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A formalism to compute the electromagnetic compatibility of complex networks

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ABSTRACT

The purpose of our work is to develop a method to compute the electromagnetic compatibility of complex networks. We starts from the tensorial analysis of networks developed by Sir Gabriel KRON in 1939[1] extended with the antennas interactions [2]. We add the hybrid tensors concept developed by A.REINEIX and O.MAURICE[3] to define a multidomain metric (hybrid metric). The result is a generalized tensorial equation representing the problem to solve. Many validations of the technique exist today, given through references. The French army project DIAMANTS (DIAkoptic Methodology for ANalysis of Disturbances on Systems) has for purpose to begin a first automatisation of the method. The paper presents the basic concepts, then details the tensorial equation to resolve for one network and in conclusion, presents the future works to construct a system of networks. The method is a topological one, using the union operation of the topology [15] to construct complex networks from simple ones.

Key words: EMC, Tensorial Analysis of Networks, topology.

INTRODUCTION

After defining a complex network, we describe a typical complex network from the electromagnetic compatibility (EMC) point of view. Then we explain how the reality can be synthesized under a symbolic graph. From this graph we define a complex metric based on a topology before to construct the tensorial equation (an hybrid one) representing all the interferences in the network. This hybrid tensor is based on both lagrangian and hamiltonian formalisms. We give a simple example to illustrate the mechanism of the method and for show how the coupling are taken into account inside the equations using the hybrid tensors. Finally we conclude on the future extension to complex systems.

Complex Networks

Compare to a complicated network, a complex one has entangled elements and multiscale behaviours[3]. The entanglement means that firstly the modelling network using classical numerical techniques would take to many resources and secondly more than ever we need a theory to understand how it works. Numerical techniques are virtual experiments, but give no way to theoretically understanding the network behaviour. An example of complex network for EMC is a computer for planes with all its peripherals and cables. Inside this subsystem, many electromagnetic interactions occur: that's the purpose of the EMC job.

A complex network from the EMC point of view

Topologically, a complex network is made from simple ones using the union operation of topology [14]. Any electronic system can be seen as a system constructed in five layers: the component layer which includes all the electronic components and the printed circuit boards, the cable layer, which includes all the wires and cables traveling information between the equipment and the structure layer which involves the structures including the equipment and cables and the last layer which is the environment around the system. A complex network is nothing more than a particular system but without movement. It can be made of many subnetworks. It means that the elements of the network still fixed. Under this hypothesis we can detailed the interactions present in the network. According to Maxwell's equations, the various electromagnetic interactions between the components of a network (or between various subnetworks) are: conducted noises, electrical influence, magnetic induction and radiated fields. Added to these fundamental processes, there is the guided waves exchange of information between subnetworks. The available modellinginvites us to distinguish the guided waves and the radiated ones. Inside the electronic network we find as usual components like resistors, inductances and capacitances. The primary observables are loads, currents and voltages. The first job in front of a complex network is to represent it under a symbolic form. This symbolic representation, which is a graph, will help us to construct the metric of the network in a topological approach.

Link between the reality and a graph as a symbolic representation

If we take a look to a real experiment in electronic, we see parts where the currents keep the same intensity and still homogeneous. For a given scale of observation and starting from a generator, we always find under this assumption, at least one closed circuit of constant current. This first basic fact gives for many simple circuits the equivalent topology. As an evident example, we can consider a battery connected to a resistor, it constitutes the simplest topology that can be defined with one mesh: the closed electrical circuit of the battery and the resistor; and two branches, one linked with the battery and another for the resistor. One can wonder why a branch is devoted to the battery and another branch to the resistor. Each branch must be a primitive network. It means that any branch can be connected to a load or a generator in order to characterize its own electrical properties. The primitive network defined by G.Kron [1] is a single branch whose properties can be extracted through analysis of its response to external excitation. For any real network and using this principle, it is possible to link topology and the associated graph by identifying these areas with constant currents to the branch in the topology. This is done from a long time in electrical representation as SPICE. What is less obvious and does not appear in the patterns of electrical circuits are the radiated interactions. In case of near field interactions, symbols like capacitance or inductance, represent electrostatic or magnetostatic field lines under the same assumption of homogeneity. But there are no symbols for the far field interactions. In how representation, we use ropes. Ropes are some Poynting's flux going from one emitter to a receptor. Anyway it gives the radiated energy received on one branch or one mesh and coming from a remote current. The formal separation of the components of near field and far field is a key point of constructing the topology from the reality of electromagnetic phenomena. This can always be done if one accepts an extended definition of the capacitance and of the inductance. We begin from the field development of an electrical dipole (p is the electrical moment of the dipole and r the distance between the point where the field is measured and the dipole)[5]:

