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# A Comparative Study of Non Linear and Linear System of Three Species Ecological Model 

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#### Abstract

The present paper is devoted to Numerical investigation of three species ecological model with a Prey ( $N_{1}$ ), a predator $\left(N_{2}\right)$ and a competitor $\left(N_{3}\right)$ to the both prey $\left(N_{1}\right)$ and Predator $\left(N_{2}\right)$. In addition to that, the species are provided with alternative food. The model is characterized by a set of first order non-linear ordinary differential equations. All the eight equilibrium points of the model are identified and study the comparison between linear system of equations and nonlinear system of equations using Numerical example with mat lab simulation.


Keywords: Prey, Predator, Competitor, Equilibrium points, Numerical example.

## INTRODUCTION

Ecology relates to the study of living beings (animals and plants) in relation to their habits and habitats. This discipline of knowledge is a branch of evolutionary biology purported to explain how or what extent the living beings are regulated in nature .Allied to the problem of population regulation is the problem of species distribution prey , predator ,competition and so on. Research in the area of theoretical ecology was initiated in 1925 by Lotka [5] and by Volterra [6]. The general concept of modeling have been presented in the treatises of Paul Colinvaux [4], Freedman [1], Kapur [2, 3] etc. The ecological symbiosis can be broadly classified recently as prey-predator, competitor, mutualism, commensalism, amensalism and so on .Recently Shiva Reddy [7, 9] R Srilatha[10,11], K.Madhu sudhan reddy [12] ,B.Srihari[13]discussed the stability analysis three species eco system consting of prey, predator and super predator and Paparao [8] discussed with a cover linearly varying with the prey population and an alternative food for the predator. Inspired from that, we discussed a more general three species model with prey, predator and competitor to the both prey and predator is described by system of non linear differential equations. All the eight equilibrium points are identified, and we study the comparison of the non linear and linear system of equations are discussed.

## 2. Basic Equations:

The model equations for a three species Prey - Predator and competitor to the predator system is given by the following system of first order ordinary differential equations employing the following notation:

$$
\begin{align*}
& \frac{d N_{1}}{d t}=a_{1} N_{1}-\alpha_{11} N_{1}^{2}-\alpha_{12} N_{1} N_{2}-\alpha_{13} N_{1} N_{3} \\
& \frac{d N_{2}}{d t}=a_{2} N_{2}-\alpha_{22} N^{2}+\alpha_{21} N_{1} N_{2}-\alpha_{23} N_{3} N_{2} \tag{2.1}
\end{align*}
$$

$\frac{d N_{3}}{d t}=a_{3} N_{3}-\alpha_{33} N_{3}^{2}-\alpha_{31} N_{1} N_{3}-\alpha_{32} N_{2} N_{3}$
The linear model equations for a three species Prey - Predator and competitor to the predator system is given by the following system of first order ordinary differential equations employing the following notation:
$\frac{d u_{1}}{d t}=-\alpha_{11} \overline{N_{1}} u_{1}-\alpha_{12} \overline{N_{1}} u_{2}-\alpha_{13} \overline{N_{1}} u_{3}$
$\frac{d u_{2}}{d t}=\alpha_{21} \overline{N_{2}} u_{1}-\alpha_{22} \overline{N_{2}} u_{2}-\alpha_{23} \overline{N_{2}} u_{3}$
$\frac{d u_{3}}{d t}=-\alpha_{31} \overline{N_{3}} u_{1}-\alpha_{32} \overline{N_{3}} u_{2}-\alpha_{33} \overline{N_{3}} u_{3}$
Where $N_{1}, N_{2}$ and $N_{3}$ are the populations of the prey and predator and a competitor to the predator with the natural growth rates $a_{1} a_{2}$ and $a_{3}$ respectively,
$\alpha_{11}$ is rate of decrease of the prey due to insufficient food and inter species competition,
$\alpha_{12}$ is rate of decrease of the prey due to inhibition by the predator,
$\alpha_{21}$ is rate of increase of the predator due to successful attacks on the prey,
$\alpha_{22}$ is rate of decrease of the predator due to insufficient food other than the prey, and inter species competition,
$\alpha_{23}$ is rate of decrease of the predator due to the competition with the third species,
$\alpha_{33}$ is rate of decrease of the Competitor to the both prey and predator due to insufficient food and inter species competition,
$\alpha_{32}$ is rate of decrease of the competitor due to the competition with the both prey and predator,
$\alpha_{13}$ is rate of decrease of the prey due to the competition with the both prey and predator,
$\alpha_{31}$ is rate of decrease of the competitor due to the competition with the both prey and predator .

