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# A Computational study of Bingham plastic flow of Blood through an artery by multiple stenoses and post dilatation

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## ABSTRACT

*The present paper deals with fully developed one dimensional Bingham plastic flow of blood through a small artery having multiple stenoses and post-stenotic dilatation. We have determined the resistance- to flow -ratio for yield stresses 0, .02, .04 n/m$^2$, blood viscosities 0.00345, 0.004, 0.00455 Pa.s and fluxes 1, 10, 100. The resistance- to -flow moves nearer to unity as yield stress increases and flux decreases. Variation in viscosities shows no significant change in Resistance –to-flow ratio. The degree of these changes is maximum for yield stress and slightest for flux. The results are discussed numerically and graphically.*

**Keywords:** Resistance-to flow, Bingham plastic, blood viscosity, yield stress, flux

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## INTRODUCTION

An elevated arterial pressure is probably the most important public health problem in developed and developing countries. It is common, asymptotic, readily and often lead to lethal complications if left untreated.

Coronary microvessels have been confirmed to be significant in the regulation of local resistance and flow. Recent studies also propose that these microvessels are more approachable to physiological and pharmacological stimuli than tool vessels. But little is known about the comparative understanding of different microvascular segments in response to flow (shear stress) and agonists.

Ikbal et al. [1] presented mathematical models that characterize non-Newtonian flow of blood through a stenosed artery in the presence of a transverse magnetic field. They estimated the effects of Hartmann number, power law index, generalized Reynolds number, severity of the stenosis on various parameters such as flow velocity, flux and wall shear stress by means of their graphical representations. Sreenadh et al. [2] developed a mathematical model to study the steady flow of casson fluid through an inclined tube of non-uniform cross section with multiple stenoses. Chakravarty and Mandal [3] developed a mathematical model of non-linear two-dimensional blood flow in tapered arteries in the presence of stenosis. Mishra et al. [4] proposed a fluid mechanical study on the effects of the permeability of the wall through an artery with a composite stenosis. They derived expressions for the blood flow characteristics, flow resistance, the wall shear stress, shearing stress at the stenosis throat. Sapna Ratan Shah [5] studied the effects of peripheral layer on blood flow characteristics due the presence of a mild stenosis. She developed a two fluid model for blood flow through abnormally constricted stenosed artery. Mukhopadhyay et al. [6] presented numerical techniques based on pressure-velocity formulation to solve approximately, the governing equations for viscous flows through a tube (simulating an artery) with a periodic constriction. They also investigated effect of constriction as well as the rigid of the tube, on the flow characteristics and its consequences for arterial diseases. Yadav and Kumar [7] studied the non-Newtonian behavior on blood flow through stenosed artery with power law fluid model. They gave the result that the resistance-to flow increases with stenosis size for different value of flow index behavior.Agarwal et al. [8] investigated the effect of the plug flow in the cystic duct on the flow characteristic of bile. They considered bile as a Casson fluid. They found that as the size of stone and the core radius increases, the resistance to flow and shear stress also increases. Srivastava and Mishra [9] investigated the effects of an overlapping stenosis on blood flow characteristics in a narrow artery. They investigated that the impedance increases with the non-Newtonian behavior of blood as well as with the stenosis size. Chan et al. [10] investigated fluid and structural responses to pulsatile non-Newtonian blood flow through a stenosed artery. They found highest stress concentration at the throat of the stenosis. Sung et al. [11] performed a numerical analysis to investigate the effect of rotation on the blood flow characteristics with four different angular velocities. Srivastava and Rastogi [12] investigated blood flow through a narrow catheterized artery with an axially nonsymmetrical stenosis. They studied that the impedance increases with the catheter size, the hematocrit and the stenosis size (height and length) but decreases with the shape parameter. Verma et al. [13] studied the blood flow through a symmetric stenosis during artery catheterization assuming blood to behave a Newtonian fluid. Pincombe et al. [14] proposed a fully developed one-dimensional casson flow through a single vessel varying radius as a model of low Reynolds number blood flow in small stenosed coronary arteries.