$$\vec{E}(r,p) = f\left(\sum_{n,q=1,0}^{N,2} \frac{\alpha_q^n}{r^n}\frac{\partial^q \vec{p}}{\partial t^q}\right)$$

The function f can be formally separated in two parts:

$$f\left(\sum_{n,q=1,0}^{N,2} \frac{\alpha_q^n}{r^n}\frac{\partial^q \vec{p}}{\partial t^q}\right) = f_n\left(\sum_{n,q=2,0}^{N,2} \frac{\alpha_q^n}{r^n}\frac{\partial^q \vec{p}}{\partial t^q}\right) + f_1\left(\sum_{q=0}^{2} \frac{\alpha_q}{r}\frac{\partial^q \vec{p}}{\partial t^q}\right)$$

The function f_1 relates with the far field emission, and is represented by a rope. The function f_n takes in charge the near field interaction. The components depending on p can be treated as capacitors. The components depending on the time derivative of p relates in fact to the near electrostatic field when we look in front of the dipole. In the detailed expression, one factor is transformed in : the magnetic permeability, by the c^{-1} which is factor of the derivative of p in these relations. When both near and far fields are created through a rope and some branches (including the mutual partial inductances between the two branches in interaction), we obtain the same function f because both energies will be add through the induced currents, following in that the superposition principle. The tricky part constituted through the adding of the far field interactions in a symbolic graph is so resolved.

Topological approach to modeling the system

At this step, a graph exists with its nodes (numbered from 1 to N), its branches (numbered from 1 to B), its meshes (numbered from 1 to M). Nodes, branches, meshes are parts of networks (numbered from 1 to R) with: M=B-N+R. Each node is located in a three dimensional space, having its coordinates in a defined referential. A branch can be defined between two nodes, whatever is its trajectory in the 3D space. Currents can be considered as flowing on branches and a current vector is created to identify each branch with one component of this vector in the description space of branches. By the fact, currents are natural projection of the branches on a space whose base (of dimension B) is made of all the vectors coming from the nodes pairs. We write: . By abuse, we note the current vector through its components I^k(this is always used from here). Now, we create a dual space from the natural space by writing: (is the Kronecker's symbol). are the base vectors of the dual space. This dual space is the space of the voltages and of the electromotive forces. From the tensorial algebra, it exists a metric relation z between the two spaces that makes a link between the current vector and the voltage vector: . By replacement, we obtain the definition of the fundamental invariant (in all this document we use the mute index technique [6]):

$$v_k I^k = z_{km} I^k I^m = S$$

S is a generalized power of the whole circuit.

If we look at the circuit with a different point of view, for example without looking at the branches, but looking at the meshes, how the circuit equation will be expressed under this new vision? Firstly we create a matrix to change of bases: C. The currents on the branches are expressed like linear combination of currents on the meshes: . In an electrical circuit, branches without any source of current but with electromotive sources have the Kirchhoff's equation given by: . In this equation, V_k is the difference voltage vector developed on each branch. In this equation we can replace the current vector by its expression in the mesh space: . Due to the duality properties, if then . Multiplying to the left by we obtain: which leads to:

$$e_\nu = z_{\nu\mu} I^\mu$$

The V voltages have disappeared because of their scalar characteristic. According to Maxwell's equationRot(Grad(V))=0, the integration of V on a closed circulation is zero. As the transformation from branch to meshes makes the sum of the voltages across the branches, the term has disappeared.We see here that the tensor algebra gives us an equation invariant circuit, but destitute of the scalar potential. The last equation is to be “complexified”, even if it is with the same form, in order to take into account some specific interactions, and static interactions will be added using a coupled expression.