## 3. Equilibrium states:

The system under investigation has eight equilibrium states. They are
I.E 1 : The extinct state $\bar{N}_{1}=0 ; \bar{N}_{2}=0, \overline{N_{3}}=0$
II.E $\mathrm{E}_{2 \text { : }}$ The state in which only the predator survives and the prey and competitor to the prey and predator are extinct

$$
\begin{equation*}
\overline{N_{1}}=0, \overline{N_{2}}=\frac{a_{2}}{\alpha_{22}}, \overline{N_{3}}=0 \tag{3.2}
\end{equation*}
$$

III. $\mathrm{E}_{3}$ : The state in which both the prey and the predators extinct and competitor to the prey and predator survive
$\bar{N}_{1}=0 ; \bar{N}_{2}=0 \quad \overline{N_{3}}=\frac{a_{3}}{\alpha_{33}}$
IV. $\mathrm{E}_{4}$ : The state in which both the predator and competitor to the prey and predator extinct and prey alone survive
$\overline{N_{1}}=\frac{a_{1}}{\alpha_{11}} ; \quad \overline{N_{2}}=0 ; \quad \overline{N_{3}}=0$
V. $\mathrm{E}_{5}$ :The state in which both the prey and the predators exist and competitor to the prey and predator extinct
$\overline{N_{1}}=\frac{\left(a_{1} \alpha_{22}-a_{2} \alpha_{12}\right)}{\alpha_{11} \alpha_{22}+\alpha_{12} \alpha_{21}} ; \overline{N_{2}}=\frac{\left(a_{2} \alpha_{11}+a_{1} \alpha_{21}\right)}{\alpha_{11} \alpha_{22}+\alpha_{12} \alpha_{21}} ; \overline{N_{3}}=0$
This case arise only when $\frac{a_{1}}{a_{2}}>\frac{\alpha_{12}}{\alpha_{22}}$
VI. $\mathrm{E}_{6}$ :The state in which both prey and competitor to the prey and predator exist and predator extinct

$$
\begin{equation*}
\overline{N_{1}}=\frac{\left(a_{1} \alpha_{33}-a_{3} \alpha_{13}\right)}{\left(\alpha_{11} \alpha_{33}-\alpha_{13} \alpha_{31}\right)} ; \overline{N_{2}}=0 ; \overline{N_{3}}=\frac{\left(a_{3} \alpha_{11}-a_{1} \alpha_{31}\right)}{\left(\alpha_{11} \alpha_{33}-\alpha_{13} \alpha_{31}\right)} \tag{3.6}
\end{equation*}
$$

The equilibrium state exist only when $\alpha_{11} \alpha_{33}>\alpha_{13} \alpha_{31}, \frac{a_{1}}{a_{3}}>\frac{\alpha_{13}}{\alpha_{33}}$ and $\frac{a_{3}}{a_{1}}>\frac{\alpha_{31}}{\alpha_{11}}$
VII. E7: The state in which both predator and competitor to the prey and predator exist and prey extinct

$$
\begin{equation*}
\overline{N_{1}}=0 ; \overline{N_{2}}=\left(\frac{a_{2} \alpha_{33}-a_{3} \alpha_{23}}{\alpha_{22} \alpha_{33}-\alpha_{23} \alpha_{32}}\right) ; \overline{N_{3}}=\left(\frac{a_{3} \alpha_{22}-a_{2} \alpha_{32}}{\alpha_{22} \alpha_{33}-\alpha_{23} \alpha_{32}}\right) \tag{3.8}
\end{equation*}
$$

