ --- (15)

Now, $C_{(k+1),m} = \max\left\{C_{(k+1),(m-1)}, C_{k,m}\right\} + A_{(k+1),m} = \max\left\{C_{(k+1),1} + A_{(k+1),2} + A_{(k+1),3} + - - - + A_{(k+1),(m-1)}, C_{k,m}\right\} + A_{(k+1),m}$

$= \max\left\{C_{k,1} + A_{(k+1),1} + A_{(k+1),2} + A_{(k+1),3} + - - - + A_{(k+1),(m-1)}, C_{k,m}\right\} + A_{(k+1),m}$

Since, $A_{k,j} = t_{k,j}$, therefore $A_{(k+1),1} + A_{(k+1),2} + - - - - - + A_{(k+1),(m-1)} = t_{(k+1),1} + t_{(k+1),2} + - - - - - + t_{(k+1),(m-1)} = G_{k+1}$

Hence, $C_{(k+1),m} = \max\left\{C_{k,1} + G_{k+1}, C_{k,m}\right\} + A_{(k+1),m}$ --- (16)

Now, $C_{k,m} = \max\left\{C_{k,(m-1)}, C_{(k-1),m}\right\} + A_{k,m} = \max\left\{C_{k,1} + A_{k,2} + A_{k,3} + - - - - - + A_{k,(m-1)}, C_{(k-1),m}\right\} + A_{k,m}$

$= \max\left\{C_{(k-1),1} + A_{k,1} + A_{k,2} + A_{k,3} + - - - - - + A_{k,(m-1)}, C_{(k-1),m}\right\} + A_{k,m}$

Also, $A_{k,1} + A_{k,2} + - - - - - - + A_{k,(m-1)} = t_{k,1} + t_{k,2} + - - - - - - + t_{k,(m-1)} = G_k$

Hence, $C_{k,m} = \max\left\{C_{(k-1),1} + G_k, C_{(k-1),m}\right\} + A_{k,m}$

$= \max\left\{C_{(k-1),1} + G_k + A_{k,m}, C_{(k-1),m} + A_{k,m}\right\}$ --- (17)

On using (17), the result (16) can be written as

$$C_{k+1,m} = \max\left\{C_{k,1} + G_{k+1}, C_{k-1,1} + G_k + A_{k,m}, C_{(k-1),m} + A_{k,m}\right\} + A_{(k+1),m}$$
$$= \max\left\{C_{k,1} + G_{k+1} + A_{(k+1),m}, C_{k-1,1} + G_k + A_{k,m} + A_{(k+1),m}, C_{(k-1),m} + A_{k,m} + A_{(k+1),m}\right\}$$
$$= \max\left\{C_{(k-1),1} + A_{k,1} + G_{k+1} + A_{(k+1),m}, C_{k-1,1} + G_k + A_{k,m} + A_{(k+1),m}, C_{(k-1),m} + A_{k,m} + A_{(k+1),m}\right\}$$
$$= \max\left\{C_{(k-1),1} + t_{k,1} + G_{k+1} + t_{(k+1),m}, C_{k-1,1} + G_k + t_{k,m} + t_{(k+1),m}, C_{(k-1),m} + t_{k,m} + t_{(k+1),m}\right\} \quad \text{--- (18)}$$

Similarly, we can obtain

$$C'_{(k+1),m} = \max\left\{C'_{(k-1),1} + G_k + t_{(k+1),1} + t_{k,m}, C'_{(k-1),1} + G_{k+1} + t_{(k+1),m} + t_{k,m}, C'_{(k-1),m} + t_{k+1,m} + t_{k,m}\right\} \quad \text{--- (19)}$$