General lagrangian approach

The energy can be separated in three parts: kinetic (T), potential (U) and losses (D). In electrical circuits, these three energies are known under the forms electrical (capacitors), magnetic (inductances) and losses through resistors. The three energies have the next form:

$$U = \frac{1}{2}\frac{q^a q^m}{C^{ma}} \quad T = \frac{1}{2} L_{am} \dot{q}^a \dot{q}^m \quad D = \frac{1}{2} R_{am} \dot{q}^a q^m$$

With using temporal operators, these expressions can be rewritten:

$$\dot{U} = \frac{1}{2C^{ma}} \int_t dt \dot{q}^a \dot{q}^m \quad \dot{T} = \frac{L_{am}}{2} \partial_t \dot{q}^a \dot{q}^m \quad \dot{D} = \frac{1}{2} R_{am} \dot{q}^a \dot{q}^m$$

These are the average power dissipated in the network, but also the temporal operators of the metric in the space of the meshes [2][7]. For this reason, we consider the expression of the metric in the space of meshes as the lagrangian expression of the network.In this space of configuration we compute the currents of meshes from whose we deduce the current of branches and the voltages through the connectivity. But it doesn't give the static loads, as they don't generate currents. To take into account this static energy, it is better to employ the hamiltonian.

Hamiltonian approach

If the hamiltonian doesn't allow to discriminate the degenerate cases, at the opposite, it allows to compute the static constraint applied to the networks. The hamiltonian is the total energy under electrostatic or magnetostatic forms. So fact it is written using the electrostatic loads and magnetostatic forces. After a derivation in time of the hamiltonian depending on the loads and of magnetomotive forces, we obtain the generalized coordinates which are the potential vector and the magnetic flux. The hamiltonian is:

$$H = \frac{q^k q^m}{2C^{km}} + \frac{F_\mu F_\sigma}{2R_{\mu\sigma}}$$

With q the electrical loads, F the magnetomotive forces, C and R respectively:respectively: the inverse of the metric and the metric obtained from the potential and dissipative energies. The derivation in time of the hamiltonian gives the generalized coordinates: the potential and the flux .

$$\frac{\partial H}{\partial q^k} = \psi_k \quad \frac{\partial H}{\partial F_k} = \phi^k$$

The equations to solve are: and .We now will see how these equations can be coupled with these of the lagrangian.

The L-H formalism

If we consider both equations and (more than two equations may be considered), we can group them in a single expression:

$$\left\{ \begin{array}{c} e_\mu \\ q^k \end{array} \right\} = \left\{ \begin{array}{cc} z_{\mu\sigma} & 0 \\ 0 & Y^{km} \end{array} \right\} \left\{ \begin{array}{c} I^\sigma \\ \psi_m \end{array} \right\}$$

Under this simple form, there are no couplings occurring between the two equations. In order to add these coupling, some non-diagonal terms appear in the matrix of the second member:

$$\left\{ \begin{array}{c} e_\mu \\ q^k \end{array} \right\} = \left\{ \begin{array}{cc} z_{\mu\sigma} & \Gamma_\mu^{\ m} \\ j^k_{\ \sigma} & Y^{km} \end{array} \right\} \left\{ \begin{array}{c} I^\sigma \\ \psi_m \end{array} \right\}$$

But if a tensorial space, even a little special can be constructed on the first expression (as the matrix made of the metric and of a inverse metric is purely diagonal, it can be inversed by blocs), this new expression doesn't allow to compute an inverse. Fortunately we can use the fact that the coupling terms are delayed in the time. We can separate these terms for write implicitly:

$$\left\{ \begin{array}{c} e_\mu \\ q^k \end{array} \right\}^{(+)} = \left\{ \begin{array}{cc} z_{\mu\sigma} & 0 \\ 0 & Y^{km} \end{array} \right\} \left\{ \begin{array}{c} I^\sigma \\ \psi_m \end{array} \right\}^{(+)} + \left\{ \begin{array}{cc} 0 & \Gamma_\mu^{\ m} \\ j^k_{\ \sigma} & 0 \end{array} \right\} \left\{ \begin{array}{c} I^\sigma \\ \psi_m \end{array} \right\}^{(-)}$$