**MATHEMATICAL FORMULATION**

Let us consider the flow of blood through a straight, rigid, axisymmetric artery containing multiple abnormal segments. The Geometry of the wall of artery is given by

$$\frac{R}{R_0} = 1 - \frac{\delta_j}{2R_0}\left[1 + COS\frac{2\pi}{l_j}\left(z - \alpha_j - \frac{l_j}{2}\right)\right]; \alpha_j \le z \le \beta_j$$

$=1$; Otherwise (1)

The distance from the origin to the start of the $j^{th}$ abnormal segment is given by

$$\alpha_j = \left[\sum_{i=1}^{j}(d_i + l_i) - l_j\right] \tag{2}$$

The distance from the origin to the end of $j^{th}$ abnormal segment is

$$\beta_j = \sum_{i=1}^{j}(d_j + l_j) \tag{3}$$

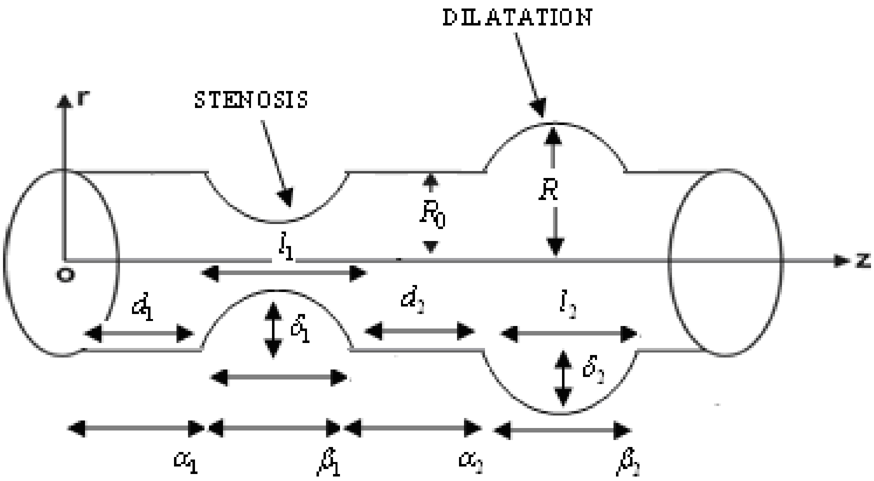


**Fig.1: Geometry of arterial wall with multiple stenoses and Post dilatation**

The constitutive equations for Bingham Plastic fluid are

$$\dot{\beta} = f(\tau) = -\frac{du}{dr} = \begin{cases} \dfrac{\tau - \tau_0}{\mu} & \tau \geq \tau_0 \\ 0 & \tau \leq \tau_0 \end{cases} \tag{4}$$

The flux $Q$ through the artery is given by

$$Q = \int_0^R 2\pi r u dr \tag{5}$$

Integrating (5) and using the no-slip boundary condition $u = 0$ when $r = R$

$$Q = \pi \int_0^R r^2 \left( -\frac{du}{dr} \right) dr \tag{6}$$

Applying (4) in (6) to obtain

$$Q = \pi \int_0^R r^2 f(\tau) dr \tag{7}$$

The expressions for $\tau$ and $\tau_R$ (the shear at the wall i.e. when $r = R$ ) is given by

$$\tau = -\frac{r}{2}\frac{dp}{dz} \text{ And } \tau_R = -\frac{R}{2}\frac{dp}{dz} \tag{8}$$

Where $p$ is the pressure. From equations (7) and (8), we get

$$Q = \frac{\pi R^3}{\tau_R{}^3} \int_0^{\tau_R} \tau^2 f(\tau) dr \tag{9}$$

Substitution of equation (4) and rearrangement gives the result

$$Q = \frac{\pi R^3}{\mu} \left[ \tau_R - \frac{\tau_0}{3} \right] \Rightarrow \tau_R = \frac{\tau_0}{3} + \frac{\mu Q}{\pi R^3} \tag{10}$$

Using the second result of equation (8)

$$\frac{dp}{dz} = -\frac{2}{R} \left[ \frac{\tau_0}{3} + \frac{\mu Q}{\pi R^3} \right] \tag{11}$$

Integrating equation (11) with respect to $z$ with the condition that

$p = p_1$ at $z = 0$ and $p = p_0$ at $z = L$

$$p_1 - p_0 = -\frac{2\tau_0}{3} \int_0^L \frac{1}{R} dz - \frac{\mu Q}{\pi} \int_0^L \frac{1}{R^4} dz \tag{12}$$

$$\lambda = \frac{p_1 - p_0}{Q} = -\frac{2\tau_0}{3R_0 Q} \int_0^L \left( \frac{R}{R_0} \right)^{-1} dz - \frac{\mu}{\pi R_0{}^4} \int_0^L \left( \frac{R}{R_0} \right)^{-4} dz$$