On using (18) and (19), result (15) becomes

$$\max\left\{C_{(k-1),1} + t_{k,1} + G_{k+1} + t_{(k+1),m}, C_{k-1,1} + G_k + t_{k,m} + t_{(k+1),m}, C_{(k-1),m} + t_{k,m} + t_{(k+1),m}\right\}$$
$$< \max\left\{C'_{(k-1),1} + G_k + t_{(k+1),1} + t_{k,m}, C'_{(k-1),1} + G_{k+1} + t_{(k+1),m} + t_{k,m}, C'_{(k-1),m} + t_{k+1,m} + t_{k,m}\right\}$$

Since, $C'_{(k-1),m} = C_{(k-1),m}$ and third term on both side become equal, hence, we have

$$\max\left\{C_{(k-1),1} + t_{k,1} + G_{k+1} + t_{(k+1),m}, C_{k-1,1} + G_k + t_{k,m} + t_{(k+1),m}\right\} <$$
$$\max\left\{C'_{(k-1),1} + G_k + t_{(k+1),1} + t_{k,m}, C'_{(k-1),1} + G_{k+1} + t_{(k+1),m} + t_{k,m}\right\}$$
$$= \max\left\{t_{k,1} + t_{(k+1),1} + t_{(k+1),2} + ---- + t_{(k+1),(m-1)} + t_{(k+1),m}, t_{k,1} + t_{k,2} + ---- + t_{k,(m-1)} + t_{k,m} + t_{(k+1),m}\right\} <$$
$$\max\left\{t_{(k+1),1} + t_{k,1} + t_{k,2} + ---- + t_{k,(m-1)} + t_{k,m}, t_{(k+1),1} + t_{(k+1),2} + ---- + t_{(k+1),m-1} + t_{(k+1),m} + t_{k,m}\right\}$$

On subtracting $\left(t_{k,1} + t_{k,2} + ---- + t_{k,m}\right) + \left(t_{k+1,1} + t_{k+1,2} + ---- + t_{k+1,m}\right)$ from each side, we get

$$\max\left\{-t_{k,2} - t_{k,3} - .... - t_{k,m}, -t_{(k+1),1} - t_{(k+1),2} - ... - t_{(k+1),(m-1)}\right\} < \max\left\{-t_{k+1,2} - t_{k+1,3} - .... - t_{k+1,m}, -t_{k,1} - t_{k,2} - ... - t_{k,m-1}\right\}$$

$$\Rightarrow \max\left\{-H_k, -G_{k+1,}\right\} < \max\left\{-H_{k+1}, -G_k\right\}$$

$$\Rightarrow \min\left\{G_{k+1}, H_k\right\} > \min\left\{G_k, H_{k+1}\right\}, \text{ i.e. } \min\left\{G_k, H_{k+1}\right\} < \min\left\{G_{k+1}, H_k\right\}$$

Hence, theorem verified.

Remark: If the structural relationship in the theorem can be taken as

$$\min A_{i,(s+1)} \geq \max A_{i,s} \quad (s = 2, 3, 4, ..., m-1)$$

then the above theorem can be verified in the same fashion.

b. The effective transportation time of jobs $T'_{i,1\to2} \le U_{iM_2} - T_{i,M_1}$ **where** $T'_{i,1\to2} = \max\{D_i - A_{i1}, E_i - A_{i2}, T_{i,1\to2}\}$

Proof: Let U_{ix} and T_{ix} denote the starting time and completion times of any job i on machine X (X = M_1, M_2; i=1,2,...,n) respectively in a sequence S.

From the definition of Start lag D_i,

we have, $U_{iM_2} - U_{iM_1} \ge D_i$

Now $T_{iM_1} = U_{iM_1} + A_{i1}$

i.e., $U_{iM_1} = T_{iM_1} - A_{i1}$

Hence, we have

$$U_{iM_2} - T_{iM_1} \ge D_i - A_{i1} \quad \dots (1)$$

i.e., $U_{iM_2} - T_{iM_1} \ge D_i - A_{i1}$

From the definition of Stop lag E_i,

we have, $T_{iM_2} - T_{iM_1} \ge E_i$

Now $T_{iM_2} = U_{iM_2} + A_{i2}$

Hence we have $U_{iM_2} + A_{i2} - T_{iM_1} \ge E_i$

i.e., $$U_{iM_2} - T_{iM_1} \ge E_i - A_{i2} \quad \dots (2)$$

Also, from definition of transportation time $T_{i,1\to2}$, we have

$$U_{iM_2} - T_{iM_1} \ge T_{i,1\to2} \quad \dots (3)$$

Let $$T'_{i,1\to2} = \max\{D_i - A_{i1}, E_i - A_{i2}, T_{i,1\to2}\} \quad \dots (4)$$