VIII. $\mathrm{E}_{8}$ :The state in which prey, predator and competitor to the prey and predator exist

$$
\begin{align*}
& \overline{N_{1}}=\frac{a_{1}\left(\alpha_{22} \alpha_{33}-\alpha_{23} \alpha_{32}\right)-\alpha_{12}\left(a_{2} \alpha_{33}-a_{3} \alpha_{23}\right)+\alpha_{13}\left(a_{2} \alpha_{32}-a_{3} \alpha_{22}\right)}{\alpha_{11}\left(\alpha_{22} \alpha_{33}-\alpha_{23} \alpha_{32}\right)+\alpha_{12}\left(\alpha_{21} \alpha_{33}+\alpha_{31} \alpha_{23}\right)-\alpha_{13}\left(\alpha_{21} \alpha_{32}+\alpha_{31} \alpha_{22}\right)} \\
& \overline{N_{2}}=\frac{\left(a_{2} \alpha_{11} \alpha_{33}+a_{1} \alpha_{21} \alpha_{33}+a_{1} \alpha_{31} \alpha_{23}\right)-\left(a_{3} \alpha_{11} \alpha_{23}+a_{3} \alpha_{21} \alpha_{13}+a_{2} \alpha_{13} \alpha_{31}\right)}{\alpha_{11}\left(\alpha_{22} \alpha_{33}-\alpha_{23} \alpha_{32}\right)+\alpha_{12}\left(\alpha_{21} \alpha_{33}+\alpha_{31} \alpha_{23}\right)-\alpha_{13}\left(\alpha_{21} \alpha_{32}+\alpha_{31} \alpha_{22}\right)} \\
& \overline{N_{3}}=\frac{\left(a_{3} \alpha_{11} \alpha_{22}+a_{3} \alpha_{21} \alpha_{12}+a_{2} \alpha_{12} \alpha_{31}\right)-\left(a_{2} \alpha_{11} \alpha_{32}+a_{1} \alpha_{21} \alpha_{32}+a_{1} \alpha_{31} \alpha_{22}\right)}{\alpha_{11}\left(\alpha_{22} \alpha_{33}-\alpha_{23} \alpha_{32}\right)+\alpha_{12}\left(\alpha_{21} \alpha_{33}+\alpha_{31} \alpha_{23}\right)-\alpha_{13}\left(\alpha_{21} \alpha_{32}+\alpha_{31} \alpha_{22}\right)} \tag{3.9}
\end{align*}
$$

The equilibrium state exist only when, $\alpha_{11}\left(\alpha_{22} \alpha_{33}-\alpha_{23} \alpha_{32}\right)+\alpha_{12}\left(\alpha_{21} \alpha_{33}+\alpha_{31} \alpha_{23}\right)>\alpha_{13}\left(\alpha_{21} \alpha_{32}+\alpha_{31} \alpha_{22}\right)$,

$$
\begin{align*}
& a_{1}\left(\alpha_{22} \alpha_{33}-\alpha_{23} \alpha_{32}\right)+\alpha_{13}\left(a_{2} \alpha_{32}-a_{3} \alpha_{22}\right)>\alpha_{12}\left(a_{2} \alpha_{33}-a_{3} \alpha_{23}\right) \\
& \left(a_{2} \alpha_{11} \alpha_{33}+a_{1} \alpha_{21} \alpha_{33}+a_{1} \alpha_{31} \alpha_{23}\right)>\left(a_{3} \alpha_{11} \alpha_{23}+a_{3} \alpha_{21} \alpha_{13}+a_{2} \alpha_{13} \alpha_{31}\right) \text { and } \\
& \left(a_{3} \alpha_{11} \alpha_{22}+a_{3} \alpha_{21} \alpha_{12}+a_{2} \alpha_{12} \alpha_{31}\right)>\left(a_{2} \alpha_{11} \alpha_{32}+a_{1} \alpha_{21} \alpha_{32}+a_{1} \alpha_{31} \alpha_{22}\right) \tag{3.10}
\end{align*}
$$