By grouping the sources and the passed terms we retrieve the previous expression where the source vector is enriched of the coupling terms. The manipulated objects are called hybrid tensors by the authors [3] and their properties are similar to these of tensors under the assumption of the previous operation. This trick is a key method to obtain the equations of a complex system, using both hamiltonian and lagrangian formalism in an approach of tensors hybrid L-H. Typically, three equations will be grouped in an unique hybrid tensor: one Lagrange's equation in the mesh space and two Hamilton's equations for the electrostatic and magnetostatic interactions. The technique can be extended to multiphysic problems as the one address in [16].

An example to illustrate the theory

Our example is here just to give a beginning of understanding of how the method works. It will appear very simple compared to complex systems! But the application of this technique in these cases is only an extension of the example with more complex models and with spaces dimensions many larger. Figure 1 give the graph of the problem considered, made of three parts:

a first one is a generator, connected through a transformer to another circuit (the second part), which radiates in the direction of one distant receiver.

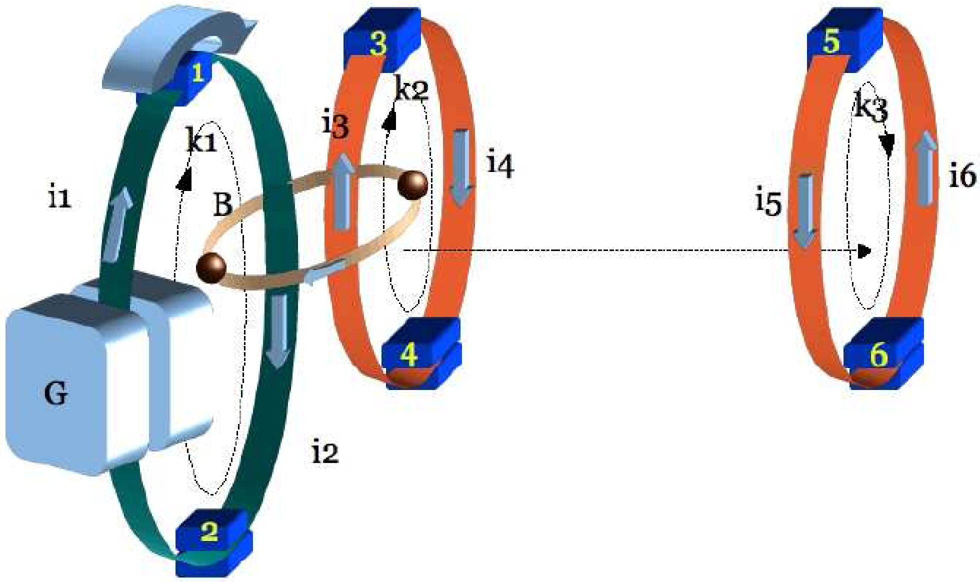

Fig1: graph of the system calculated

On the figure we see three networks made with two branches and two nodes numbered from 1 to 6. The first network is equipped a power supply generator G. Between the first and the second network there is a magnetic interaction symbolised by a closed line B that cross two nodes. Between the second and the third network, there is a far field interaction symbolised by a dashed line. We suppose that the topology is sufficiently intelligible to avoid any confusion in its representation by a symbolic graph.