From (1), (2) and (3), it is obvious that

$$T'_{i,1\to2} \le U_{iM_2} - T_{i,M_1}$$

Hence, result.

Algorithm

The following algorithm is proposed to find the optimal sequence of jobs processing:

Step 1: Find the average high ranking (AHR) $A_{ij} (i = 1,2,...,n; j = 1,2,...,m)$ of the processing time of jobs.

Step 2: Check the condition Min $A_{is} \ge$ Max $A_{i(s+1)}$; s = 1, 2, 3, 4………

If the conditions are satisfied then go to Step 3, else the data is out of scope of the present algorithm.

Step 3: Introduce the two fictitious machines G and H with processing times G_i and H_i as

$G_i = A_{i1} + A_{i2} + A_{i3} + ... + A_{i(m-1)}$ and $H_i = A_{i2} + A_{i3} + A_{i4} + ... + A_{im}$ for all *i*.

Step 4: Calculate the effective transportation times $T'_{i,s\to s+1}$ as

$$T'_{i,s\to s+1} = \max(D_i - G_i, E_i - H_i, T_{i,s\to s+1}); s = 1,2,3,....(m-1)$$

Step 5: Define the two fictitious machines G' and H' having respective processing times for job i as G'_i and H'_i are defined by

$$G'_i = G_i + T'_{i,s\to s+1}; H'_i = H_i + T'_{i,s\to s+1}; s = 1,2,3,...(m-1)$$

Step 6: Find the optimal sequence(s) by Johnson's [2] procedure for 2 machines, n jobs problem on the reduced problem in step 3.

Step 7: Prepare In-Out tables for the optimal sequence S and calculate the total elapsed time.

Numerical Illustration

Consider 5 jobs, 4 machine flow shop problem with processing time described by triangular fuzzy numbers as given in the following table. Our objective is to obtain optimal schedule to minimize total elapsed time subject to some specified lag constraint.

Table 1: Machines with processing fuzzy processing time

Jobs	Machine M_1	Machine M_2	Machine M_3	Machine M_4	$T_{i,s\to s+1}$	Start lag	Stop lag
i	a_{i1}	a_{i2}	a_{i3}	a_{i4}		D_i	E_i
1	(11,12,13)	(8,10,12)	(6,7,9)	(2,3,4)	2/3	98/3	24
2	(12,13,14)	(9,10,11)	(7,8,9)	(4,5,7)	8/3	100/3	70/3
3	(6,7,21)	(8,9,11)	(5,6,8)	(3,4,5)	4/3	97/3	71/3
4	(10,11,12)	(6,7,20)	(6,7,8)	(4,5,6)	5/3	89/3	76/3
5	(8,11,12)	(9,10,11)	(8,9,10)	(2,4,6)	9	95/3	25

Solution: As per step 1. The AHR of processing time of jobs are as given in table

Table 2: Machines with AHR for processing time

Jobs	Machine M_1	Machine M_2	Machine M_3	Machine M_4	$T_{i,s\to s+1}$	Start lag	Stop lag
i	A_{i1}	A_{i2}	A_{i3}	A_{i4}		D_i	E_i
1	38/3	34/3	24/3	11/3	2/3	98/3	24
2	41/3	32/3	26/3	18/3	8/3	100/3	70/3
3	36/3	30/3	21/3	14/3	4/3	97/3	71/3
4	35/3	35/3	23/3	17/3	5/3	89/3	76/3
5	37/3	32/3	29/3	16/3	9	95/3	25

Here Min $A_{is} \geq$ Max $A_{i(s+1)}$; for s = 1, 2, 3, .i.e. the structural condition is satisfied.