Let $$f_1 = \frac{2\tau_0}{3R_0 Q}; \; f_2 = \frac{\mu}{\pi R_0{}^4} \tag{13}$$

$$\lambda = -f_1 \int_0^L \left(\frac{R}{R_0}\right)^{-1} dz - f_2 \int_0^L \left(\frac{R}{R_0}\right)^{-4} dz \tag{14}$$

$$\lambda = -f_1 \left[ \int_0^{\alpha_j} dz + \sum_{j=1}^{k} \int_{\alpha_j}^{\beta_j} \left(\frac{R}{R_0}\right)^{-1} dz + \sum_{j=1}^{k-1} \int_{\beta_j}^{\alpha_{j+1}} dz + \int_{\beta_k}^{L} dz \right]$$

$$- f_2 \left[ \int_0^{\alpha_j} dz + \sum_{j=1}^{k} \int_{\alpha_j}^{\beta_j} \left(\frac{R}{R_0}\right)^{-4} dz + \sum_{j=1}^{k-1} \int_{\beta_j}^{\alpha_{j+1}} dz + \int_{\beta_k}^{L} dz \right] \tag{15}$$

$$I_1 = \sum_{j=1}^{k} \int_{\alpha_j}^{\beta_j} \left(\frac{R}{R_0}\right)^{-1} dz; I_2 = \sum_{j=1}^{k} \int_{\alpha_j}^{\beta_j} \left(\frac{R}{R_0}\right)^{-4} dz \tag{16}$$

$$\lambda = -f_1 \left[ \sum_{j=1}^{k+1} d_j + I_1 \right] - f_2 \left[ \sum_{j=1}^{k+1} d_j + I_2 \right]$$

$$\lambda = -(f_1 + f_2) \sum_{j=1}^{k+1} d_j - (f_1 I_1 + f_2 I_2) \tag{17}$$

If there is no abnormal segments,

$$\lambda_N = -(f_1 + f_2) L$$

$$\bar{\lambda} = \frac{\sum_{j=1}^{k+1} d_j}{L} + \frac{(f_1 I_1 + f_2 I_2)}{(f_1 + f_2) L} \tag{18}$$

Substitute in equation (1) to obtain

$$a_j = 1 - \frac{\delta_j}{2R_0}, b_j = \frac{\delta_j}{2R_0}, \theta = \pi - \frac{2\pi}{l_j}\left(z - \alpha_j - \frac{l_j}{2}\right)$$

$$\frac{R}{R_0} = 1 - b_j (1 - \cos\theta) \tag{19}$$

As $z = \alpha_j$ implies that $\theta = 2\pi$ and $z = \beta_j$ implies that $\theta = 0$ Then (16) reduces to

$$I_1 = \sum_{j=1}^{k} \frac{l_j}{2\pi} \int_0^{2\pi} \frac{1}{a_j + b_j \cos\theta} d\theta = \sum_{j=1}^{k} \frac{l_j}{\sqrt{a_j^2 - b_j^2}}$$

$$I_2 = \sum_{j=1}^{k} \frac{l_j}{2\pi} \int_0^{2\pi} \frac{1}{(a_j + b_j \cos\theta)^4} d\theta = \sum_{j=1}^{k} \frac{a_j (a_j^2 + \frac{3}{2} b_j^2) l_j}{(a_j^2 - b_j^2)^{7/2}}$$