Metric

For a known problem, the metric is also known. We don't represent the characteristics of the branches in the symbolic graph. Just to recall that these properties can be any function and not restricted to those of classical electrical circuits. We suppose that the first two branches are for resistive branches (R1, R2). The third branch is a resistive branch (R3) also but the fourth is a capacitor (C4). The branches number 5 and 6 are resistive branches (R5, R6). Each mesh (linked to the currents of meshes 1 to 3) has its own inductance: L1 or L2. In the lagrangian description, the previous information are sufficient to construct the metric z in the space of the branches. As it is purely diagonal (under our assumptions of metric, no interaction is described in this space.), we can express it by a matrix of a single line:

$$z_{ab} = \begin{pmatrix} R1 & R2 & R3 & \frac{1}{pC4} & R5 & R6 \end{pmatrix}$$

(p is the Laplace's operator). The real metric is easily obtained by making from this matrix a diagonal matrix. With our choices of directions for the currents, we can establish the relations between the currents of the branches and the currents of the meshes. If we take for example the first branch, the impedance R1 sees the current I1 of the branch 1, or, it doesn't change anything of its point of view, it sees the current k1 of the mesh 1. Using the same process for all the currents, we obtain the next matrix for connectivity between the space of the branches and the space of the meshes (We agree to use latin letters for the indices of the tensors in the space of the branches, and greec letters for the tensors in the space of the meshes):

$$C^b{}_\sigma = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

As said before, we operate a change of base to transform the metric z of the space of the branches to a new description of it in the space of the meshes. By making we obtain:

$$z_{\mu\sigma} = C_\mu{}^a z_{ab} C^b{}_\sigma = \begin{pmatrix} R2 + R1 & 0 & 0 \\ 0 & R3 + \frac{1}{pC4} & 0 \\ 0 & 0 & R6 + R5 \end{pmatrix}$$

We must define the sources of energy. In the graph there is only one generator. The source vector in the space of the branches has a dimension of 6 with only its first element different from zero, which is the function of the generator defined in the time domain or in the frequency domain: . The transformation of e_a in the space of the meshes gives the vector: .

If we compute the equation we should obtain zero for all the currents of meshes, except the first one and the result obtained for the first current will be incorrect. This is completely normal by the fact that firstly there is no exchange of energy between the first mesh and the meshes two and three due to the absence of any coupling, secondly the inductances of the meshes are not still taken into account. As discussed previously, the rotational part of the electric field is computed in the space of the meshes. For this reason, the inductances of the free magnetic energy must be added in this space and not before. In a first step we create a metric for the magnetic energy. It is purely diagonal and each element is the inductance of each mesh. As for z_{ab} we can write it as a matrix with a single line: .
Now, the metric completely defined for the space of the meshes is obtained by adding to .
The previous equation still gives zero for the meshes: 2 and 3, because if our operation gives a correct metric, it doesn't yet defined the functions of coupling between the meshes. These functions of coupling are about to be defined through other kind of descriptions.

Coupling through the model of reluctances

The reluctance, coined by Heaviside in 1888, gives the ratio between the magnetomotive force and the magnetic flux. A parallel network to the electrical one, made of reluctances, can be created in order to take into account the distribution of energy through magnetostatic processes. It will give the opportunity to benefit of the powerful models coming from the reluctances. In this space, the dimension of the generalized coordinates is no more the ampere but the weber. The sources are the magnetomotive force in ampere-turns, and the reluctances are the components of the metric (ampere-turns per weber). From the currents in the space of meshes we define the source in the space of reluctances; from the magnetic flux in the same space, we compute electromotive force in the meshes space. The mechanism follows a two steps process: the current in the mesh 1 creates a magnetic flux. This flux is transported to the mesh 2 by reluctance. Finally, this flux generates an electromotive force in the mesh 2. A reciprocal process exists and creates an electromotive force in mesh 1 coming from the current in the mesh 2. Our equation of the coupled network is now given by:

$$\begin{Bmatrix} e_\mu \\ 0 \end{Bmatrix}^{(+)} - \begin{Bmatrix} t_{\mu\sigma} & (p)_\mu{}^m \\ j_{k\sigma} & 0 \end{Bmatrix} \begin{Bmatrix} I^\sigma \\ \phi_m \end{Bmatrix}^{(-)} = \begin{Bmatrix} C_\mu{}^a z_{ab} C^b{}_\sigma + M_{\mu\sigma} & 0 \\ 0 & R_{km} \end{Bmatrix} \begin{Bmatrix} I^\sigma \\ \phi_m \end{Bmatrix}$$