## Numerical Investigation:

In fig 1.1 we present the diagram of the values of parameters which are present in system of equations (2.1) .which correspond to the conditions analytically established for the existence and stability of nonlinear system of equations (2.1) which is asymptotically stable .The two dimensional and three dimensional graphs for this model are shown in fig1.1 and fig 1.3 respectively For the same parameters we present the diagram of the values of parameters which are present in system of equations (2.2) .which correspond to the conditions analytically established for the existence and stability of linearized system of equations (2.2) which is asymptotically stable the two dimensional and three dimensional graphs for this model are shown in fig1.2 and fig 1.4 respectively. Fig 1.1 shows the change in N1, N2 \& N3 with respective time t , which shows that $\mathrm{N} 1, \mathrm{~N} 2 \& \mathrm{~N} 3$ are asymptotically stable, where as Fig 1.2 shows the change of perturbed variables $u_{1}, u_{2}$ and $u_{3}$ with respective time $t$.For the same set of values as taken for non linear system of equations, which shows that they are also asymptotically stable. Fig 1.3 and 1.4 shows the relation between non linear variables $N 1, N 2 \& N 3$ and perturbed variables $u_{1}, u_{2}$ and $u_{3}$ respectively both the diagrams shows that, the non linear and linear systems are asymptotical stable.

The following values of parameters are present shows asymptotically stable for equations (2.1) and (2.2) in which the roots of the system are complex
4.1. Let $a_{1}=1.5 ; a_{2}=2.65 ; a_{3}=3.45 ; \alpha_{11}=0.1 ; \alpha_{12}=0.3 ; \alpha_{13}=0.01 ; \alpha_{22}=0.2 ; \alpha_{21}=0.3 ; \alpha_{23}=0.2 ; \alpha_{33}=0.2 ; \alpha_{31}=0.01 ; \alpha_{32}=0.2$


Fig 1.1: The Variation of N1, N2 and N3 with respective Time (t) for system of Eq (2.1)


Fig 1.2: The Variation of $u_{1}, u_{2}$ and $u_{3}$ with respective Time (t) for system of Eq (2.2)


Fig 1.3: The Phage diagram of N1, N2, N3 for system of Eq (2.1)


Fig 1.4: The Phage diagram of $\mathbf{u}_{1}, \mathbf{u}_{2}$ and $\mathbf{u}_{3}$ for system of $\operatorname{Eq}$ (2.2)
The above graphs shows the variation with initial strengths $10,8,25$ of prey, predator and competitor populations respectively

The comparison between the systems of equations (2.1) and (2.2) with respective stability shown in fig (1.1), (1.2) and (1.3), (1.4) shows the variation between the stability of non liner and linear systems. In fig (2.1), (2.2), (2.3) and (2.4) we present the diagram of the values of parameters which are in the system of equations (2.1) and (2.2), which correspond to the conditions analytically established for the existence and stability of nonlinear \& quasi linearized system of equations (2.1) and (2.2) respectively are asymptotically stable.

The following values of parameters are present shows stable for equations (2.1) and (2.2) In which the roots of the system are real
4.2. Let $\mathrm{a}_{1}=1 ; \mathrm{a}_{2}=1 ; \mathrm{a}_{3}=1.5 ; \alpha_{11}=0.2 ; \alpha_{12}=0.1 ; \alpha_{13}=0.1 ; \alpha_{22}=0.2 ; \alpha_{21}=.1 ; \alpha_{23}=0.1 ; \alpha_{33}=0.2 ; \alpha_{31}=0.1 ; \alpha_{32}=0.2$


Fig 2.1: The Variation of $\mathbf{N} 1, \mathrm{~N} 2$ and N 3 with respective Time (t) for system of $\mathbf{E q}$ (2.1)


Fig 2.2: The Variation of $u_{1}, u_{2}$ and $u_{3}$ with respective Time (t) for system of $E q$ (2.2)


Fig 2.3: The Phage diagram of N1, N2, N3 for system of Eq (2.1)


Fig 2.4: The Phage diagram of $u_{1}, u_{2}$ and $u_{3}$ for system of $E q$ (2.2)
The above graph shows the variation with initial strengths $20,20,5$ of prey, predator and competitor populations respectively

## CONCLUSION

In the analysis of the considered prey, predator and a competitor model, we compare the non linear and linear models using Numerical Methods. We also established the values of parameters for which the equations (2.1) and (2.2) exhibit asymptotically stable we observe from the graphs that the non linear behavior of the system almost coincide with the linearzed model. In general solving non linear system is very difficult or some times impossible so to study the analytical behavior of non linear system we can construct the linearized which can easily solvable. Through by this example we conclude that, a non linear system can be replaced with quasi linearized equations for detailed study and analysis.

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