## NUMERICAL RESULTS

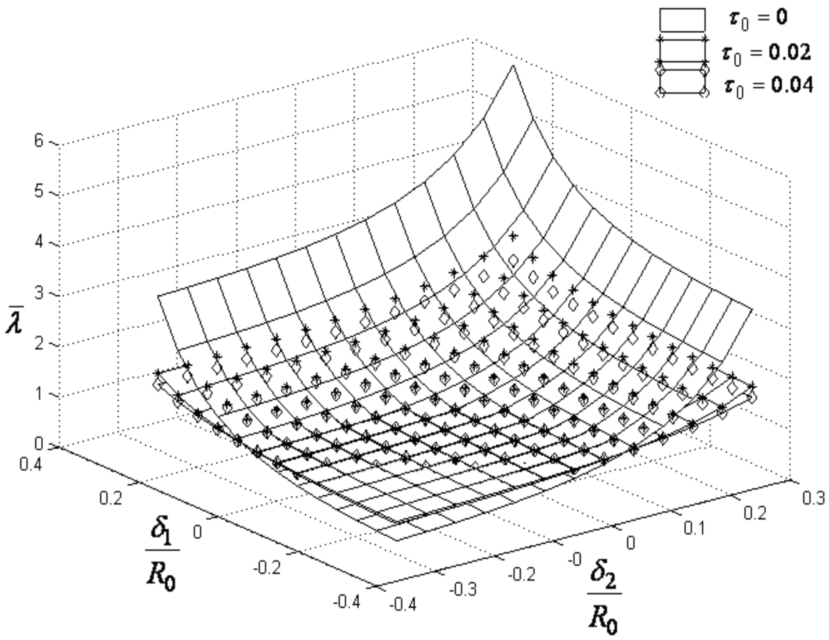


**Fig.2: Variation in resistance to flow for different yield stress**

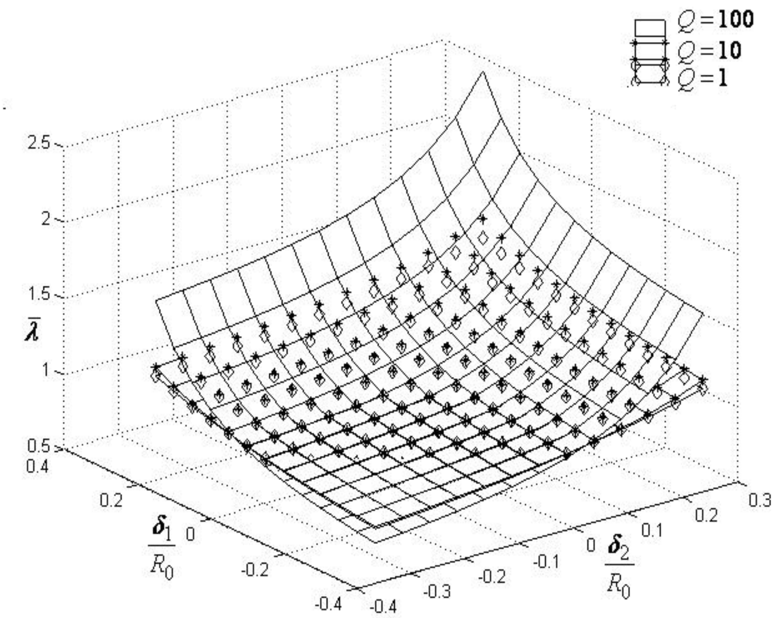


**Fig.3: Variation in resistance to flow for different Flux**

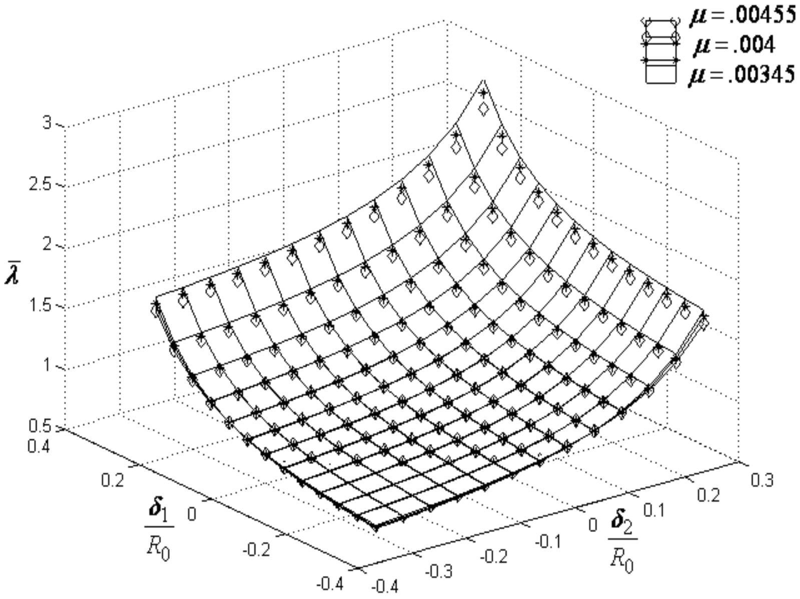


**Fig.4: Variation in resistance to flow for different blood viscosities**

## CONCLUSION

Resistance to flow ratio is a calculation of how far the resistance to flow in an artery of abnormal cross-section deviates from normal condition. Here we have considered the influence of blood yield stress, viscosity and flux on the resistance –to-flow ratio for Bingham plastic flow of blood through vessels contaning abnormal segments. The study reveals that as the yield stress increases, the resistance to flow ratio moves further from one. Resistance to flow shows no significant variation for variable blood viscosity and it decreases and moves closer to one as flux decreases. It is important for medical purpose that increase in flux elevate the Reynolds number and losing the blood viscosity.

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