t is a tensor that connect the passed of the currents on the meshes as generators of the next step to compute.p is a matrix made with time derivative to transform the passed magnetic flux into electromotive force.jis the connectivity that makes the relation between the currents on the meshes and the magnetomotive forces that create the magnetic flux. If we resolve this system, we will find currents on the meshes 1 and 2, but not on the mesh 3. As the meshes 2 and 3 interact through a far field process, there still to add this interaction in the moments space.

The moment space

When a current (i) traverses a loop, it radiates a far field given by [10] (at a distance r, a surfaceA_e, and a current I in the mesh):

$$B = -\mu_0 p^2 \frac{\pi A_e Sin\theta}{4\pi^2 c^2 r} I$$

If the two loops are aligned, the loop in reception (with a surfaceA_r) develops an electromotive force given by:

$$e = -pA_r B = p^3 \mu_0 \frac{\pi A_e A_r Sin\theta}{4\pi^2 c^2 r} I$$

by making the ratio between the induced electromotive force and the current, we define a coupling impedance that can be added in the metric. This previous impedance relation can be many more complicated, coming from 3D codes, analytical computation with vectors, etc. This what will change nothing on the fact that finally, it is always possible to reduce those computations into this kind of coupling impedance. The tensor is enriched of the components M_{32} and M_{23}, which are the far field coupling between the meshes 2 and 3. This kind of coupling can be deduced from a moment space where some special connections make a link between the meshes and their normalized moment [11]. Let us define connectivity A between the surfaces of the meshes and the meshes themselves. The radiation diagram is enclosed in a functionF_{xy}that gives all the relation of the radiation between the point of emission (x) and any point on the 3D sphere in reception (y). The previous relation can always be written under this assumption:

$$M_{\mu\sigma}(radiation) = A_\mu{}^x \left(p^3 \mu_0 \frac{Sin\theta}{4\pi c^2 r} \right) A^y{}_\sigma = A_\mu{}^x F_{xy} A^y{}_\sigma$$

F is the radiation function of the antenna, obtained from the fields and of Maxwell's equations.

Conclusion about the example

The formulation used with the Laplace's formalism is a general formulation. We have to translate it in the time domain or in the frequency domain. It is important to remark that both domains can be treated simultaneously. One network can be treated in the time domain, while another can be treated in the frequency domain, like if the network is devoted to another physical domain. There are matrices of coupling exchange the information between the two networks working in the two domains. The usual problems linked with the numerical application of the calculations are encountered, but the method can take benefit of all the knowledge in this domain, it is not particular of this point of view and even reduced the number of unknowns in the spaces of the meshes or of the junctions for example.

CONCLUSION

Even if we don't have detailed all the mechanisms involved in the tensorial analysis of networks and its capabilities (including the capability to calculate the interactions between moving objects through Christoffel's symbols [2]), we hope having given the reader a good image of the formalism. The technique of the "hybrid tensors" presented allows coupling some equations coming from both formalisms: the Lagrange's formalism and Hamilton's formalism. Many applications [12][13] were made with the Lagrange's form, more particularly they concern sometimes very complex problems of power systems like [17] where it can be many moreefficient than the classical techniques. In all the cases, exceptional results compared to the complexity of the problems were obtained. These performances were principally due to the capacity to couple models from various scales of descriptions of the physics problems. For example, to compute the near field interaction of two dipoles, comparisons between the measurement and the computation has give 0,6% of divergence [10]. In another case, to compute the interaction between two antennas in a closed volume [11], the comparison between the measurement and the computation was less than 1 dB from 200MHz to 2GHz. Today around twenty publications have presented studies using the Kron's method applied to EMC. Our next step is to couple the L-H formalism with a space of the junctions. This last connection will give to the method a capability for changing of scales at the infinity. Some particular applications as began could concerns some very specific domain like spins [14]

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