On using the Step 3 to Step 6 of the proposed algorithm, we obtain S = 5 – 2 – 4 – 3 – 1 as an optimal sequence of jobs processing.

As per step 7: The In-Out table for the sequence S is as follows

Table 3: The In – Out table for the optimal sequence S is

Jobs	Machine M_1	Machine M_2	Machine M_3	Machine M_4	$T_{i,s\to s+1}$
i	In – Out	In – Out	In – Out	In – Out	
5	0 – 37/3	64/3 – 96/3	123/3 – 152/3	179/3 – 195/3	9
2	37/3 – 78/3	96/3 – 128/3	152/3 – 178/3	195/3 – 213/3	8/3
4	78/3 – 113/3	128/3 – 163/3	178/3 – 201/3	213/3 – 230/3	5/3
3	113/3 – 149/3	163/3 – 193/3	203/3 – 224/3	234/3 – 248/3	10/3
1	149/3 – 187/3	193/3 – 227/3	230/3 – 254/3	257/3 – 268/3	1

Here CT(S) = Total elapsed time = 268/3 for this optimal sequence 5 – 2 – 4 – 3 – 1 .

It may be observed that

$$D_1 = \frac{98}{3} \le \frac{257}{3} - \frac{149}{3} = \frac{108}{3} \qquad E_1 = 24 \le \frac{268}{3} - \frac{187}{3} = \frac{31}{3} \qquad T_{1,s\to s+1} = \frac{2}{3} \le \frac{257}{3} - \frac{187}{3} = \frac{70}{3}$$

$$D_2 = \frac{100}{3} \le \frac{195}{3} - \frac{37}{3} = \frac{158}{3} \qquad E_2 = \frac{70}{3} \le \frac{213}{3} - \frac{78}{3} = \frac{135}{3} \qquad T_{2,s\to s+1} = \frac{8}{3} \le \frac{195}{3} - \frac{78}{3} = \frac{117}{3}$$

$$D_3 = \frac{97}{3} \le \frac{234}{3} - \frac{113}{3} = \frac{121}{3} \qquad E_3 = \frac{71}{3} \le \frac{248}{3} - \frac{149}{3} = \frac{99}{3} \qquad T_{3,s\to s+1} = \frac{4}{3} \le \frac{234}{3} - \frac{149}{3} = \frac{85}{3}$$

$$D_4 = \frac{89}{3} \le \frac{213}{3} - \frac{78}{3} = \frac{135}{3} \qquad E_4 = \frac{76}{3} \le \frac{230}{3} - \frac{113}{3} = \frac{117}{3} \qquad T_{4,s\to s+1} = \frac{5}{3} \le \frac{213}{3} - \frac{113}{3} = \frac{100}{3}$$

$$D_5 = \frac{95}{3} \le \frac{179}{3} - 0 = \frac{179}{3} \qquad E_5 = 25 \le \frac{195}{3} - \frac{37}{3} = \frac{158}{3} \qquad T_{5,s\to s+1} = 9 \le \frac{179}{3} - \frac{37}{3} = \frac{142}{3}$$

CONCLUSION

Production scheduling, with the objective of minimizing the makespan is an important task in manufacturing systems. In the past, the processing time for each job was usually assumed to be exactly known, but in many real world applications, processing times may vary dynamically due to human factors or operating faults. Fuzzy programming techniques have been developed to deal with uncertain processing times. In this paper the concept of transportation time, arbitrary lags i.e. Start lag and Stop lag are introduced in addition to fuzzy processing time. The proposed algorithm yields an optimal schedule of job processing with minimum total elapsed time. The present work can further be extended by taking trapezoidal fuzzy numbers, considering weighted jobs and by introducing the concept of setup time, job block criteria and breakdown of machines etc.

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